Universal Hedging: Optimizing Currency Risk and Reward in International Equity Portfolios

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In a world where everyone can hedge against changes in the value of real exchange rates (the relative values of domestic and foreign goods), and where no barriers limit international investment, there is a universal constant that gives the optimal hedge ratio—the fraction of your foreign investments you should hedge. The formula for this optimal hedge ratio depends on just three inputs:

- The expected return on the world market portfolio.
- The volatility of the world market portfolio.
- Average exchange rate volatility.

The formula in turn yields three rules:

- Hedge your foreign equities.
- Hedge equities equally for all countries.
- Don't hedge 100 per cent of your foreign equities.

This formula applies to every investor who holds foreign securities. It applies equally to a U.S. investor holding Japanese assets, a Japanese investor holding British assets, and a British investor holding U.S. assets. That's why we call this method "universal hedging."

WHY HEDGE AT ALL?

You may consider hedging a "zero-sum game." After all, if U.S. investors hedge their Japanese investments, and Japanese investors hedge their U.S. investments, then when U.S. investors gain on their hedges, Japanese investors lose, and vice versa. But even though one side always wins and the other side always loses, hedging *reduces risk* for both sides.

More often than not, when performance is measured in local currency, U.S. investors gain on their hedging when their portfolios do badly, and Japanese investors gain on their hedging when their portfolios do badly. The gains from hedging are similar to the gains from international diversification. Because it reduces risk for both sides, currency hedging provides a "free lunch."

Why Not Hedge *All*?

If investors in all countries can reduce risk through currency hedging, why shouldn't they hedge 100 per cent of their foreign investments? Why hedge less?

The answer contains our most interesting finding. When they have different consumption baskets, investors in different countries can all add to their expected returns by taking some currency risk in their portfolios.

To see how this can be, imagine an extremely simple case, where the exchange rate between two countries is now 1:1 but will change over the next year to either 2:1 or 1:2 with equal probability. Call the consumption goods in one country "apples" and those in the other "oranges."

Imagine that the world market portfolio contains equal amounts of apples and oranges. To the apple consumer, holding oranges is risky. To the orange consumer, holding apples is risky.

The apple consumer could choose to hold only apples, and thus bear no risk at all. Likewise, the orange consumer could decide to hold only oranges. But, surprisingly enough, each will gain in expected return by trading an apple and an orange. At year-end, an orange will be worth either two apples or 0.5 apples. Its expected value is 1.25 apples. Similarly, an apple will have an expected value of 1.25 oranges. So each consumer will gain from the swap.

This isn't a mathematical trick. In fact, it's sometimes called "Siegel's paradox."[1] It's real, and it means that investors generally want to hedge less than 100 per cent of their foreign investments.

To understand Siegel's paradox, consider historical exchange rate data for deutschemarks and U.S. dollars. Table 1 shows the quarterly percentage changes in the exchange rates and their averages. Note that, in each period and for the average, the gain for one currency exceeds the loss for the other currency.

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Table 1. Siegel's Paradox

	State-of-Quarter Exchange Rates		Percentage Changes in Exchange Rates	
	mark	dollar	mark	dollar
Quarter	dollar	mark	dollar	mark
1Q84	2.75	.362	−5.58	5.90
2Q84	2.60	.384	7.18	−6.69
3Q84	2.79	.358	9.64	−8.79
4Q84	3.06	.326	3.66	−3.52
1Q85	3.17	.315	−1.83	1.84
2Q85	3.11	.321	−2.25	2.30
3Q85	3.04	.328	−13.04	15.01
4Q85	2.64	.377	−7.59	8.21
1Q86	2.44	.408	−4.46	4.67
2Q86	2.33	.427	−6.80	7.29
3Q86	2.17	.459	−7.16	7.73
4Q86	2.02	.494	−5.19	5.46
1Q87	1.91	.521	−5.11	5.41
2Q87	1.81	.549	0.49	−0.49
3Q87	1.82	.547	1.09	−1.08
4Q87	1.84	.541	−14.00	16.28
1Q88	1.58	.629	4.29	−4.12
2Q88	1.65	.603	9.83	−8.95
3Q88	1.82	.549	2.27	−2.22
4Q88	1.86	.537	−4.88	5.12
Average			−1.97	2.47

Why *Universal* Hedging?

