Improving the Outlook for a Successful Retirement: A Case for Using Downside Hedging

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Current Draft: November 30, 2014

JEL Classification: G11, G17

Key Words: Retirement Income Sustainability, Sequence-of-Returns Risk, Mortality Risk, Downside Risk Hedging

* We are grateful for the comments of Dan Cassidy, Jeremy Crean, and Leonard Glynn. The opinions and analyses presented herein are those of the authors and do not necessarily represent the views of Putnam Investments.

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Abstract

One of the biggest risks to a successful retirement is the exposure of savings to one or more adverse negative investment returns in the early stages of the retirement. As such, the viability of downside hedging to mitigate this "sequence-of-returns" risk is an important investment question. In this study, we conduct extensive simulation analysis to show that for sustainable withdrawal rates, hedging with costless collars or with put options can eliminate or significantly reduce funding shortfall risk for a retirement portfolio. In addition, we demonstrate with a few examples that, for a given level of shortfall risk, hedging can increase the income generated by retirement savings by almost 40%. Thus, downside hedging strategies within retirement portfolios appear to offer attractive benefits to the retiree worried about outliving their income resources.

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1. Introduction

Recent record-setting volatility in equity markets and steep declines in retirement portfolios have caused many investors to question the conventional wisdom of simply diversifying across asset classes and holding on for the long-term regardless of what markets may do. In particular, the global financial market sell-off that began in October of 2007 and hit stocks, bonds and real estate around the world throughout 2008 provided an emphatic example—indeed, a "black swan" event—of how severe price shocks can erode portfolio wealth in a relatively short period of time. If nothing else, this dramatic downturn highlighted the special potential that some alternative investment strategies may have in curbing volatility and reducing the risk of depletion of retirement portfolios meant to be drawn on for essential lifelong income.[1]

As an alternative to trying to manage retirement planning problems solely with allocation schemes, investment strategies that are designed to directly limit the downside exposure of a portfolio are also available to investors. We broadly refer to these as downside hedging (DH) strategies, and they often employ a wide variety of instruments and tactics—such as derivatives, short-selling and tactical asset allocation—to hopefully reshape the distribution of potential future return outcomes. Well-executed DH funds or strategies, therefore, have at least the potential of lowering volatility in down markets. That potential has especially great promise if the hedging strategy can be successfully applied to investment portfolios intended to be drawn on during retirement.

In this paper, we address the question of whether the use of DH strategies can add measurable value to lifelong retirement portfolios. In this effort, we use two specific

[1] Dating to the work of Samuelson (1969) and Merton (1969), there is a substantial literature that has examined the issue of how asset allocation strategies should be designed to solve the retirement planning problem in an optimal manner; for example, see also Bodie and Crane (1997), Poterba and Samwick (2001), Schleef and Eisinger (2007), and Farhi and Panageas (2007). However, as the "Great Recession" of 2008 showed, it is difficult to control the deleterious effects of rapid and severe market declines on retirement portfolio wealth using diversification strategies alone.

derivatives-based approaches to model the potential return distributions that DH strategies are capable of generating. We then look at different retirement scenarios to compare the implications of the use of DH strategies to improve the investment characteristics of a portfolio versus relying on traditional unhedged asset allocation strategies alone.

We make these comparisons within the context of a Retirement Present Value (RPV) analytical framework. As described in Harlow and Brown (2013), the RPV computational structure is a very effective way of capturing the effects of both mortality risk and investment uncertainty over an individual's whole savings accumulation and retirement drawdown periods; in fact, one explicit advantage of the RPV process is that it expresses results in current dollars. We show that a DH strategy potentially provides significant improvements in the long-term viability of many retirement plans, provided that spending or withdrawal rates are held at reasonable and sustainable levels. Indeed, compared to reliance on traditional, long-only investment portfolios, the application of DH strategies may significantly lower the risk of an unsuccessful retirement outcome. Thus, since the threat of a severe financial stress event occurring in late old age is the greatest fear of most retirement savers, DH strategies appear to potentially offer great emotional value along with their more direct practical benefits.

2. Defining the Retirement Investment Problem and "Sequence of Returns" Risk

To fully appreciate the value of reducing downside risk in retirement accounts, we should first understand the most significant investment risk that a retiree faces, namely "sequence-of-returns" risk. Sequence-of-returns risk (or more simply "sequence risk") relates to the timing or sequence of a series of adverse investment returns.[2] Exhibit 1 provides a clear and dramatic illustration of this risk. In this example, two funds, A and B, each begin with $100,000. From each, $7,000 is withdrawn each year. Further, we

[2] With few exceptions (e.g., Dichev (2007)), sequence risk does not appear to be a widely recognized phenomenon in the broad-based academic literature. However, it is a topic that has received recent scrutiny in more narrowly defined studies focusing on retirement investment management issues (e.g., Frank, Mitchell, and Blanchett (2011), Basu, Doran, and Drew (2013), Kenigsberg, Mazumdar, and Feinschreiber (2014)).

assume that each fund experiences exactly the same returns over a 20-year period, but they experience those returns in the exact opposite order than one another. This difference in return ordering—or sequencing—is the only thing differentiating these two portfolios. Thus, their average return and volatility over the investment horizon are the same, despite the fact that the timing of their annual returns is different.

Of course, the fact that the portfolios have the same average return with the same level of volatility of over the 20-year period means that owners of these portfolios will end up with exactly the same ending-level of wealth, providing they did not withdraw any assets from the funds during the investment horizon. However, this does not describe the reality for retirees who must routinely draw down their portfolios to support their income needs. In such a scenario, the sequencing of returns matters greatly; a portfolio experiencing negative returns early on in the investment period will be more severely impacted than a portfolio with positive initial returns when withdrawals are made on a routine basis. In this example, Fund A has the misfortune of having a sequence of negative returns in its early years and is completely depleted by year 12. Fund B, in stark contrast, has the good luck of a few positive returns in its early years and ends up two decades on with more than triple the assets with which it began.

Clearly, retirees cannot predict how markets will fare in the initial years after they cease working for labor-generating income and begin drawing most of their needed funds from their retirement investments.[3] The question, then, is whether DH investment strategies or funds can mitigate the impact of highly negative market movements in the early years of an investor's retirement. If so, such strategies should also increase the chances that the portfolio will last long enough to provide the retiree with income for life, thereby avoiding financial ruin.[4]

3 A related issue that we do not consider explicitly in this study is the sustainable withdrawal rate than investors can withdraw from their retirement portfolios without facing financial ruin (i.e., exhausting their financial resources) before the end of their lives. For more on this problem, see Bengen (1994), Pye (2000), Guyton (2004), Milevsky and Robinson (2005), Scott, Sharpe, and Watson (2009), and Blanchett, Kowara, and Chen (2012).

4 An interesting analog to the financial ruin problem faced by individual investors in retirement is the spending decisions faced by university endowment funds; see Ho, Mozes, and Greenfield (2010) for examinations of the sustainability of the traditional 4-5% annual spending rate by university endowments.

3. Modeling Downside Hedging Strategies

To address the question of managing downside sequence risk in retirement portfolios, we first need to come up with a reasonable way to model a DH strategy in order to compare it to a traditional portfolio withdrawal pattern that relies only on simple, long-only investment strategies. A DH approach is one whose net effect is to alter the future downside distribution of a portfolio's investment returns. In other words, the extreme negative "tails" of the potential distribution are explicitly truncated through specific investment tactics or securities selection.

