Credit Ratings: Strategic Issuer Disclosure and Optimal Screening*

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Abstract

We consider a model in which a security issuer can manipulate information observed by a credit rating agency (CRA). We show that stricter screening by the CRA can sometimes lead to increased manipulation by the issuer. Accounting for the issuer's behavior pulls optimal CRA screening towards the extremes of laxness or stringency. Surprisingly, an improvement in prior asset quality can result in more rating errors. In a two-period version of the model, stricter screening can result in more short-run rating errors. Our results suggest complex interplay between issuer and CRA behavior, complicating the evaluation of CRA policy effectiveness.

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1 Introduction

Investors depend on credit ratings to evaluate trillions of dollars of financial instruments, including corporate and government bonds, mortgage-backed securities (MBS), and other asset-backed securities. However, credit rating agencies (CRAs) may lack incentives to screen diligently since issuers pay for ratings – a concern that many commentators have blamed for the failure of MBS ratings precipitating the 2008 financial crisis. Less appreciated is the fact that an issuer produces much of the information that a CRA uses to determine a rating, and the issuer has incentives to manipulate this information in an effort to obtain a more favorable rating. Complicating matters is the possibility that CRA policies and issuer incentives interact: The level of scrutiny applied by a CRA may affect the issuer's gains from manipulating the information on which the CRA relies, and expectations about its effect on issuer behavior may in turn feed back into optimal CRA policy. Thus, a comprehensive evaluation of the effectiveness of CRA diligence and the development of effective vetting policies requires understanding the interaction between the behavior of a CRA and an issuer.

In this paper, we construct a theoretical model to investigate the implications of an issuer's ability to manipulate information used to rate securities. An issuer is looking to sell a claim against an asset that may be of high or low quality. An issuer with a low-quality asset can manipulate the information observed by the CRA, increasing the likelihood of obtaining a high rating. The CRA invests in screening technology that provides a noisy report about the issuer's type. We show that an increase in CRA screening can sometimes strengthen the issuer's incentives to manipulate. More generally, accounting for the issuer's anticipated response causes the CRA to shade screening towards extremes, further weakening optimal screening when it is lax and strengthening it when it is strict. We also show that an improvement in the prior distribution of asset quality strengthens incentives to manipulate to the extent that it can actually increase the incidence of inflated ratings. Finally, we show in a two-period version of the model that more thorough screening can increase rating errors in the short run even as it reduces errors in the long run, implying that patience is required to assess the effectiveness of efforts to improve ratings accuracy.

The asset in our model takes the form of a project for which the issuer requires funding, and the claim issued takes the form of a security that the issuer sells to investors. The issuer privately observes project quality, creating a role for a third-party certifier, the CRA. The CRA observes a signal of project quality and relays that signal to investors through a rating. The accuracy of the CRA's rating depends on actions taken by both the issuer and the CRA. The CRA first chooses a publicly-observable screening intensity at a private cost. If project quality is low, the issuer then chooses an unobserved and costly manipulation intensity. The CRA subsequently observes either a high or low signal of project quality and issues a rating, also high or low, consistent with its signal.[1] If project quality is high, the CRA always observes a high signal. If it is low, the CRA may observe either a high or low signal. A higher issuer manipulation intensity increases the probability that the CRA is successfully fooled and incorrectly observes a high signal. Higher screening intensity attenuates this effect, diminishing the effectiveness of manipulation.

The issuer's expected payoff consists of the expected proceeds from selling a security (net of a commission it pays the CRA) minus any manipulation cost. Given our assumptions about the CRA's signal, a low rating reveals that the project has low quality. A low-quality project has weakly negative NPV, so investors only buy a security if it receives a high rating. Expected proceeds from issuance then equal the product of the probability of receiving a high rating and the price of the security conditional on a high rating. The CRA's payoff is the commission it receives minus the sum of its screening cost and a penalty if its rating turns out to be incorrect.

We begin by fixing the CRA's screening intensity and investigating the issuer's choice of manipulation intensity. Higher screening intensity reduces the probability that a low-quality project obtains a high rating, which weakens incentives to manipulate. However, it also increases the average quality, and hence the price, of a high-rated security, which strengthens incentives to manipulate. The probability that manipulation succeeds and the price of a high-rated security are complements from the issuer's standpoint. Therefore, the former effect dominates when the price of a high-rated security is high, while the latter effect dominates when the probability that manipulation succeeds is high. All else equal, manipulation intensity increases (decreases) with screening intensity when the probability of a high-quality project, manipulation cost, and screening intensity

[1] We abstract away from intentional inflation of ratings by the CRA, in order to focus on the issuer's strategic behavior.

are low (high), as these factors cause a relatively high (low) probability that manipulation succeeds and/or a relatively low (high) price for a high-rated security. The relationship with screening intensity implies that manipulation intensity is an inverted U-shaped function of screening intensity.

Even though manipulation intensity can increase with better screening, the equilibrium incidence of rating errors unambiguously decreases with screening intensity. On the other hand, rating errors can increase with the ex ante probability that project quality is high. The direct effect of an increase in this probability is to reduce rating errors, since high-quality projects are always rated correctly. However, better ex ante project quality also increases the price of a high-rated security, which strengthens incentives to manipulate. Indeed, it can do so to the point where the incidence of erroneously-rated low-quality projects can actually increase. Intuitively, because the pool of high-rated securities includes more high-quality projects, the pool can also absorb more incorrectly-rated low-quality projects without a large drop in price, which would otherwise restrain incentives to manipulate.