Why is the optimal hedge ratio identical for investors everywhere? The answer lies in how exchange rates reach equilibrium.

Models of international equilibrium generally assume that the typical investor in any country consumes a single good or basket of goods.[2] The investor wants to maximize expected return and minimize risk, measuring expected return and risk in terms of his own consumption good.

Given the risk-reducing and return-enhancing properties of international diversification, an investor will want to hold an internationally diversified portfolio of equities. Given no barriers to international investment, every investor will hold a share of a fully diversified portfolio of world equities. And, in the absence of government participation, some investor must lend when another investor borrows, and some investor must go long a currency when another goes short.

Whatever the given levels of market volatility, exchange rate volatilities, correlations between exchange rates and correlations between exchange rates and stock, in equilibrium, prices will adjust until everyone is willing to hold all stocks and until someone is willing to take the other side of every exchange rate contract.

Suppose, for example, that we know the return on a portfolio in one currency, and we know the change in the exchange rate between that currency and another currency. We can thus derive the portfolio return in the other currency. We can write down an equation relating expected returns and exchange rate volatilities from the points of view of two investors in the two different currencies.

Suppose that Investor A finds a high correlation between the returns on his stocks in another country and the corresponding exchange rate change. He will probably want to hedge in order to reduce his portfolio risk. But suppose Investor B in that other country would increase his own portfolio's risk by taking the other side of A's hedge. Investor A may be so anxious to hedge that he will be willing to pay B to take the other side. As a result, the exchange rate contract will be priced so that the hedge reduces A's expected return but increases B's.

In equilibrium, both investors will hedge. Investor A will hedge to reduce risk, while Investor B will hedge to increase expected return. But they will hedge equally, in proportion to their stock holdings.

THE UNIVERSAL HEDGING FORMULA

By extending the above analysis to investors in all possible pairs of countries, we find that the proportion that each investor wants to hedge depends on three averages: the average across countries of the expected excess return on the world market portfolio; the average across countries of the volatility of the world market portfolio; and the average across all pairs of countries of exchange rate volatility. These averages become inputs for the universal hedging formula:[3]

$$\frac{\mu_m - \sigma_m^2}{\mu_m - \frac{1}{2}\sigma_e^2},$$

where

μ_m = the average across investors of the expected excess return (return above each investor's riskless rate) on the world market portfolio (which contains stocks from all major countries in proportion to each country's market value)

σ_m = the average across investors of the volatility of the world market portfolio (where variances, rather than standard deviation, are averaged)

σ_e = the average exchange rate volatility (averaged variances) across all pairs of countries

Neither expected changes in exchange rates nor correlations between exchange rate changes and stock returns or other exchange rate changes affect optimal hedge ratios. In equilibrium, the expected changes and the correlations cancel one another, so they do not appear in the universal hedging formula.

In the same way, the Black-Scholes option formula includes neither the underlying stock's expected return nor its beta. In equilibrium, they cancel one another.

The Capital Asset Pricing Model is similar. The optimal portfolio for any one investor could depend on the expected returns and volatilities of all available assets. In equilibrium, however, the optimal portfolio for any investor is a mix of the market portfolio with borrowing or lending. The expected returns and volatilities cancel one another (except for the market as a whole), so they do not affect the investor's optimal holdings.

Inputs for the Formula

Historical data and judgment are used to create inputs for the formula. Tables 2 through 8 give some historical data that may be helpful.

Table 2 lists weights that can be applied to different countries in estimating the three averages. Japan, the United States and the United Kingdom carry the most weight.

Tables 3 to 5 contain statistics for 1986–88 and Tables 6 to 8 contain statistics for 1981–85. These subperiods give an indication of how statistics change from one sample period to another.

When averaging exchange rate volatilities over pairs of countries, we include the volatility of a country's exchange rate with itself. Those volatilities are always zero; they run diagonally through Tables 3 and 6. This means that the average exchange rate volatilities shown in Tables 5 and 8 are lower than the averages of the positive numbers in Tables 3 and 6.