While there are many—if not infinite—ways to model this type of investment strategy, we will look at two economically legitimate approaches that could be executed through the use of option contracts. Options, of course, are highly liquid financial instruments that provide the holder of the contract with the right either to sell a stock or bond at some pre-determined price by exercising a put option, or to buy a stock or bond at a set price by exercising a call option.

We will look at two ways to use options to potentially mitigate risk in a retirement income portfolio. Our first example involves purchasing put options to limit a portfolio's downside risk and financing that purchase by selling call options of equivalent value on the same portfolio. The result is a so-called costless collar position with the portfolio's downside protection fully financed by the sale of the call option written on the same portfolio's upside potential. This relationship is illustrated in Exhibit 2, which shows that the downside risk "insurance" inherent in the put option is effectively paid for by surrendering some of the upside price appreciation potential in the underlying portfolio (i.e., beyond the call option's exercise price) that comes with selling the call contract.

In this particular example, the exhibit depicts the payoff relationship of a collar around an asset with a current price of 1.00. The put option has a strike price of 0.95 and the call option of equivalent value has a strike price of 1.1142. Returns (as of the joint expiration date of the two options) on the underlying asset between -5.00% and +11.42%, therefore, are unaffected by the collar. Returns of less than -5.00% are protected by a floor and locked in at that level with the exercise of the put option insurance. Conversely, returns in excess of +11.42% are surrendered as the call option written against the portfolio is exercised by the contract holder.

For alternative examples, Exhibit 3 provides the strike prices of the puts and calls used to form the costless collar under the assumption that the asset has a current price of 1.00. These strikes prices are provided for various levels of the underlying asset's volatility. We assume that the option has a maturity of twelve months and that the corresponding risk-free interest rate and dividend yield are 3.00% and 1.00%, respectively. Notice that when the collar arrangement is structured to provide more downside protection (i.e., at .95 versus .90), more of the upside is also forfeited. That is because the cost of the more conservative put option is higher so that more upside potential has to be sold with the call option to ensure that the collar has no up-front expense to the investor.[5]

Since our goal is to compare the impact of DH strategies to traditional, long-only portfolio strategies, the underlying distribution of returns in our analysis will treated as identical. But as we will see, the DH collar strategies will curtail the extreme upper and lower range of returns to varying degrees depending on the specific collar that is used. Clearly, the reduced downside exposure provided by the put option within the collar should benefit a retirement portfolio over time. What is far from obvious, however, is the net impact of forfeiting the larger upside returns on the overall ability of the portfolio to deliver income in the present as well as potentially providing future bequests to heirs.

To look more closely at that issue, we will examine at a second example which also serves as a reasonable proxy for a DH strategy: the direct purchase of the put option without the associated sale of the call option. Exhibit 4 shows the payoff schedule for

[5] To use an insurance interpretation, the difference between the current value of the asset (1.00 in this example) and the strike price on the put option providing the downside risk protection (say, 0.95) is equivalent to the deductible on the insurance contract (e.g., 5.00% in this example). So, the investor is effectively insured against any losses in the portfolio after absorbing the first 5.00% decline. Of course, insurance policies with smaller deductible levels will be more expensive to purchase, all other things equal.

this protective put strategy, which Brown and Statman (1987) labeled an "insured stock" position to make it plain that the point of the put option is to provide the desired risk mitigation for the underlying portfolio. In this approach, the cost of the put option has to be deducted from the value of the retirement portfolio each year. In exchange for this cost, however, the portfolio enjoys substantially more upside potential in that, unlike the costless collar strategy it has not surrendered any portion of the right-hand side of the future return distribution.[6]

4. Comparison of the Strategies in Retirement Funding

We have now defined two legitimate ways to model DH strategies as applied to a retirement income portfolio and to compare their potential outcomes to traditionally-allocated unhedged portfolios.[7] It is useful to begin with a couple of simple examples in order to get a feel for the mechanics of the hedging strategies and their effect on a retirement portfolio.

Let us assume an individual aged 65 retires at the end of 1972. Our retiree has saved $1,000,000 and wishes to spend $60,000 each year, adjusted for inflation. For simplicity, we will assume that retirement expenses are paid at the end of the year and that the savings are invested in the S&P 500. Exhibit 5 details and compares the results of two different investment strategies for the individual's retirement savings: an unhedged portfolio and a portfolio utilizing costless collars. For the unhedged strategy, the columns on the left side of the table indicate the increasing nominal amount of the retirement withdrawals due to the effects of inflation. Moreover, the table shows the declining value of the savings as distributions are withdrawn and the portfolio is subject

[6] By way of summarizing the two DH strategies, an insurance market analogy is once again useful. Both costless collar and protective put positions are designed to purchase the same put option as a means of providing the downside risk protection. The difference between the strategies is how the investor pays for that insurance. With the costless collar, no up-front cash payment is required, but a portion of the future potential gains to the portfolio may need to be surrendered due to the sale of the call option. With the direct purchase of the put option, no future portfolio gains will be surrendered, but the investor will have to pay for the insurance in cash now. Thus, the tradeoff of between the two strategies can be viewed as the possibility of surrendering future portfolio gains versus the certainty of paying an up-front premium.

[7] Of course, the two DH strategies are already well-established in the risk management literature. In particular, Milevsky (2006) and Milevsky and Kyrychenko (2008) have considered variations of these hedging concepts in the context of retirement income planning.

to market returns. In this example, our retiree's savings are depleted after only 12 years, in part, due to the adverse impact of the large negative market returns in 1973 and 1974 as well as the high inflation in the 1970s. Notice that both these events expose portfolios on which withdrawals are taken to sequence risk.[8]

The columns on the right side of the Exhibit 5 contrast the effects of a DH strategy employing costless collars. For the purpose of this example, we assume that the collar employs a put option with strike price of 0.95 of the current value of the underlying asset portfolio. Further, the collar is reinstated at the beginning of each year, with the sale of a new one-year call option being used to finance the acquisition of a new one-year put option. Using the Black-Scholes valuation model, both options are priced based on the historical logarithmic volatility of the S&P 500 using the prior 12 months of return data. (We will look at the sensitivity of results to this volatility assumption in a moment.) Consequently, the collars have a maturity of one year and also based on the one-year risk free rate at the time of purchase.

Notice in Exhibit 5 that protecting the downside of the retirement portfolio had a significant and beneficial impact on the ability of the portfolio to support retirement withdrawals. Specifically, the retiree was able to extend the duration of his or her savings by roughly an additional 10 years of retirement, or out to about age 88. While these results are specific to one sequence of historical returns, they do represent an important extension of the life of the retirement savings.

As an additional example, Exhibit 6 looks at the same retiree but compares the unhedged results with those of our second DH approach using protective put options. As with the costless collars, the put contracts are purchased at the beginning of each year. However, the dollar costs of these direct purchases are deducted from the value of the portfolio at the time they are made.

[8] The assumed conditions for the investor modeled in this example are easily generalized but are typical for retirement problems of this nature; see, for example, Davidoff, Brown, and Diamond (2005).

The results of this DH strategy are very similar to those using the costless collars. The life of the retirement savings for this particular period of time is extended by approximately 10 years relative to the unhedged portfolio. Based on this comparison, the use of the DH strategy potentially provides valuable risk mitigation around the critical retirement date and a hedge against sequence risk in the future return stream to the underlying portfolio. This should not actually be too surprising of a result. By eliminating, or at least reducing, the primary cause of retirement shortfall through a DH strategy, we would expect some improvement in the metrics used to evaluate the overall health of a retirement plan. The important aspect of this analysis is that the cost of providing such downside protection seems reasonable in light of the improvement, regardless of how the investor chooses to pay for that protection.