Next, we consider the CRA's optimal screening intensity, which takes into account the effect of screening on issuer behavior. When manipulation increases with screening intensity, the CRA optimally reduces screening relative to the case where manipulation is held fixed to avoid exacerbating the issuer's incentives to manipulate. Similarly, when manipulation intensity decreases with screening intensity, the CRA optimally intensifies screening to further deter manipulation. Recall that manipulation is generally an inverted U-shaped function of screening. Putting these facts together, accounting for the effect of screening on issuer behavior tends to pull optimal screening intensity towards the extremes, relaxing screening further when it would already optimally be lax and further strengthening it when it would already be strict. To demonstrate these tendencies, we consider an alternative version of the model where screening is unobserved and hence cannot affect the issuer's choice of manipulation intensity. An implication of our results is that lax screening does not necessarily imply deference to sellers, nor does aggressive screening necessarily imply excessive diligence.

Surprisingly, we find that optimal screening can increase with the prior probability that the issuer's project is high quality. This result obtains despite the fact that a lower probability of a low-quality project reduces the risk of a rating error. It follows directly from the fact that issuer manipulation intensity can increase with the prior probability of a high quality project to the point that the risk of a rating error actually increases. While the CRA increases screening intensity in response to a higher prior probability of a high quality project in this case, it does not do so sufficiently to result in a net decrease in the probability of a rating error because of an increasing marginal cost of screening. Thus, rating errors can increase with the prior probability of a high quality project, even accounting for the CRA's optimal response.[2]

In addition, we show that optimal screening intensity increases with the CRA's commission rate. The probability that the issuer can sell a security backed by a low-quality project decreases with screening intensity. Since the CRA only receives a commission when the issuer sells a security, this reduced probability would appear on the surface to decrease the CRA's payoff. However, a lower probability of successful manipulation also increases the price of a high-rated security, which increases the CRA's commission if a security is issued. The CRA's expected commission is ultimately a fixed fraction of the value created by investment in equilibrium. When the low-quality project is strictly negative NPV, this value, and hence the CRA's commission, strictly decreases with the probability that a low-quality project is funded. Thus, an increase in the CRA's commission rate strengthens incentives to screen.

Finally, we add a second period of issuance to the model in order to evaluate the dynamics of manipulation and screening behavior. The issuer can either be an opportunistic or truthful type, where only the former can manipulate, which creates issuer reputational concerns. The issuer's type is fixed across periods, but project quality is not. We assume that the CRA faces a convex cost of adjusting screening intensity in the second period, capturing the idea that such a change may require redesigning the underlying credit rating model. While rating errors always decrease with screening intensity in the one-period model, we show that first-period rating errors can increase with a higher first-period screening intensity in the two-period model. Intuitively, higher anticipated future screening because of adjustment costs reduces the reputational benefit of not being revealed

[2]Note that the expected quality of a project receiving a high rating and hence being funded still improves in this case. The improvement in the prior probability that the project is high quality more than overcomes the greater incidence of low-quality projects receiving a high rating.

as opportunistic, strengthening incentives to manipulate. This conclusion suggests that efforts to reduce rating errors through more intense screening may produce disappointing results in the short run, even if they are effective at reducing errors in the long run.

Our paper contributes most directly to the literature on the accuracy and usefulness of credit ratings. Research following failures of credit ratings to predict defaults during the financial crisis of 2008 focuses on CRA incentives to inflate ratings, either explicitly or by weakening screening, in order to maximize fee income (Mathis et al., 2009; Bolton et al., 2012; Fulghieri et al., 2014; Frenkel, 2015; Bouvard and Levy, 2018). While these concerns are certainly valid, our paper shifts the focus to issuers' incentives to distort the information on which CRAs base their ratings.[3] We show that CRA screening can either mitigate or exacerbate these incentives. In the latter case, a CRA may optimally limit the stringency of screening, even if the CRA's incentives are well-aligned with those of investors and it could intensify screening at little cost. Endogenous issuer behavior also potentially blunts the effectiveness of regulatory efforts to improve ratings quality by encouraging more thorough screening, and it complicates efforts to learn about the effectiveness of more thorough screening from changes in ratings accuracy, especially in the short run.[4]

To our knowledge, our paper is the first to demonstrate the theoretical ambiguity in the predicted effect of enhanced screening on a certification seeker's incentives to behave deceptively in the context of credit ratings. Others have highlighted similar effects in different settings. For example, research in the accounting literature shows that stricter auditing, perhaps due to greater auditor liability, can increase a firm's incentives to inflate reported performance when raising capital (Hillegeist, 1999; Strobl, 2013). Our paper builds on the general insight in several ways. First, we consider its implications for optimal certifier screening policy and show that accounting for the certification seekers' manipulative incentives tends to make screening that otherwise optimally be lax (strict) even laxer (stricter). Second, we show in the two-period version of our model that increased screening can actually degrade the informativeness of certification in the short run. Third,

[3]Another strand of the literature focuses on inaccuracies due to "ratings shopping," where an issuer can sample multiple CRAs and report only the most favorable ratings (Skreta and Veldkamp, 2009; Sangiorgi and Spatt, 2017).

[4]In a different context, Berk and van Binsbergen (2022) find that tighter regulation to filter out charlatans in the provision of a service can reduce consumer surplus by weakening competition. In our model, *total* surplus can decline with more screening. Daley et al. (2020) study signaling through retention in a setting with ratings.

we demonstrate that endogenous certification-seeker behavior can cause the frequency of certification errors to increase in response to an improvement in the prior distribution of certification-seeker quality.