The excess returns in Tables 4 and 7 are averages for the world market return in each country's currency, minus that country's riskless interest rate. The average excess returns differ

Table 2. Capitalizations and Capitalization Weights

	Domestic Companies Listed on the Major Stock Exchange as of December 31, 1987*		Companies in the FT-Actuaries World Indices™ as of December 31, 1987†	
	Capitalization (U.S. $ billions)	Weight (%)	Capitalization (U.S. $ billions)	Weight (%)
Japan	2700	40	2100	41
United States	2100	31	1800	34
United Kingdom	680	10	560	11
Canada	220	3.2	110	2.1
Germany	220	3.2	160	3.1
France	160	2.3	100	2.0
Australia	140	2.0	64	1.2
Switzerland	130	1.9	58	1.1
Italy	120	1.8	85	1.6
Netherlands	87	1.3	66	1.3
Sweden	70	1.0	17	0.32
Hong Kong	54	0.79	38	0.72
Belgium	42	0.61	29	0.56
Denmark	20	0.30	11	0.20
Singapore	18	0.26	6.2	0.12
New Zealand	16	0.23	7.4	0.14
Norway	12	0.17	2.2	0.042
Austria	7.9	0.12	3.9	0.074
Total	6800	100	5300	100

* From "Activities and Statistics: 1987 Report" by Federation Internationale des Bourses de Valeurs (page 16).

† The FT-Actuaries World Indices™ are jointly compiled by The Financial Times Limited, Goldman, Sachs & Co., and County NatWest/Wood Mackenzie in conjunction with the Institute of Actuaries and the Faculty of Actuaries. This table excludes Finland, Ireland, Malaysia, Mexico, South Africa and Spain.

Table 3. Exchange Rate Volatilities, 1986–1988

	Japan	U.S.	U.K.	Canada	Germany	France	Australia	Switzerland	Italy	Netherlands	Sweden	Hong Kong	Belgium	Denmark	Singapore	New Zealand	Norway	Austria
Japan	0	11	9	12	7	7	14	7	8	7	7	11	9	8	10	17	9	8
United States	11	0	11	5	11	11	11	12	10	11	8	4	11	11	6	15	10	11
United Kingdom	9	10	0	11	8	8	14	9	8	8	7	11	9	8	10	16	9	9
Canada	12	5	11	0	12	11	12	13	11	11	9	6	12	11	8	15	10	12
Germany	7	11	8	12	0	3	15	4	3	2	5	11	6	4	10	17	8	5
France	7	11	8	11	2	0	14	5	3	3	5	11	6	4	10	17	7	5
Australia	14	11	14	12	14	14	0	15	14	14	12	11	14	14	12	14	14	14
Switzerland	7	12	9	13	4	5	15	0	5	5	7	12	8	6	11	18	9	7
Italy	8	10	8	11	3	3	14	5	0	3	5	11	6	4	10	17	7	5
Netherlands	7	11	8	11	2	3	14	5	3	0	5	11	6	4	10	17	7	5
Sweden	7	8	7	9	5	5	12	7	5	5	0	8	6	4	8	16	6	5
Hong Kong	11	4	11	6	11	11	11	12	10	11	8	0	11	11	5	14	10	11
Belgium	9	11	9	12	6	6	14	8	6	6	6	11	0	6	10	17	8	6
Denmark	8	11	8	11	4	4	14	6	4	4	4	11	6	0	10	17	7	5
Singapore	10	6	10	8	10	10	12	11	10	10	8	5	10	10	0	15	10	10
New Zealand	17	15	16	15	17	17	14	18	17	17	15	14	17	17	15	0	16	17
Norway	9	10	9	10	7	7	13	9	7	7	5	10	8	7	10	16	0	7
Austria	8	11	9	12	5	5	15	7	5	5	5	11	6	5	10	17	8	0

Source: FT-Actuaries World Indices™ data base.

between countries because of differences in exchange rate movements.

The excess returns are *not* national market returns. For example, the Japanese market did better than the U.S. market in 1987, but the world market portfolio did better relative to interest rates in the United States than in Japan.

Because exchange rate volatility contributes to average stock market volatility, σ_m^2 should be greater than $\frac{1}{2}\sigma_e^2$. Exchange rate volatility also contributes to the average return on the world market, so μ_m should be greater than $\frac{1}{2}\sigma_e^2$, too.