While the prior examples are instructive as to the potential benefits of employing DH strategies to manage retirement outcomes, it is important that we expand our analysis for additional insights into the range of situations where they may be beneficial. Moreover, it will be useful to make the analysis more general since our prior examples omit mortality risk which is an important element of the realty of retirement. In addition, it would be helpful to have a single statistic or metric that summarizes the overall financial viability and well-being of a given retirement plan. It is to that topic that we now turn our attention.

5. Calculating Retirement Present Value

5.1 A Descriptive Overview of the RPV Process

As Harlow and Brown (2013) have noted, the most complete method to make these desired comparisons is to look at the retirement present value (RPV) of a projected saving and retirement spending plan. The RPV statistic captures the probabilistic nature of retirement success through a forecasting process that integrates uncertain future investment returns with the mortality risk of a specific individual investor.

The RPV method is analogous to the Monte Carlo simulations used in most retirement planning analyses. However, rather than simulating returns to project the future value of a retirement portfolio at some pre-specified “expiration” date for the investor (e.g., at age 85), the simulated returns are used as discount factors to compute the present value of future retirement cash flows. Accordingly, savings represent cash flows into the retirement portfolio while withdrawals or distributions are treated as cash outflows. Mortality risk is captured by weighting these net cash flows by the probability of the investor being alive at any point in the future. The cumulative effect of these computations is that a positive RPV indicates the likelihood that the investor will have at least some assets left over at the end of his or her life. On the other hand, a negative RPV implies the depletion of all retirement assets at some point before death. Thus, the RPV statistic simply represents the value of a retirement plan’s current assets and projected cash flows in today’s dollars.

If we leave aside the question of whether or not an investor wishes to leave a bequest, the ideal retirement plan would therefore be one in which the RPV would be exactly zero, meaning that a person would have *exactly* the right amount of retirement funds to spend before passing away. Real-world retirement plans, of course, have a wide distribution of possible outcomes, ranging from dying early with a large surplus to investors who outlive their retirement assets long before they expire. For planning purposes, an important possible goal would be to significantly curtail the latter possibility; that is, avoiding a negative RPV (probability of ruin) or minimizing the overall severity of a shortfall in retirement funds.

5.2 The Mechanics of the RPV Process

As the preceding discussion suggests, the RPV method is simply an expression of the monetized value of a retirement plan in current dollars. It captures both mortality risk and the uncertainty around investment returns by discounting the cash inflows and outflows of the retirement plan in the appropriate manner. The calculation of the RPV is straightforward and merely an adaptation of the familiar method of determining the discounted present value of a series of future cash flows. Mathematically, the equation for the probability-weighted discounted cash flows can be expressed as:

$$RPV = \sum_{t=0}^{\infty} \frac{p_t CF_t}{(1+R_t)^t} \quad (1)$$

where:

t = years into the future,

p_t = probability of being alive at time t,

CF_t = cash flow at time t, and

R_t = the risk-adjusted discount rate.

The cash flows of the retirement plan, CF_t in equation (1), represent savings inflows into the portfolio prior to retirement age and the outflows from living expenses deducted after retirement. CF_0 in the RPV analysis represents the individual's current savings at any desired time t=0. The probability of being alive at future time t, p_t, can be obtained directly from actuarial tables or through standard mathematical models specified to approximate the actual probability values.[9]

In order to determine the discount rate applicable at each time t (i.e., R_t,), the returns on the retirement fund investment portfolio available in each year are used. These returns, denoted r_t, can be obtained from historical time series or through Monte Carlo simulation. Assuming for convenience (but without loss of generality) that the asset classes included in the investment portfolio are stocks, bonds, and cash equivalents, the discount rate can be expressed:

$$(1+R_t)^t = (1+r_1)(1+r_2)(1+r_3) \ldots (1+r_t) \quad (2)$$

where:

$$r_t = (w_S \times r_{St}) + (w_B \times r_{Bt}) + (w_C \times r_{Ct}). \quad (3)$$

In (2) and (3), we have the following definitions:

[9] In the analyses presented in this paper, retirement plan cash flows and simulated returns are estimated from the individual's current age out to age 110. Mortality effects are based on the Social Security Administration's period life tables.

w_S, w_B, and w_C = portfolio weights in stocks, bonds, and cash, respectively;

and

r_{St}, r_{Bt}, and r_{Ct} = returns on stocks, bonds, and cash at time t, respectively.

In this investigation, we only consider the situation in which the retirement portfolio is in a state of net withdrawals. In other words, we assume that the individual is already in retirement and so all future cash flows, CF_t, are negative. Said differently, we do not include any periods of accumulation prior to retirement. It is also important to note that these spending amounts are exogenous, pre-specified values represented in constant, inflation-adjusted dollars.

5.3 Illustrating the RPV Approach

Exhibit 7 provides the RPV distribution of a representative 65 year-old retiree's projected saving and spending plan. In this example, our retiree has saved $1,000,000 and wishes to spend $70,000 each year, adjusted for inflation. This corresponds to an initial spending rate of 7% of savings (or, $70,000 divided by $1,000,000). For simplicity, let us assume that our retiree is currently relying on a long-only, unhedged portfolio. This includes a substantial allocation of stocks which gives the portfolio an expected return of 6% in real terms with a volatility of 20% per annum to represent a potentially turbulent equity market.[10]

This chart shows the probability spread of 10,000 varying portfolio outcomes over the course of retirement, which have been driven by interacting the various market actions and mortality scenarios. Consequently, the wide distribution reflects the uncertainty of future investment returns as well as uncertainty about how long an individual will live.

The outcomes shown in blue—to the right of the "$0 RPV" point—are scenarios in which this portfolio lasts for the investor's entire life and then leaves a surplus with which he or she can leave a bequest. The display indicates that there is a combined probability of more than 70% for achieving such a successful result. The outcomes shown in red

[10] During 2008, the volatility of the US equity market as measured by the VIX Index averaged 31.6%.

represent failure scenarios in which the portfolio is exhausted at some point before the retiree's death. The mean results indicate the average bequest that retirees would be expected to leave after successfully drawing income from the portfolio through to the end of their lives, as expressed in today's dollars.

Unfortunately, in this example, 29.9% of the outcomes have a negative present RPV value. This is consistent with a more than one-in-four chance of depleting the portfolio and enduring financial ruin (in actuarial terms) before the end of life. Further, some of these negative outcomes can be severe; for instance, Exhibit 7 includes some potential realizations that are close to - $1,600,000. That means there are some combinations of market and mortality events that would have actually required $2,600,000 (i.e., the original $1,000,000 plus the $1,600,000 shortfall) in initial savings at age 65 to completely fund a successful retirement that begins with distributions of just $70,000 per year. Such "limit" cases could be due to a combination of poor market returns early in the retirement period (i.e., sequence-of-returns risk) compounded by exceptional longevity, which in turn leads to an extremely long retirement period. On average, the expected shortfall of retirement funds for the traditional strategy is -$352,000.

In contrast to the previous example, Exhibit 8 provides a summary of the potential scenarios for the same portfolio using a DH strategy based on a costless collar that is created by buying an out-of-the-money put (i.e., exercise price below the current portfolio value) and selling an out-of-the-money call option. In this example, we assume that at the beginning of each year into the future, our investor buys a put with a strike price of 0.95 of the value of portfolio and sells a call option at a strike price of 1.1069 so as to have the same price as the put. As the display indicates, the impact on the range of potential outcomes is dramatic.