In the context of CEO compensation, Goldman and Slezak (2006) show that the possibility of manipulation by the agent diminishes the performance sensitivity of optimal contract. In a similar vein, Frankel and Kartik (2022) find that a decision-maker using possibly manipulated data may optimally commit to under-utilizing the data. However, there is no screening in these models and thus no scope for exploring the interaction of manipulation and screening. The analog in our model would be investors committing to relying less on the credit rating when pricing a security, which would unsurprisingly reduce incentives to manipulate. Similarly, Perez-Richet and Skreta (2022) show that an optimal test may reward the agent in some states where the agent supplies negative information in order to diminish incentives to manipulate the information provided. They also show that a falsification-detection technology that devalues more-manipulated signals reduces incentives to manipulate. This technology has a similar effect to the screening technology in our model. The important difference in our model is the connection between issuer payoffs and beliefs about issuer quality conditional on a given rating through a market price, which can cause manipulation to increase with screening.[5]

2 Single-Period Model

We first consider a single-period model, which is our main subject of analysis and produces most of our results. We then extend the model to two periods in the next section to derive additional insights. The model features an issuer who seeks financing for a project of privately-known quality, a CRA who observes a signal of the quality of the issuer's project and rates the security, and investors who can buy a financial claim backed by the project. The model can also be interpreted more generally as a model of third-party certification, with the issuer representing a seller, investors representing a buyer, and the CRA representing a certification agent. We consider other specific

[5]Our paper is also tangentially related to the economics of crime literature, which considers how enforcement responds to incentives (e.g., Tsebelis, 1990).

applications in Section 4.

The issuer's project is high quality (h) with probability η and low quality (ℓ) with probability $1-\eta$. A high-quality project has an NPV $v_h > 0$, whereas a low-quality good has an NPV $v_\ell \leq 0$. The issuer has no financial resources. To undertake the project, it must issue a security backed by the project. If the issuer fails to sell a security to investors, the project becomes worthless.

The issuer privately observes the quality of its project. By providing a credit rating based on its signal, the CRA can reduce the information asymmetry between the issuer and investors. Formally, the game begins with the CRA choosing an observable screening intensity $\alpha \in [0,1]$ at a private cost $c(\alpha)$. The assumption that the CRA's screening intensity is observable is central to our analysis, as it allows us to investigate the effects of screening intensity on issuer behavior. This assumption seems realistic, since CRAs tend to rely on standardized models to produce credit ratings, and, even though these models are proprietary, market participants are likely to be able to infer aspects of a model from observing the history of CRA interactions with issuers and the ratings that they produce.[6] A higher α represents more effort put into vetting information and increases the accuracy of ratings in a way that we make clear shortly. We assume that $c(0) = c'(0) = 0$ and that $c'(\alpha) > 0$ and $c''(\alpha) > 0$ for all feasible $\alpha > 0$. In addition, we assume that there is a screening level $\alpha_{\max}$ arbitrarily close to, but strictly less than, one such that $\lim_{\alpha \to \alpha_{\max}} c'(\alpha) = \infty$.[7]

The issuer can be one of two types, opportunistic (type O) or truthful (type T). The issuer is truthful with probability $\mu \in [0,1)$ and opportunistic with probability $1-\mu$. After observing project quality and the CRA's screening intensity α, an opportunistic issuer chooses an unobserved manipulation intensity $m \in [0,1]$ at a private cost $qk(m)$, where $q \geq 0$, $k(0) = k'(0) = 0$, and, for all $m \in (0,1]$, $k'(m) > 0$ and $k''(m) \geq 0$.[8] We also assume that $k'(m)$, $k''(m)$, and $k'''(m)$ are finite for all feasible m.[9] A truthful issuer cannot manipulate. The presence of the truthful issuer plays

[6]For example, Begley (2016) finds that corporate bond ratings depend heavily on the debt-to-EBITDA ratio of an issuer.

[7]This is a technical assumption used in Lemma 3 below to ensure that the optimal screening intensity remains bounded away from one.

[8]Note that manipulation intensity m being bounded above by 1 has a natural interpretation: Once the issuer's manipulation intensity reaches a certain threshold, the CRA effectively ignores any information provided by the issuer that the CRA cannot independently verify. Beyond this point, further efforts to manipulate have no effect on the outcome of screening.

[9]Assuming that $k'(1) < \infty$ ensures that $qk'(1)$ is well-defined for $q = 0$, allowing us to analyze this case more straightforwardly.

little role in the one-period model but becomes relevant in the two-period model, where reputation affects period-2 outcomes.

The CRA observes a signal $g \in \{g_h, g_\ell\}$. If the project is high quality, the CRA always observes the signal g_h. If the project is low quality and the issuer is the truthful type, the CRA observes the signal g_ℓ. However, if the project is low quality and the issuer is the opportunistic type, the CRA observes g_h with probability $m(1-\alpha)$ and g_ℓ with probability $1-m(1-\alpha)$. The logic behind the multiplicative functional form of these probabilities is that manipulation is less effective when the CRA screens more intensely, and screening is more useful when the issuer manipulates more. Since $\alpha, m \in [0,1]$, the probabilities of observing g_h and g_ℓ lie between 0 and 1. Regardless of the manipulation intensity m, the CRA observes g_h with probability one when the project quality is high. Thus, the opportunistic issuer only manipulates (i.e., potentially chooses $m > 0$) when project quality is low. That is, only an opportunistic issuer with a low-quality project has a strategic choice to make. With a slight abuse of terminology, and for the sake of brevity, we refer to such an issuer as a "low-quality" issuer and use m specifically to refer to a low-quality issuer's manipulation intensity.