Table 4. World Market Excess Returns and Return Volatilities in Different Currencies, 1986–1988

	Excess Return			Return Volatility		
Currency	1986	1987	1988	1986	1987	1988
Japan	8	−12	21	14	26	15
United States	29	12	14	13	25	11
United Kingdom	23	−14	16	14	26	15
Canada	26	4	5	14	24	11
Germany	8	−5	30	15	27	14
France	11	−7	27	14	26	14
Australia	23	−2	−6	19	25	14
Switzerland	8	−8	36	15	27	15
Italy	2	−6	23	15	27	14
Netherlands	8	−7	30	15	27	14
Sweden	16	−6	19	13	25	13
Hong Kong	30	13	17	13	25	11
Belgium	7	−8	28	15	27	14
Denmark	8	−10	26	15	27	14
Singapore	36	6	16	12	25	12
New Zealand	15	−22	13	20	29	14
Norway	19	−11	15	14	26	12
Austria	7	−6	30	15	27	14

Source: FT-Actuaries World Indices™ data base.

An Example

Tables 5 and 8 suggest one way to create inputs for the formula. The average excess return on the world market was 3 per cent in the earlier period and 11 per cent in the later period. We may thus estimate a future excess return of 8 per cent.

The volatility of the world market was higher in the later period, but that included the crash, so we may want to use the 15 per cent volatility from the earlier period. The average exchange rate volatility of 10 per cent in the earlier period may also be a better estimate of the future than the more recent 8 per cent.

This reasoning leads to the following possible values for the inputs:

$$\mu_m = 8\%,$$
$$\sigma_m = 15\%,$$
$$\sigma_e = 10\%.$$

Given these inputs, the formula tells us that 77 per cent of holdings should be hedged:

$$\frac{0.08 - 0.15^2}{0.08 - \frac{1}{2}(0.10)^2} = 0.77.$$

Table 5. World Average Values, 1986–1988

	Excess Return	Return Volatility	Exchange Rate Volatility
1986	17	14	9
1987	−3	26	8
1988	18	13	8
1986–88	11	18	8

Table 6. Exchange Rate Volatilities, 1981–1985

	Japan	United States	United Kingdom	Canada	Germany	France	Australia	Switzerland	Italy	Netherlands
Japan	0	12	13	11	10	10	12	11	9	10
United States	11	0	12	4	12	13	11	13	10	12
United Kingdom	12	13	0	12	10	11	14	12	11	10
Canada	11	4	11	0	11	12	10	12	10	11
Germany	10	12	10	12	0	5	13	7	5	2
France	10	13	11	12	4	0	12	8	5	5
Australia	12	10	13	10	12	12	0	13	11	12
Switzerland	11	14	12	13	7	8	14	0	8	7
Italy	9	10	11	10	5	5	12	8	0	5
Netherlands	10	12	10	11	2	5	12	7	5	0

Source: FT-Actuaries World Indices™ data base.

To compare the results of using different inputs, we can use the historical averages from both the earlier and later periods:

$$\mu_m = 3\% \text{ or } 11\%,$$
$$\sigma_m = 15\% \text{ or } 18\%,$$
$$\sigma_e = 10\% \text{ or } 8\%.$$

With the historical averages from the earlier period as inputs, the fraction hedged comes to 30 per cent:

$$\frac{0.03 - 0.15^2}{0.03 - \frac{1}{2}(0.10)^2} = 0.30.$$

Using averages from the later period gives a fraction hedged of 73 per cent:

$$\frac{0.11 - 0.18^2}{0.11 - \frac{1}{2}(0.08)^2} = 0.73.$$

Generally, straight historical averages vary too much to serve as useful inputs for the formula. Estimates of long-run average values are better.

Table 7. World Market Excess Returns and Return Volatilities in Different Currencies, 1981–1985

Currency	Excess Return	Return Volatility
Japan	3	17
United States	−1	13
United Kingdom	10	16
Canada	2	13
Germany	8	15
France	7	16
Australia	7	18
Switzerland	9	16
Italy	4	15
Netherlands	8	15

Optimization

The universal hedging formula assumes that you put into the formula your opinions about what investors around the world expect for the future. If your own views on stock markets and on exchange rates are the same as those you attribute to investors generally, then you can use the formula as it is.

If your views differ from those of the consensus, you may want to incorporate them using optimization methods. Starting with expected returns and covariances for the stock markets and exchange rates, you would find the mix that maximizes the expected portfolio return for a given level of volatility.