Most importantly, the distribution of potential RPV values is, as we might expect, significantly curtailed, with much less dispersion on both sides of the distribution. Additionally, the likelihood of a successful retirement outcome—defined as a lifelong income and some positive bequest—is raised from just over 70% to more than 88%.

Conversely, the probability of failure—defined as portfolio depletion before death—is reduced from over 29.9% to just 11.5%. Perhaps more significantly, however, the downside risk in retirement funds (i.e., the average expected shortfall level) is reduced substantially from -$352,000 to only -$38,900.

Clearly, the use of the DH strategy provides valuable risk mitigation around the critical retirement date as well as an effective hedge against sequence-of-returns risk. In fact, by eliminating, or at least reducing, the primary cause of retirement shortfall through a DH strategy, we would expect significant improvement in the metrics used to evaluate the overall health of a retirement plan. However, it is important to point out that there are tradeoffs to reducing the risk of retirement failure by using a costless collar. Not surprisingly, most of the extreme potential upside seen in the previous example has been sacrificed in exchange for significantly cutting the downside risk; indeed, this surrender of upside potential is how the investor pays for the put option insurance in the collar strategy. This lower upside potential is reflected in the lower median of the DH distribution versus the traditional unhedged portfolio ($178,000 versus $220,000) as well as in the much lower range of outcomes on the upside of the distribution. Yet the average RPV of the collared strategy is actually *higher* than that of the unhedged portfolio ($166,000 versus $110,000). Thus, the net effect is that while the average bequest is somewhat higher for the well-hedged retiree's heirs, those benefactors also have much smaller chances of receiving an outsized inheritance.

For a more complete picture of the role that a DH strategy with a costless collar can play in retirement, Exhibit 9 summarizes the probabilities of a retirement portfolio achieving a failure state as well as its downside risk for varying assumptions regarding rates of spending and volatility. The most salient thing to note is that the hedged strategy offers a significant reduction in the downside risk of the retirement portfolio. For example, in the case where asset volatility is assumed to be 15% and the inflation-adjusted spending rate is $7 per $100 in savings, the downside risk for the traditional portfolio is $13.36. Hedging decreases this risk to $2.42, which is an 82% reduction. Notice that for all scenarios presented in the table, the magnitude of the shortfall is either reduced or eliminated altogether through the use of the costless collar.

There are other relationships in these data that are worth noting. Specifically, for high withdrawal rates, the probability of experiencing a shortfall is actually *higher* for the hedged strategy even though the overall downside risk is lower. Returning to the 15% volatility scenarios, the traditional portfolio has a probability of ruin of 30.21% for a $9 spending rate versus a 55.00% for the hedged portfolio. This reflects the fact that the hedged portfolio has a more concentrated range of outcomes (i.e., a narrower distribution of potential future outcomes, as we saw in Exhibit 8) and that the distribution is shifting slightly toward the more negative values. Thus, for very high spending rates, the hedged portfolio experiences a higher rate of failure because of the way in which the collar truncates the possibility of extreme fortuitous outcomes in the underlying portfolio return distribution. However, when such a failure does occur, it is not likely to be as severe because the downside has been protected by the collar.

Another interesting result in Exhibit 9 is that for lower levels of asset volatility (i.e., in the 10% to 15% range), the reduction in downside risk comes with a reduced average RPV. In other words, the cost of providing the protection by forfeiting the upside results in a lower average value of the retirement plan as measured by the present value of cash flows. On the other hand, for higher levels of volatility, protecting the portfolio from adverse returns actually leads to higher average RPVs. This suggests that curtailing the large negative returns when volatility is high becomes more important than giving up the upside potential when assessing the overall impact of the hedging strategy on the distribution of retirement plan values.

The reduction in downside risk presented in Exhibit 9 also suggests an alternative perspective of the value of hedging. To see this, Exhibit 10 plots the relationship between downside risk and spending rates of the traditional unhedged and collar- hedged portfolios for the case when asset volatility is 15%. As we have already noted, the hedged portfolio always exhibits a lower downside risk than the unhedged portfolio, as evidenced by the fact that its curve is positioned lower on the graph. What is interesting in this display, however, is the significant increase in retirement income that be gained for the same level of overall shortfall risk.

As an illustration, the horizontal reference line in Exhibit 10 indicates a downside risk value of $8.06 which is associated with the traditional unhedged portfolio for a $6.00 initial spending rate. For this same level of risk, the hedged portfolio provides a much larger income of $8.21. Hedging in this scenario, therefore, enabled our hypothetical retiree to take a 37% pay increase (i.e., $8.21 versus $6.00) in retirement without incurring any additional downside risk.[11]

As a final part of the analysis, let us turn our attention to the second DH strategy which employs the direct purchase of a put option to limit the downside risk of the retirement portfolio. Exhibit 11 summarizes the same RPV analysis as we saw before but with the protective put-only approach. Like the costless collar, the put option does improve the chances for a successful retirement for a wide range of withdrawal rates. However, the reductions in the probabilities of ruin are somewhat smaller in magnitude. This suggests that forfeiting the upside with the collar in a retirement portfolio may well be a less expensive way of obtaining the downside protection. Of course, both sets of results are dependent on the interest rate and volatility assumptions used in the analyses.

Of course, there may be combinations of assumptions where the put option approach has stronger results than that of the collar. However, both approaches seem likely candidates to improve the likelihood of retirement success.

6. Conclusions

Successfully translating an investor's savings over a lifetime into a reliable source of sustainable income in retirement is a daunting challenge. Indeed, for millions of

[11] The 37% pay increase in the example is somewhat arbitrary. From the relationships depicted in Exhibits 9 and 10, it is easy to identify cases in which hedging, for the same level of risk, would increase retirement income by 100% or even 200%.

investors, as well for the multitude of financial professionals and advisors who serve those investors, this is the crucial question that must be addressed. Perhaps the greatest challenge that the financial market meltdown of 2008 has posed for plan participants is in finding ways to insulate retirement portfolios from the risk that a similar period of volatility and decline might occur in the initial years of their retirements.

One positive development for those facing this challenge is that the emergence of '40 Act mutual funds that employ absolute return strategies that offer the possibility of dampening volatility and enhancing returns, no matter how securities markets move. In this paper, however, we have suggested the viability of a different approach through our examination of the role that DH strategies might play in increasing the prospects for retirement success and mitigating the risk of financial ruin.

To the extent that the costless collar and put option models used in our analyses resemble the way that DH-type funds might behave, our analysis suggests that these strategies have the potential to significantly improve a retirement portfolio's prospects for success, mainly by significantly mitigating sequence-of-returns risk. A foreseeable challenge to implementing these strategies, however, will be the availability of exchange-traded and over-the-counter option contracts based on security portfolios that approximate those that investors actually hold.

Finally, as we have demonstrated, the main trade-off an investor must consider when considering the use of a hedge position alongside his or her underlying retirement portfolio to capture its many advantages is the sacrifice of some potential asset returns on the upside, the principle result of which comes in the form of lower likely bequests to heirs. As always, though, a key proviso in securing these very real benefits from DH strategies is that a retiree maintains the discipline to draw down his or her retirement assets at sustainable annual rates. In that respect, at least, there is no free lunch.