After observing its signal, the CRA publicly reports a rating $r \in \{r_h, r_\ell\}$.[10] We assume that the CRA reports its signal faithfully; that is, it reports $r = r_h$ if $g = g_h$ and $r = r_\ell$ if $g = g_\ell$. We do not consider intentional ratings inflation on the part of the CRA, though in equilibrium in our model, the CRA's rating may be inflated on average (relative to the true project quality). The CRA's error rate (i.e., the probability that the CRA's rating disagrees with the true quality of the issuer's project) plays an important role in the analysis that follows. Given the signal structure, the CRA never reports r_ℓ when $v = v_h$. Thus, the error rate is the probability of a high rating

[10]For the sake of simplicity, we model the CRA as always rating the issuer's security and do not allow the issuer to opt out. We could equivalently allow the issuer to opt out but restrict attention to equilibria in which the issuer always applies for a rating. Such equilibria are supported by investors' belief that an issuer who does not apply for a rating has a low-quality project. In such equilibria, an unrated security is not sold, and so an issuer who does not apply for a rating and an issuer who obtains a low rating receive the same payoff of zero. An issuer with a high-quality project has a stronger incentive to apply for a rating than one with a low-quality project. Thus, an equilibrium supported by out-of-equilibrium beliefs that an issuer not obtaining a rating has a high-quality project would not survive the D1 equilibrium refinement.

when project quality is, in fact, low. The error rate is

$$\gamma(\alpha, m) = (1-\mu)(1-\eta)(1-\alpha)m. \tag{1}$$

After the CRA produces its rating, investors potentially buy a security from the issuer. Investors form rational expectations and operate in a perfectly-competitive financial market. As a result, the price of a security equals its expected value given their information. When pricing a security, investors know the potential project payoffs v_h and v_ℓ, the distribution of project quality η, and the distribution of issuer types μ. They also observe the CRA's screening intensity α as well as the CRA's rating r. They do not observe a low-quality issuer's manipulation intensity m. We denote by $\tilde{m}$ their belief about the value of m given their information. Since investors and the CRA share the same information, we assume that they both form the same belief $\tilde{m}$. In equilibrium, of course, this belief must match a low-quality issuer's actual manipulation intensity.

By construction, a low rating reveals that the project has low quality. Since a low-quality project is weakly negative NPV (recall that $v_\ell \leq 0$), a low-rated security is never sold.[11] A high-rated security is sold at a price $p(\alpha, \tilde{m}) > 0$. Note that p is a function of $\tilde{m}$ (investors' beliefs about manipulation by the issuer) and not m (the actual manipulation intensity of a low-quality issuer), since investors do not observe m. If the issuer sells a security, it pays a fraction $\phi \in [0, 1)$ of the selling price to the CRA as a commission. The parameter ϕ represents the bargaining power of the CRA relative to the issuer.

The game unfolds in the following stages:

1. The issuer draws and observes its truthfulness type (O or T).

2. Project quality $v \in \{v_h, v_\ell\}$ is realized and privately observed by the issuer.

3. The CRA chooses a screening intensity α.

4. If the issuer is opportunistic and has a low-quality project (i.e., is a low-quality issuer), it chooses a manipulation intensity m.

[11]Technically, if the low-quality project is zero-NPV, investors would be willing to buy a low-rated security at a price of zero. Since neither the seller nor investors benefit from a sale in this case, we ignore this possibility.

5. The CRA observes its signal g and issues a rating r.

6. If the rating is high ($r = r_h$), investors purchase the security at a price p at which they break even given their posterior beliefs, and the issuer pays a commission ϕp to the CRA. If the rating is low ($r = r_\ell$), the issuer does not sell the security, and the game ends.

7. Project cash flow is revealed.

We refer to stages 4 through 7 as the *issuer's continuation game* and the game unfolding across all 7 stages as the *overall game*.

2.1 Payoffs

Recall that the issuer only has a strategic decision to make if it is a low-quality issuer. The expected payoff to the issuer in this case is a function of three components. The first is the price it obtains if it sells a security, net of the commission it pays to the CRA. The second is the cost of manipulation. The third is a continuation payoff, which depends on whether it *succeeds* in obtaining a high rating or *fails* (i.e., it receives a low rating). We refer to these two "states" as s and f. Let π_s and π_f denote the continuation payoffs in states s and f, respectively. For now, we take π_s and π_f to be exogenous. These continuation payoffs play little role in the analysis in this section. However, they do play an important role in the two-period model in Section 3, where we endogenize them. We include them here as exogenous variables for the sake of continuity. Formally, a low-quality issuer's expected payoff is

$$\begin{aligned}\Pi(\alpha, m) &= (1-\alpha)m[(1-\phi)p + \pi_s] + \{1-(1-\alpha)m\}\pi_f - qk(m) \\ &= \pi_f + (1-\alpha)m[(1-\phi)p(\alpha, \tilde{m}) - \Delta] - qk(m), \qquad (2)\end{aligned}$$

where $\Delta = \pi_f - \pi_s$ is the difference in the opportunistic issuer's continuation payoffs between the failure and success states. We make the following assumption about Δ, which allows us to focus on the interesting case where equilibrium manipulation intensity is not always maximal (i.e., one) or always minimal (i.e., zero).

Assumption 1. $v_\ell < \frac{\Delta}{1-\phi} < v_h$.

Note that this range includes 0 when $v_\ell < 0$. When v_ℓ is strictly negative, Δ too may be negative. However, in the two-period model in Section 3, we show that a positive Δ emerges naturally, since successful manipulation in period 1 reveals the issuer to be the opportunistic type, which reduces the issuer's period-2 payoffs.

The CRA's payoff is a function of four components. First, the CRA derives a fixed benefit $\Lambda > 0$ from the exercise of rating the issuer. This benefit may be thought of as a combination of a fixed fee and a reputational benefit the CRA earns from being in the rating business. We assume that Λ is high enough to ensure that the CRA's equilibrium expected payoff is non-negative. The second component is the commission ϕp paid by the issuer if it sells a security. Third, screening entails a cost $c(\alpha)$. Fourth, the CRA incurs a cost $\lambda > 0$ whenever it issues an erroneous rating. This cost represents a combination of regulatory, litigation, and reputational costs. Formally, the CRA's expected payoff is

$$\Psi(\alpha, m) = \Lambda + [\eta + \gamma(\alpha, m)]\phi p(\alpha, \tilde{m}) - c(\alpha) - \lambda\gamma(\alpha, m). \tag{3}$$

2.2 Equilibrium

A Perfect Bayesian Equilibrium (PBE) of the game is given by an optimal screening intensity for the CRA, α^*, a best response function for a low-quality issuer, $m^*(\alpha)$, and a manipulation conjecture for the CRA and investors, $\tilde{m}(\alpha)$, such that:

(i) For the CRA, α^* maximizes $\Psi(\alpha, m^*(\alpha))$, subject to $0 \leq \alpha \leq \alpha_{\max}$.