The optimization approach is fully consistent with the universal hedging approach. When you put the expectations of investors around the world into the optimization approach, you will find that the optimal currency hedge for any foreign investment will be given by the universal hedging formula.

A Note on the Currency Hedge

The formula assumes that investors hedge real (inflation-adjusted) exchange rate changes, not changes due to inflation differentials between countries. To the extent that currency changes are the result of changes in inflation, the formula is only an approximation.

In other words, currency hedging only approximates real exchange rate hedging. But most

Table 8. World Average Values, 1981–1985

Excess Return	Return Volatility	Exchange Rate Volatility
3	15	10

changes in currency values, at least in countries with moderate inflation rates, are due to changes in real exchange rates. Thus currency hedging will normally be a good approximation to real exchange rate hedging.

In constructing a hedging basket, it may be desirable to substitute highly liquid currencies for less liquid ones. This can best be done by building a currency hedge basket that closely tracks the basket based on the universal hedging formula. When there is tracking error, the fraction hedged should be reduced.

In practice, then, hedging may be done using a basket of a few of the most liquid currencies and using a fraction somewhat smaller than the one the formula suggests.

The formula also assumes that the real exchange rate between two countries is defined as the relative value of domestic and foreign goods. Domestic goods are those consumed at home, not those produced at home. Imports thus count as domestic goods. Foreign goods are those goods consumed abroad, not those produced abroad.

Currency changes should be examined to see if they track real exchange rate changes so defined. When the currency rate changes between two countries differ from *real* exchange rate changes, the hedging done in that currency can be modified or omitted.

If everyone in the world eventually consumes the same mix of goods and services, and prices of goods and services are the same everywhere, hedging will no longer help.

APPLYING THE FORMULA TO OTHER TYPES OF PORTFOLIOS

How can you use the formula if you don't have a fully diversified international portfolio, or if foreign equities are only a small part of your portfolio? The answer depends on why you have a small amount in foreign equities. You may be

(a) wary of foreign exchange risk;
(b) wary of foreign equity risk, even if it is optimally hedged; or
(c) wary of foreign exchange risk and foreign equity risk, in equal measure.

In case (a), you should hedge more than the formula suggests. In case (b), you should hedge less than the formula suggests. In case (c), it probably makes sense to apply the formula as given to the foreign equities you hold.

If the barriers to foreign investment are small, you should gain by investing more abroad and by continuing to hedge the optimal fraction of your foreign equities.

Foreign Bonds

What if your portfolio contains foreign bonds as well as foreign stocks?

The approach that led to the universal hedging formula for stocks suggests 100 per cent hedging for foreign bonds. A portfolio of foreign bonds that is hedged with short-term forward contracts still has foreign interest rate risk, as well as the expected return that goes with that risk.

Any foreign bonds you hold unhedged can be counted as part of your total exposure to foreign currency risk. The less you hedge your foreign bonds, the more you will want to hedge your foreign stocks.

At times, you may want to hold unhedged foreign bonds because you believe that the exchange rate will move in your favor in the near future. In the long run, though, you will want to hedge your foreign bonds even more than your foreign equities.

CONCLUSION

The formula's results may be thought of as a base case. When you have special views on the prospects for a certain currency, or when a currency's forward market is illiquid, you can adjust the hedging positions that the formula suggests.

When you deviate from the formula because you think a particular currency is overpriced or underpriced, you can plan to bring your position back to normal as the currency returns to normal. You may even want to use options, so that your effective hedge changes automatically as the currency price changes.

FOOTNOTES

1. J.J. Siegel, "Risk, Interest Rates, and the Forward Exchange," *Quarterly Journal of Economics* (May 1972).
2. See, for example, B.H. Solnik, "An Equilibrium Model of the International Capital Market," *Journal of Economic Theory*

(August 1974); F.L.A. Grauer, R.H. Litzenberger, and R.E. Stehle, "Sharing Rules and Equilibrium in an International Capital Market Under Uncertainty," *Journal of Financial Economics* (June 1976); P. Sercu, "A Generalization of the International Asset Pricing Model," *Revue de l'Association Francaise de Finance,* (June 1980); and R. Stulz, "A Model of International Asset Pricing," *Journal of Financial Economics* (December 1981).

3. The derivation of the formula is described in detail in F. Black, "Equilibrium Exchange Rate Hedging," National Bureau of Economic Research Working Paper No. 2947 (April 1989).