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Exhibit 1

Demonstrating Sequence-of-Returns Risk

Year	Fund A	Balance	Fund B	Balance
0		$100,000		$100,000
1	-19.05%	$73,950	27.53%	$120,530
2	-18.30%	$53,417	23.65%	$142,035
3	-5.63%	$43,410	7.83%	$146,157
4	19.29%	$44,784	14.56%	$160,437
5	8.45%	$41,568	35.72%	$210,745
6	12.61%	$39,809	27.43%	$261,553
7	-14.77%	$26,930	14.62%	$292,792
8	15.52%	$24,109	-3.91%	$274,344
9	10.83%	$19,720	15.88%	$310,909
10	17.79%	$16,228	16.69%	$355,800
11	16.69%	$11,937	17.79%	$412,097
12	15.88%	$6,832	10.83%	$449,727
13	-3.91%	$0	15.52%	$512,525
14	14.62%	$0	-14.77%	$429,825
15	27.43%	$0	12.61%	$477,026
16	35.72%	$0	8.45%	$510,334
17	14.56%	$0	19.29%	$601,778
18	7.83%	$0	-5.63%	$560,898
19	23.65%	$0	-18.30%	$451,253
20	27.53%	$0	-19.05%	$358,290
Arithmetic Mean	10.34%		10.34%	
Geometric Mean	9.25%		9.25%	
Standard Deviation	15.33%		15.33%	

Note: The analysis assumes that each fund begins with $100,000 and that $7,000 is withdrawn each year.

Exhibit 2

Costless Collar for Self-Financing Downside Protection

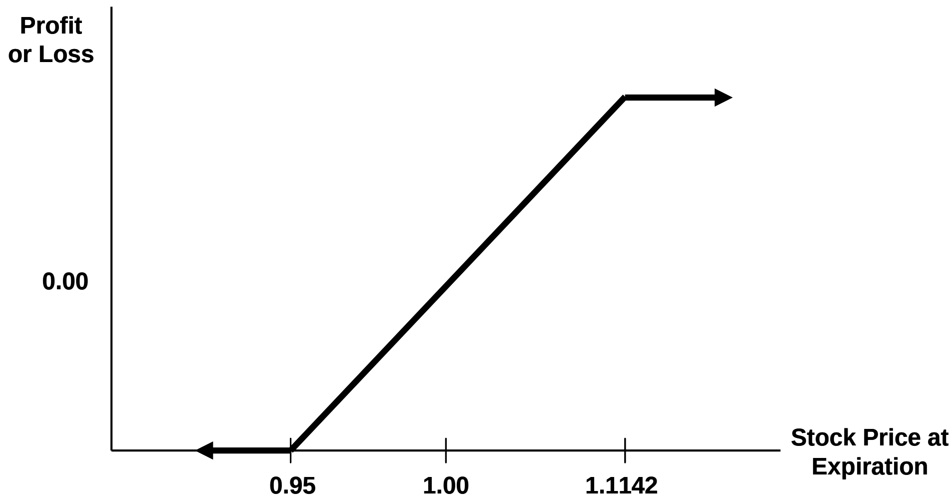


Exhibit 3

Different Combinations of Put and Call Strike Prices forming a Costless Collar at Different Volatility Levels for the Underlying Asset

Volatility	Put Strike Price	Call Strike Price
10%	0.95	1.1005
15%	0.95	1.1035
20%	0.95	1.1069
30%	0.95	1.1142
10%	0.90	1.1635
15%	0.90	1.1689
20%	0.90	1.1750
30%	0.90	1.1881

Exhibit 4

Put Option for Downside Protection

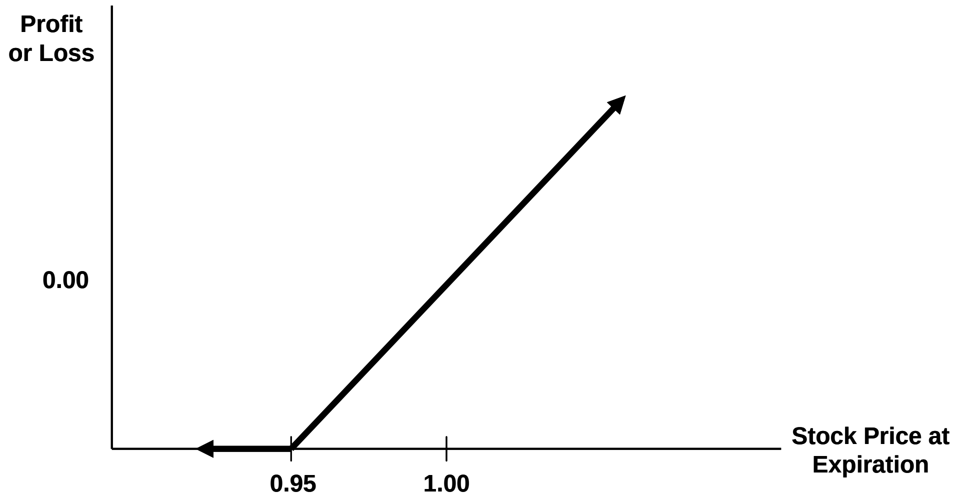


Exhibit 5

Example of the Effects of Hedging Using Costless Collars

				Unhedged Retirement Plan			Hedged Retirement Plan - Costless Collars				
Date	Year	Age	Inflation	Unhedged Return	Inflation Adjusted Withdrawal	Retirement Savings	Put Strike Price	Call Strike Price	Hedged Return	Inflation Adjusted Withdrawal	Retirement Savings
1972	0	65				**1,000,000**	0.9500	1.1824			**1,000,000**
1973	1	66	8.35%	-14.69%	-65,009	**788,104**	0.9500	1.2310	-5.00%	-65,009	**884,991**
1974	2	67	11.63%	-26.47%	-72,572	**506,942**	0.9500	1.2436	-5.00%	-72,572	**768,170**
1975	3	68	6.71%	37.23%	-77,439	**618,231**	0.9500	1.2054	24.36%	-77,439	**877,881**
1976	4	69	4.75%	23.93%	-81,117	**685,038**	0.9500	1.1694	20.54%	-81,117	**977,094**
1977	5	70	6.49%	-7.16%	-86,378	**549,635**	0.9500	1.2169	-5.00%	-86,378	**841,861**
1978	6	71	8.63%	6.57%	-93,836	**491,914**	0.9500	1.3193	6.57%	-93,836	**803,342**
1979	7	72	12.48%	18.61%	-105,548	**477,908**	0.9500	1.3449	18.61%	-105,548	**847,290**
1980	8	73	11.79%	32.50%	-117,996	**515,250**	0.9500	1.4144	32.50%	-117,996	**1,004,695**
1981	9	74	8.55%	-4.92%	-128,080	**361,803**	0.9500	1.3896	-4.92%	-128,080	**827,151**
1982	10	75	3.76%	21.55%	-132,894	**306,866**	0.9500	1.2715	21.55%	-132,894	**872,481**
1983	11	76	3.72%	22.56%	-137,839	**238,241**	0.9500	1.2962	22.56%	-137,839	**931,431**
1984	12	77	3.87%	6.27%	-143,177	**110,011**	0.9500	1.2789	6.27%	-143,177	**846,690**
1985	13	78	3.73%	31.73%	-148,515	**-3,601**	0.9500	1.2349	27.89%	-148,515	**934,296**
1986	14	79	1.09%	18.67%	-150,136		0.9500	1.2006	18.67%	-150,136	**958,550**
1987	15	80	4.34%	5.25%	-156,651		0.9500	1.2494	5.25%	-156,651	**852,227**
1988	16	81	4.32%	16.61%	-163,425		0.9500	1.2689	16.61%	-163,425	**830,343**
1989	17	82	4.54%	31.69%	-170,849		0.9500	1.2394	26.89%	-170,849	**882,806**
1990	18	83	5.93%	-3.10%	-180,975		0.9500	1.2232	-3.10%	-180,975	**674,438**
1991	19	84	3.02%	30.46%	-186,437		0.9500	1.1535	22.32%	-186,437	**638,567**
1992	20	85	2.86%	7.62%	-191,768		0.9500	1.1351	7.62%	-191,768	**495,454**
1993	21	86	2.71%	10.08%	-196,968		0.9500	1.1346	10.08%	-196,968	**348,421**
1994	22	87	2.64%	1.32%	-202,167		0.9500	1.2236	1.32%	-202,167	**150,855**
1995	23	88	2.51%	37.58%	-207,235		0.9500	1.1698	22.36%	-207,235	**-22,644**
1996	24	89	3.27%	22.96%	-214,008		0.9500	1.1825	16.98%	-214,008	
1997	25	90	1.69%	33.36%	-217,621		0.9500	1.1875	18.25%	-217,621	
1998	26	91	1.60%	28.58%	-221,101		0.9500	1.1699	18.75%	-221,101	
1999	27	92	2.65%	21.04%	-226,958		0.9500	1.1961	16.99%	-226,958	
2000	28	93	3.33%	-9.10%	-234,517		0.9500	1.1841	-5.00%	-234,517	
2001	29	94	1.54%	-11.89%	-238,129		0.9500	1.1109	-5.00%	-238,129	
2002	30	95	2.35%	-22.10%	-243,722		0.9500	1.0916	-5.00%	-243,722	