(ii) For a low-quality issuer, for any $\alpha \in [0, \alpha_{\max}]$, $m^*(\alpha)$ maximizes $\Pi(\alpha, m)$ subject to $0 \leq m \leq 1$.

(iii) Investors' and the CRA's conjecture of a low-quality issuer's manipulation intensity is correct; that is, $\tilde{m}(\alpha) = m^*(\alpha)$ for any $\alpha \in [0, \alpha_{\max}]$.

(iv) Investors break even in expectation given their information; that is, $p(\alpha, \tilde{m}) = \mathbb{E}[v \,|\, r_h, \alpha, \tilde{m}(\alpha)]$ for any $\alpha \in [0, \alpha_{\max}]$, where the posterior probability that the project is high quality given r_h, α, and $\tilde{m}(\alpha)$ is determined by Bayes' rule.

Given (ii), (iii), and (iv), the equilibrium price of a high-rated security for a given value of α must satisfy

$$p(\alpha, m^*(\alpha)) = \frac{\eta v_h + \gamma(\alpha, m^*(\alpha))v_\ell}{\eta + \gamma(\alpha, m^*(\alpha))}. \tag{4}$$

We first characterize the equilibrium in the issuer's continuation game, taking the CRA's choice of screening intensity α as given. This equilibrium is characterized by $m^*(\alpha)$. We then analyze the equilibrium of the overall game. This equilibrium is characterized by $(\alpha^*, m^*(\alpha^*))$.

2.3 Equilibrium in the issuer's continuation game

We first fix $\alpha \in [0, \alpha_{\max}]$ and characterize the issuer's equilibrium manipulation intensity at stage 4. A low-quality issuer chooses m to maximize its expected payoff Π as defined in equation (2), given an observed α. We begin by establishing that, holding α fixed, the equilibrium manipulation intensity $m^*(\alpha)$ at stage 4 is unique. If $m^*(\alpha) \in (0, 1)$, then the issuer's first-order condition implies that $\frac{\partial \Pi}{\partial m} = 0$ or, equivalently, that

$$(1 - \alpha)[(1 - \phi)p(\alpha, \tilde{m}) - \Delta] - qk'(m) = 0. \tag{5}$$

It is straightforward to see that the second-order condition is satisfied. Since investors' conjecture must match the actual manipulation intensity, we can write the equilibrium condition when $m^*(\alpha) \in (0, 1)$ as

$$(1 - \alpha)\left[(1 - \phi)p(\alpha, m^*(\alpha)) - \Delta\right] - qk'(m^*(\alpha)) = 0. \tag{6}$$

Fixing the screening intensity α, the equilibrium manipulation intensity, $m^*(\alpha)$, is unique. Further, when $m^*(\alpha) \in (0, 1)$, $m^*(\alpha)$ is strictly decreasing in the manipulation cost parameter q.

Lemma 1. *Fix $\alpha \le \alpha_{\max}$ and consider the continuation game starting at stage 4. Then:*

(i) The equilibrium manipulation intensity, $m^*(\alpha)$, is uniquely determined; there exists a threshold manipulation cost parameter $\underline{q}$ (possibly negative) such that $m^*(\alpha) = 1$ if $q \le \underline{q}$ and $m^*(\alpha) \in (0, 1)$ if $q > \underline{q}$.

(ii) For $q > \underline{q}$, the equilibrium manipulation intensity $m^*(\alpha)$ is strictly decreasing in the manipulation cost parameter q.

The threshold $\underline{q}$ is explicitly defined in equation (23) in the proof of the lemma and depends on the screening intensity α and the exogenous parameters. The threshold is increasing in η and in α. Our assumption that $k'(0) = 0$ ensures that $m^* > 0$ in equilibrium.

2.3.1 Comparative statics of m^* with respect to α

Consider the effects of a small increase in the screening intensity α. One might naturally expect that greater screening intensity would discourage manipulation by an issuer. However, we show that the optimal manipulation intensity can actually increase in α if the prior probability that a project is high quality (η), screening intensity (α), and the cost of manipulation (q) are all sufficiently small.

As a preliminary step, we first show that the optimal manipulation intensity can increase in α when the (endogenous) rating error rate is sufficiently high.

Lemma 2. *Fix $\alpha \leq \alpha_{\max}$ and suppose that in the equilibrium of the continuation game $m^*(\alpha) \in (0, 1)$. Then, $\frac{dm^*}{d\alpha} > 0$ if and only if $\gamma(\alpha, m^*(\alpha)) > \bar{\gamma} \equiv \eta \left(\sqrt{1 + \frac{v_h - \frac{\Delta}{1-\phi}}{\frac{\Delta}{1-\phi} - v_\ell}} - 1 \right)$.*

Intuitively, an increase in the screening intensity α has two effects on the issuer's incentives, one direct and one indirect. Directly, an increase in α reduces the probability that a low-quality issuer obtains a high rating, which reduces the marginal benefit of manipulation by $(1 - \phi)p - \Delta$. However, more intense screening also increases the price of the high-rated security by decreasing the rating error rate. This indirect price effect increases the marginal benefit of manipulation by $(1 - \alpha)(1 - \phi)\frac{\partial p}{\partial \alpha}$. It is easy to see from equation (4) that the equilibrium price is low when the equilibrium rating error rate is high.[12] As a result, the direct effect is small, and the indirect effect dominates, causing manipulation to increase with screening intensity. Intuitively, when investors

[12] The ability to affect the equilibrium price through manipulation may be interpreted as an incentive for the opportunistic issuer with a low-quality project to manipulate. In a somewhat different setting with moral hazard, Goldman and Slezak (2006) show that high-powered incentives may induce a manager to misrepresent information.

are skeptical about the informativeness of a high rating, the benefit of receiving a high rating is small, so a change in the probability of receiving a high rating is second order.

We now completely characterize the region of the parameter space in which $\frac{dm^*}{d\alpha} > 0$. Necessary and sufficient conditions are (i) $m^* \in (0,1)$ and (ii) $\gamma(\alpha, m^*(\alpha)) > \bar{\gamma}$. The following proposition characterizes the parameter region for which these conditions hold.

Proposition 1. *There exist thresholds $\bar{\eta} > 0$, $\bar{\alpha}(\eta) > 0$, and $\bar{q}(\eta, \alpha) > 0$ such that the equilibrium manipulation intensity $m^*(\alpha)$ is strictly increasing in the screening intensity α if and only if $\eta < \bar{\eta}$, $\alpha < \bar{\alpha}(\eta)$, and $q \in (\underline{q}(\eta, \alpha), \bar{q}(\eta, \alpha))$.*

The thresholds $\bar{\eta}$, $\bar{\alpha}(\eta)$, and $\bar{q}(\eta, \alpha)$ are all defined explicitly in the proof of the proposition. Intuitively, the rating error rate is large (and hence the condition in Lemma 2 is satisfied) when the ex ante probability that the project is low-quality is high, the cost of manipulating is low (and hence manipulation intensity is high), and the CRA's screening is lax. The condition $q > \underline{q}$ is necessary to ensure that $m^*(\alpha) < 1$. Note that this threshold is the same as the threshold $\underline{q}$ in Lemma 2, though we write it as a function of η and α in Proposition 1 to make its dependence on these parameters clear. The proof also demonstrates that when $\eta < \bar{\eta}$, we have $\bar{\alpha}(\eta) > 0$. Further, when $\eta < \bar{\eta}$ and $\alpha < \bar{\alpha}(\eta)$, the threshold $\bar{q}(\eta, \alpha)$ strictly exceeds $\underline{q}(\eta, \alpha)$. Thus, the set of parameters at which m^* is strictly increasing in α is non-empty.

2.3.2 Comparative statics of γ with respect to α, ϕ, and η

Proposition 1 shows that the issuer may manipulate with greater intensity in response to an increase in the CRA's screening intensity. Because of this countervailing force, it is possible in principle that the accuracy of ratings *decreases* rather than increases in the CRA's screening intensity. However, we show in the next proposition that this is never the case – the rating error rate always at least weakly decreases in screening intensity. In contrast, we show in Section 3 that when the model is extended to two periods, more intense screening may indeed result in less accurate ratings in the short run.

Proposition 2. *Fix the screening intensity $\alpha \le \alpha_{\max}$ and consider the continuation game starting at stage 4. Then,*

(i) If $q = 0$ and $\alpha < 1 - \frac{\eta}{(1-\mu)(1-\eta)} \left(\frac{v_h - \frac{\Delta}{1-\phi}}{\frac{\Delta}{1-\phi} - v_\ell} \right)$, the equilibrium error rate $\gamma(\alpha, m^*(\alpha))$ is invariant to the screening intensity α.

(ii) In all other cases, $\gamma(\alpha, m^*(\alpha))$ strictly decreases in α.

First, suppose that the conditions in case (i) are satisfied. The condition on α ensures that $m^* < 1$. As seen from equation (6), when $q = 0$, the equilibrium manipulation intensity is pinned down by the condition $p(\alpha, m^*(\alpha)) = \frac{\Delta}{1-\phi}$. Since the right-hand side is constant, the manipulation intensity $m^*(\alpha)$ must adjust following a small change in screening intensity α in a manner that keeps the price of a high-rated security, p, constant. This invariance further requires that the rating error rate γ be invariant as α changes since, holding fixed the parameters η and μ, the price depends solely on the error rate. That is, manipulation intensity increases exactly enough to offset the effect of more intense screening on the error rate in this case.

In case (ii) of the proposition, there are two possible scenarios. First, it may be that $m^*(\alpha) = 1$. In this case, a small change in α leaves m^* unchanged, so the only effect of an increase in α is a direct reduction in the rating error rate. The other scenario is that $m^*(\alpha) \in (0, 1)$ and $q > 0$. Suppose, in contradiction to the statement in part (ii) of Proposition 2, that the error rate increases with the CRA's screening intensity. Then, the price of the high-rated security must strictly decrease with screening intensity. In addition, more intense screening reduces the marginal effect of an increase in manipulation intensity on the probability of successful manipulation. Both effects weaken the issuer's incentive to manipulate, reducing equilibrium manipulation intensity. However, if screening intensity increases and manipulation intensity decreases, then the error rate must decrease, which contradicts the assumption that the error rate is increasing. Nevertheless, part (i) of the proposition and continuity imply that the relationship may be close to flat when q is small.

Next, consider the comparative statics of the error rate $\gamma(\alpha, m^*(\alpha))$ with respect to the CRA's commission rate, ϕ, and the quality of projects in the economy, as measured by η, when $m^* \in (0, 1)$. As ϕ increases, the issuer shares more of the proceeds from selling a security with the CRA, so the benefit to an issuer of obtaining a high rating decreases, leading to less manipulation in equilibrium. Keeping the manipulation intensity m constant, an increase in η leads to a decrease in the error rate since only low-quality projects are subject to possible rating errors. However, a lower error rate leads to an increase in the price of a high-rated security, which strengthens a low-quality issuer's incentives to manipulate, and increased manipulation tends to increase the error rate. The overall change in the error rate depends on which of these two effects dominates. As we show in part (ii) of the following proposition, when the manipulation cost is low, the second effect dominates, so that the equilibrium error rate in fact increases as η increases.