Note: The analysis assumes the retirement portfolio is invested in the S&P 500. The returns are collared with the purchase of a twelve month put option with a strike price of .95 and the sale of a call option with the indicated strike. The individual retires in 1972 at age 65 with $1,000,000 in savings and spends $60,000 per year adjusted for inflation. The analysis ignores taxes and transaction costs.

Exhibit 6

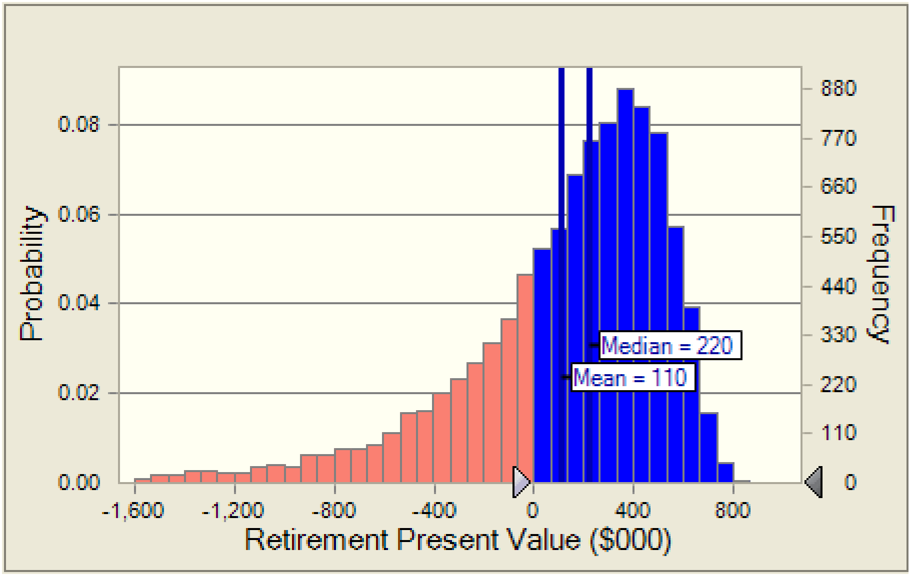
Example of the Effects of Hedging Using Put Options

				Unhedged Retirement Plan			Hedged Retirement Plan - Put Options			
Date	Year	Age	Inflation	Unhedged Return	Inflation Adjusted Withdrawal	Retirement Savings	Put Strike Price	Hedged Return	Inflation Adjusted Withdrawal	Retirement Savings
1972	0	65				**1,000,000**	0.9500			**1,000,000**
1973	1	66	8.35%	-14.69%	-65,009	**788,104**	0.9500	-5.00%	-65,892	**884,108**
1974	2	67	11.63%	-26.47%	-72,572	**506,942**	0.9500	-5.00%	-86,225	**753,678**
1975	3	68	6.71%	37.23%	-77,439	**618,231**	0.9500	37.23%	-108,876	**925,386**
1976	4	69	4.75%	23.93%	-81,117	**685,038**	0.9500	23.93%	-104,842	**1,041,960**
1977	5	70	6.49%	-7.16%	-86,378	**549,635**	0.9500	-5.00%	-104,120	**885,741**
1978	6	71	8.63%	6.57%	-93,836	**491,914**	0.9500	6.57%	-97,513	**846,429**
1979	7	72	12.48%	18.61%	-105,548	**477,908**	0.9500	18.61%	-117,610	**886,334**
1980	8	73	11.79%	32.50%	-117,996	**515,250**	0.9500	32.50%	-123,488	**1,050,937**
1981	9	74	8.55%	-4.92%	-128,080	**361,803**	0.9500	-4.92%	-141,195	**858,002**
1982	10	75	3.76%	21.55%	-132,894	**306,866**	0.9500	21.55%	-136,233	**906,640**
1983	11	76	3.72%	22.56%	-137,839	**238,241**	0.9500	22.56%	-158,260	**952,872**
1984	12	77	3.87%	6.27%	-143,177	**110,011**	0.9500	6.27%	-145,485	**867,169**
1985	13	78	3.73%	31.73%	-148,515	**-3,601**	0.9500	31.73%	-156,794	**985,498**
1986	14	79	1.09%	18.67%	-150,136		0.9500	18.67%	-157,783	**1,011,663**
1987	15	80	4.34%	5.25%	-156,651		0.9500	5.25%	-184,342	**880,436**
1988	16	81	4.32%	16.61%	-163,425		0.9500	16.61%	-227,117	**799,546**
1989	17	82	4.54%	31.69%	-170,849		0.9500	31.69%	-173,517	**879,374**
1990	18	83	5.93%	-3.10%	-180,975		0.9500	-3.10%	-188,267	**663,821**
1991	19	84	3.02%	30.46%	-186,437		0.9500	30.46%	-204,329	**661,718**
1992	20	85	2.86%	7.62%	-191,768		0.9500	7.62%	-208,151	**503,986**
1993	21	86	2.71%	10.08%	-196,968		0.9500	10.08%	-199,028	**355,753**
1994	22	87	2.64%	1.32%	-202,167		0.9500	1.32%	-202,840	**157,610**
1995	23	88	2.51%	37.58%	-207,235		0.9500	37.58%	-208,176	**8,660**
1996	24	89	3.27%	22.96%	-214,008		0.9500	22.96%	-214,012	**-203,363**
1997	25	90	1.69%	33.36%	-217,621		0.9500	33.36%	-215,874	
1998	26	91	1.60%	28.58%	-221,101		0.9500	28.58%	-210,216	
1999	27	92	2.65%	21.04%	-226,958		0.9500	21.04%	-188,342	
2000	28	93	3.33%	-9.10%	-234,517		0.9500	-5.00%	-218,610	
2001	29	94	1.54%	-11.89%	-238,129		0.9500	-5.00%	-202,015	
2002	30	95	2.35%	-22.10%	-243,722		0.9500	-5.00%	-173,756	

Note: The analysis assumes the retirement portfolio is invested in the S&P 500. The returns are hedged with the purchase of a twelve month put option with a strike price of .95. The individual retires in 1972 at age 65 with $1,000,000 in savings and spends $60,000 per year adjusted for inflation. The analysis ignores taxes and transaction costs.

Exhibit 7

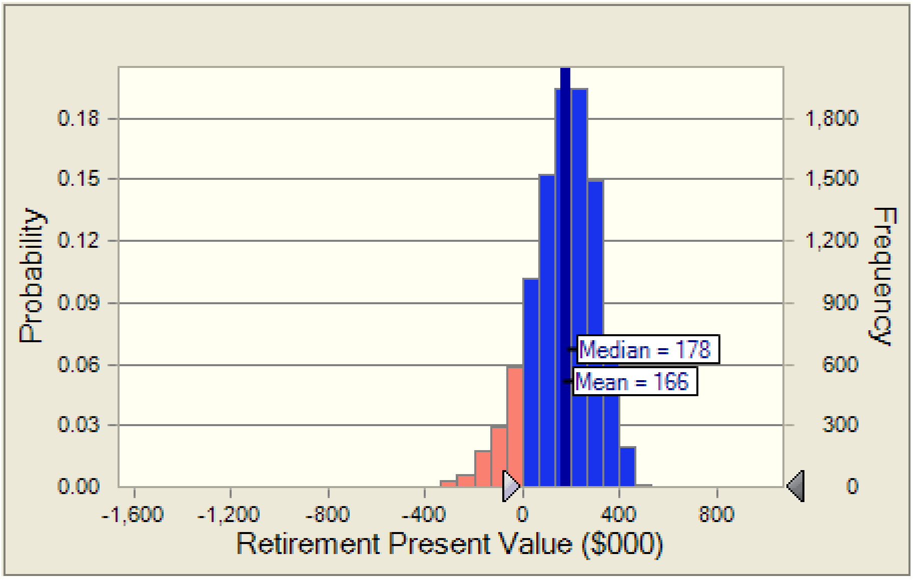
RPV Analysis of a Traditional Equity-Oriented Portfolio



Note: The analysis assumes the retirement portfolio is invested in assets with an expected real return of 6% and a volatility of 20%. The individual retires at age 65 with $1,000,000 in savings and spends $70,000 per year adjusted for inflation. The analysis ignores taxes and transaction costs. Mortality is modeled with a Gompertz-Makeham fit to the Social Security Administration's 2005 Period Life Table.

Exhibit 8

RPV Analysis of a Downside-Protected Equity-Oriented Portfolio



Note: The analysis assumes the retirement portfolio is invested in assets with an expected real return of 6% and a volatility of 20%. The returns are collared with the purchase of a twelve month put option with a strike price of .95 and the sale of a call option with a strike price of 1.1069. The individual retires at age 65 with $1,000,000 in savings and spends $70,000 per year adjusted for inflation. The analysis ignores taxes and transaction costs. Mortality is modeled with a Gompertz-Makeham fit to the Social Security Administration's 2005 Period Life Table.

Exhibit 9

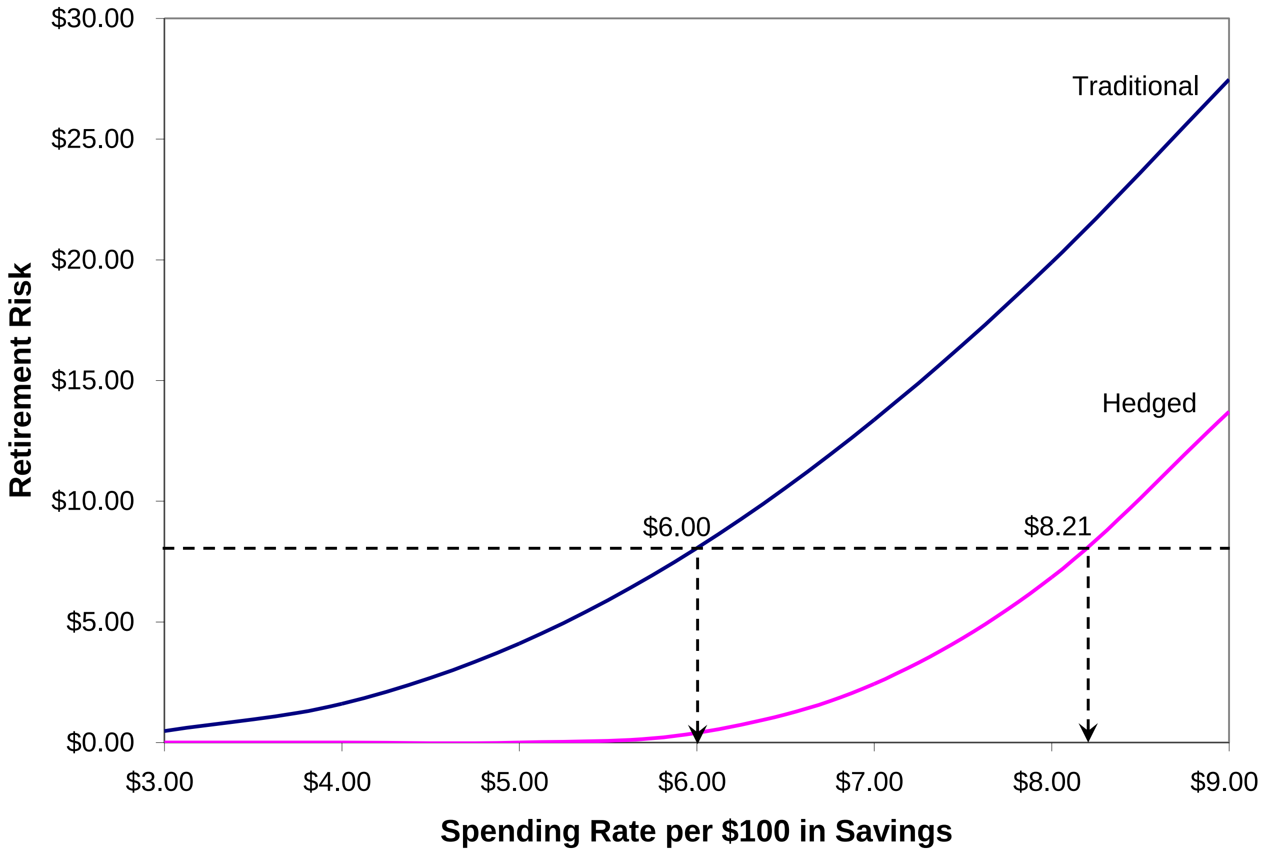
Costless Collars: Comparison of Traditional and Hedged Strategy for Different Volatilities and Spending Rates in Retirement for a 65 Year Old Male