Proposition 3. *Fix the screening intensity $\alpha \leq \alpha_{\max}$ and suppose that $q > \underline{q}$. Then:*

(i) The error rate $\gamma(\alpha, m^*(\alpha))$ is strictly decreasing in the CRA's commission rate ϕ.

(ii) There exists a threshold manipulation cost parameter $\bar{q}_\eta$ such that, if $q \in (\underline{q}, \bar{q}_\eta)$, the error rate $\gamma(\alpha, m^*(\alpha))$ is strictly increasing in the prior project quality η.

The threshold $\bar{q}_\eta$ in part (ii) of the proposition is explicitly defined in equation (43) in the proof. Comparing the expression to the expression for $\underline{q}$ as defined in equation (23), it can be shown that, holding the other parameters fixed, $\bar{q}_\eta > \underline{q}$ when the prior project quality η is sufficiently low, with the converse holding when η is high.

The intuition for part (ii) of Proposition 3 is as follows. Holding α fixed, the equilibrium error rate in the continuation game is $\gamma(\alpha, m^*(\alpha)) = (1-\mu)(1-\alpha)(1-\eta)m^*(\alpha)$. Here, the last two terms depend on the prior asset quality η. An increase in η causes $m^*(\alpha)$ to increase more rapidly than $1-\eta$ decreases when q is small. To see why this is the case, observe that we can rewrite the equilibrium price of a high-rated security as

$$p(\alpha, m^*(\alpha)) = \frac{\frac{\eta}{1-\eta} v_h + (1-\mu)(1-\alpha)m^*(\alpha)v_\ell}{\frac{\eta}{1-\eta} + (1-\mu)(1-\alpha)m^*(\alpha)}. \tag{7}$$

From equation (7), it is apparent that the price depends on the ratio of high-quality to low-quality projects, $\frac{\eta}{1-\eta}$, which increases with η at a rate greater than one. Intuitively, an increase in η simultaneously causes the probability that the project is high quality to increase *and* the probability that the project is low quality to decrease, and both of these effects lead to an increase in price. When q is small, the sharp increase in price with η causes the issuer's optimal manipulation intensity to increase sharply with η as well, and the error rate increases. However, the increased incentive to manipulate is tempered when q is large, and the error rate may therefore decrease with η when q is large; hence, the upper threshold $\bar{q}_\eta$ in part (ii) of Proposition 3.

2.4 Equilibrium in the overall game

To this point, we have treated the screening intensity α as exogenous and analyzed the continuation game in which a low-quality issuer chooses its manipulation intensity m. We next characterize the equilibrium of the overall game, where the CRA chooses α to maximize its expected payoff Ψ as given by equation (3). In a PBE, in making its choice the CRA accounts for the effect of α on the manipulation intensity m^*. Using the expression for the equilibrium price $p(\alpha, m^*(\alpha))$ in equation (4), we can rewrite the CRA's expected payoff in equation (3) as

$$\Psi(\alpha, m^*(\alpha)) = \Lambda + \phi[\eta v_h + \gamma(\alpha, m^*(\alpha))v_\ell] - \gamma(\alpha, m^*(\alpha))\lambda - c(\alpha). \tag{8}$$

Writing the CRA's expected payoff in this form provides a useful insight. The expected value created by investment in equilibrium is $\eta v_h + \gamma(\alpha, m^*(\alpha))v_\ell$. When $v_\ell < 0$, only high-quality projects are funded in the first-best outcome. The CRA receives a commission whenever the issuer sells a security, regardless of the security's true quality. However, the CRA accounts for the effect of its screening intensity on the average price of securities being sold, which decreases when the error rate is higher. Since the CRA's commission is effectively a fixed fraction of expected value created, the CRA fully accounts for the value destroyed by lower-quality projects being funded in equilibrium, $\gamma(\alpha, m^*(\alpha))v_\ell$.

2.4.1 Existence of equilibrium in the overall game

We first verify existence of an equilibrium in the overall game. Taking the derivative of the CRA's expected payoff with respect to the screening intensity α, we have

$$\Psi'(\alpha, m^*(\alpha)) = -(\lambda - \phi v_\ell)\frac{d\gamma}{d\alpha} - c'(\alpha). \tag{9}$$

Since $\frac{d\gamma}{d\alpha} \leq 0$ by Proposition 2 and $v_\ell \leq 0$ by assumption, the term $-(\lambda - \phi v_\ell)\frac{d\gamma}{d\alpha}$ is weakly positive. This expression reflects the benefits to the CRA of a small increase in screening intensity due to a lower expected error penalty and a higher expected commission. The CRA trades these benefits off against the higher cost of screening, as reflected in $c'(\alpha)$.

We focus on pure strategy equilibria.[13] From Lemma 1, an equilibrium may involve $m^*(\alpha) \in (0,1)$ or $m^*(\alpha) = 1$. In principle, it is also possible that the CRA's problem has multiple local maxima involving different values of $m^*(\alpha) \in (0,1)$. We rule this possibility out to make the problem well behaved by assuming that the CRA's screening cost function $c(\alpha)$ is sufficiently convex, which ensures that the CRA's problem is globally concave in α.