Initial Spending Rate per $100	Parameters: Expected Return	Parameters: Volatility	Traditional Portfolio: Probability of Ruin	Traditional Portfolio: Downside Risk	Traditional Portfolio: Average RPV	Hedged Portfolio: Probability of Ruin	Hedged Portfolio: Downside Risk	Hedged Portfolio: Average RPV
$3.00	6.00%	10.00%	0.00%	$0.00	$71.00	0.00%	$0.00	$68.00
$4.00	6.00%	10.00%	0.00%	$0.00	$61.00	0.00%	$0.00	$57.00
$5.00	6.00%	10.00%	0.05%	$0.29	$51.00	0.00%	$0.00	$46.00
$6.00	6.00%	10.00%	0.78%	$1.00	$41.00	0.04%	$0.10	$35.00
$7.00	6.00%	10.00%	3.72%	$2.76	$32.00	1.72%	$0.87	$24.00
$8.00	6.00%	10.00%	11.17%	$5.82	$22.00	12.28%	$3.44	$13.00
$9.00	6.00%	10.00%	23.72%	$10.27	$12.00	38.45%	$8.39	$3.00
$3.00	6.00%	15.00%	0.05%	$0.47	$67.00	0.00%	$0.00	$66.00
$4.00	6.00%	15.00%	0.67%	$1.61	$57.00	0.00%	$0.00	$54.00
$5.00	6.00%	15.00%	2.99%	$4.10	$46.00	0.00%	$0.00	$43.00
$6.00	6.00%	15.00%	7.74%	$8.06	$35.00	0.52%	$0.40	$31.00
$7.00	6.00%	15.00%	15.88%	$13.36	$24.00	6.74%	$2.42	$20.00
$8.00	6.00%	15.00%	26.83%	$19.89	$13.00	25.82%	$6.84	$8.00
$9.00	6.00%	15.00%	39.21%	$27.46	$2.00	55.00%	$13.70	($3.00)
$3.00	6.00%	20.00%	1.70%	$4.31	$62.00	0.00%	$0.00	$64.00
$4.00	6.00%	20.00%	5.28%	$9.65	$49.00	0.00%	$0.00	$52.00
$5.00	6.00%	20.00%	11.37%	$16.80	$36.00	0.03%	$0.07	$40.00
$6.00	6.00%	20.00%	19.93%	$25.41	$24.00	1.72%	$0.90	$29.00
$7.00	6.00%	20.00%	29.86%	$35.20	$11.00	11.54%	$3.89	$17.00
$8.00	6.00%	20.00%	39.96%	$45.94	($2.00)	34.69%	$9.42	$5.00
$9.00	6.00%	20.00%	50.28%	$56.90	($14.00)	62.49%	$17.32	($7.00)
$3.00	6.00%	30.00%	15.97%	$53.57	$35.00	0.00%	$0.00	$63.00
$4.00	6.00%	30.00%	25.40%	$80.11	$14.00	0.00%	$0.00	$50.00
$5.00	6.00%	30.00%	35.07%	$108.01	($8.00)	0.18%	$0.27	$38.00
$6.00	6.00%	30.00%	44.08%	$136.79	($28.00)	4.19%	$1.90	$26.00
$7.00	6.00%	30.00%	52.62%	$166.18	($51.00)	18.36%	$6.11	$13.00
$8.00	6.00%	30.00%	59.47%	$196.01	($73.00)	44.51%	$12.90	$1.00
$9.00	6.00%	30.00%	65.60%	$226.16	($94.00)	69.61%	$21.87	($12.00)

Note: The analysis assumes the retirement portfolio is invested in assets with the depicted expected real return and volatility. The returns are collared with the purchase of a twelve month put option with a strike price of .95 and the sale of an appropriate call option. The individual retires at age 65 and begins withdrawing funds at the noted initial spending rate per year adjusted for inflation. The analysis ignores taxes and transaction costs. Mortality is modeled with a Gompertz-Makeham fit to the Social Security Administration's 2005 Period Life Table.

Exhibit 10

Costless Collars: Comparison of the Downside Risk for a Traditional and Hedged Strategy 65 Year Old Male and 15% Volatility



Note: The analysis assumes the retirement portfolio is invested in assets with a 6% real return and 15% volatility. The returns are collared with the purchase of a twelve month put option with a strike price of .95 and the sale of an appropriate call option. The individual retires at age 65 and begins withdrawing funds at the noted initial spending rate per year adjusted for inflation. The analysis ignores taxes and transaction costs. Mortality is modeled with a Gompertz-Makeham fit to the Social Security Administration's 2005 Period Life Table.

Exhibit 11

Put Options: Comparison of Traditional and Hedged Strategy for Different Volatilities and Spending Rates in Retirement for a 65 Year Old Male

Initial Spending Rate per $100	Parameters: Expected Return	Parameters: Volatility	Traditional Portfolio: Probability of Ruin	Traditional Portfolio: Downside Risk	Traditional Portfolio: Average RPV	Hedged Portfolio: Probability of Ruin	Hedged Portfolio: Downside Risk	Hedged Portfolio: Average RPV
$3.00	6.00%	10.00%	0.00%	$0.00	$71.00	0.00%	$0.00	$59.00
$4.00	6.00%	10.00%	0.00%	$0.00	$61.00	0.00%	$0.00	$51.00
$5.00	6.00%	10.00%	0.05%	$0.29	$51.00	0.00%	$0.00	$43.00
$6.00	6.00%	10.00%	0.78%	$1.00	$41.00	0.04%	$0.28	$35.00
$7.00	6.00%	10.00%	3.72%	$2.76	$32.00	1.44%	$1.02	$26.00
$8.00	6.00%	10.00%	11.17%	$5.82	$22.00	8.04%	$2.96	$18.00
$9.00	6.00%	10.00%	23.72%	$10.27	$12.00	23.02%	$6.63	$10.00
$3.00	6.00%	15.00%	0.05%	$0.47	$67.00	0.00%	$0.00	$47.00
$4.00	6.00%	15.00%	0.67%	$1.61	$57.00	0.00%	$0.00	$40.00
$5.00	6.00%	15.00%	2.99%	$4.10	$46.00	0.16%	$0.23	$34.00
$6.00	6.00%	15.00%	7.74%	$8.06	$35.00	1.60%	$1.07	$27.00
$7.00	6.00%	15.00%	15.88%	$13.36	$24.00	6.92%	$2.97	$20.00
$8.00	6.00%	15.00%	26.83%	$19.89	$13.00	18.80%	$6.10	$13.00
$9.00	6.00%	15.00%	39.21%	$27.46	$2.00	32.95%	$10.66	$6.00
$3.00	6.00%	20.00%	1.70%	$4.31	$62.00	0.00%	$0.00	$37.00
$4.00	6.00%	20.00%	5.28%	$9.65	$49.00	0.04%	$0.11	$32.00
$5.00	6.00%	20.00%	11.37%	$16.80	$36.00	1.08%	$0.82	$27.00
$6.00	6.00%	20.00%	19.93%	$25.41	$24.00	5.20%	$2.41	$21.00
$7.00	6.00%	20.00%	29.86%	$35.20	$11.00	13.80%	$5.05	$15.00
$8.00	6.00%	20.00%	39.96%	$45.94	($2.00)	26.20%	$8.75	$9.00
$9.00	6.00%	20.00%	50.28%	$56.90	($14.00)	38.34%	$13.52	$3.00
$3.00	6.00%	30.00%	15.97%	$53.57	$35.00	0.20%	$0.14	$25.00
$4.00	6.00%	30.00%	25.40%	$80.11	$14.00	1.44%	$1.03	$21.00
$5.00	6.00%	30.00%	35.07%	$108.01	($8.00)	5.84%	$2.69	$17.00
$6.00	6.00%	30.00%	44.08%	$136.79	($28.00)	14.36%	$5.26	$13.00
$7.00	6.00%	30.00%	52.62%	$166.18	($51.00)	24.92%	$8.67	$8.00
$8.00	6.00%	30.00%	59.47%	$196.01	($73.00)	35.28%	$12.78	$3.00
$9.00	6.00%	30.00%	65.60%	$226.16	($94.00)	44.42%	$17.54	($2.00)

Note: The analysis assumes the retirement portfolio is invested in assets with the depicted expected real return and volatility. The returns are hedged with the purchase of a twelve month put option with a strike price of .95. The individual retires at age 65 and begins withdrawing funds at the noted initial spending rate per year adjusted for inflation. The analysis ignores taxes and transaction costs. Mortality is modeled with a Gompertz-Makeham fit to the Social Security Administration's 2005 Period Life Table.