Lemma 3. *There exists a pure strategy equilibrium $(\alpha^*, m^*(\alpha^*))$. Further, there is at most one equilibrium with $m^*(\alpha^*) = 1$, and, if $c''(\alpha) > \underline{c}$ for all $\alpha \in [0, \alpha_{\max}]$ (where $\underline{c}$ is a constant defined in the proof), at most one equilibrium with $m^*(\alpha^*) \in (0,1)$.*

The proof of Lemma 3 uses the fact that $\alpha_{\max} < 1$ (instead, if α can approach one, the marginal benefit of screening can potentially become large). For the rest of this section, we assume that the conditions of Lemma 3 are satisfied, so that there is at most one equilibrium with $m^*(\alpha^*) \in (0,1)$ and at most one with $m^*(\alpha^*) = 1$.

As Lemma 3 suggests, there can be two local maxima of the CRA's payoff function, say α_A with $m^*(\alpha_A) = 1$ and α_B with $m^*(\alpha_B) \in (0,1)$. Thus, in principle, there can be up to two pure-strategy equilibria in the game. However, cases where $\Psi(\alpha_A) = \Psi(\alpha_B)$ are knife-edge cases. That is, starting with any set of parameter values for which there are two equilibria, a small perturbation in any

[13] As the screening intensity α is observed by the issuer, there is no benefit to the CRA from mixing.

of η, v_h, v_ℓ, λ, ϕ or Δ leads to one local maximum being greater than the other one. Thus, the equilibrium is generically unique (i.e., unique except for a measure zero set of parameter values).

2.4.2 Comparative statics of equilibrium in the overall game

We next consider the comparative statics of the CRA's optimal screening intensity α in the overall game. Suppose that, in an equilibrium of the overall game, we have $m^*(\alpha^*) \in (0,1)$. We show that optimal screening intensity α^* generally increases with the CRA's commission rate ϕ and can increase with the probability that project quality is high, η.

Proposition 4. *Suppose that, in an equilibrium of the overall game, $m^* \in (0,1)$. Then:*

(i) If $v_\ell < 0$ and $q > 0$, the optimal screening intensity α^* is strictly increasing in the CRA's commission rate ϕ, whereas if $v_\ell = 0$ or $q = 0$, then α^* is invariant to ϕ.

(ii) There exists a threshold manipulation cost $\hat{q} > 0$ such that, if $q < \hat{q}$, the optimal screening intensity α^* increases in η.

Consider first the effect of a small increase in ϕ, the CRA's commission as a fraction of the price of any security sold. One might naturally expect an increase in ϕ to weaken the CRA's incentives to screen since more intense screening reduces the probability that the security receives a high rating and therefore that the CRA collects a fee. However, as noted after equation (8), the CRA's expected commission when choosing α increases with the expected value created by investment since the CRA internalizes the effect of screening on the price of a high-rated security. Expected value created decreases with the rating error rate, so increasing the CRA's commission rate has the same effect as punishing the CRA more for a rating error.

Next, consider the effect of a small increase in the quality of projects in the economy, as reflected by the parameter η, on the equilibrium of the overall game. One might naturally intuit that an increase in η results in lower screening intensity since there are fewer low-quality projects for the CRA to screen out through the rating process. However, we show that optimal screening intensity can, in fact, increase with η. That is, optimal screening increases precisely when it would appear there is less need for it. The intuition follows directly from Proposition 3 part (ii). When q is low, the error rate increases with η, holding α fixed. As a result, the CRA optimally screens more intensely.

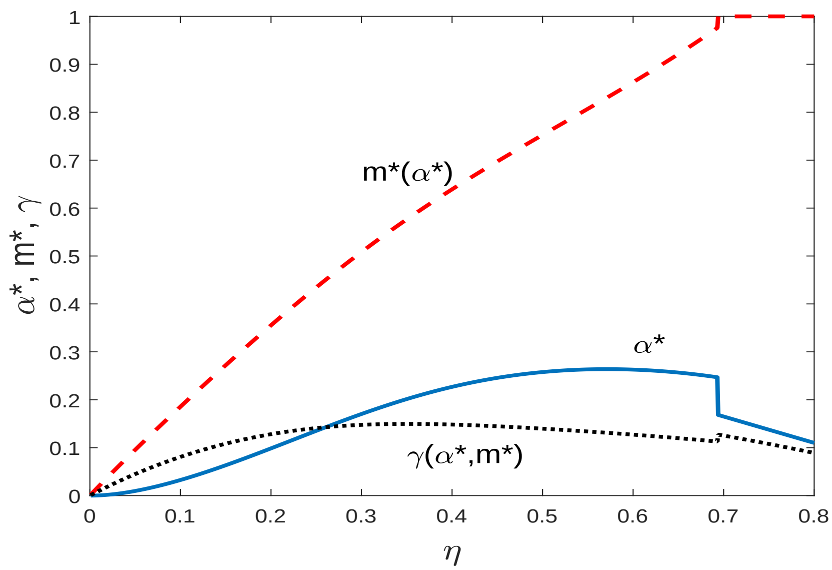
Proposition 4 assumes that, in the equilibrium of the overall game, $m^* \in (0,1)$. From Lemma 1 part (i), a sufficient condition to ensure that m^* is in the interior is that $q > \underline{q}$. As $\underline{q}$ increases in η and α^*, the condition is more likely to be satisfied when η is low and α^* is low (i.e., the cost function $c(\alpha)$ is steep). Of course, the optimal screening intensity may increase (decrease) discontinuously in η if there is a change from an equilibrium in which $m^* \in (0,1)$ ($m^* = 1$) to one in which $m^* = 1$ ($m^* \in (0,1)$).

2.5 Numerical example

We now consider a numerical example to illustrate equilibrium behavior in the overall game. We make two points in the example: (i) The error rate γ can increase in prior project quality η, and (ii) the issuer's response to a change in screening intensity pulls the CRA's optimal screening policy towards the extremes — that is, because the CRA anticipates the issuer's response, it screens less intensely when α is optimally low and more intensely when α is optimally high. We illustrate this second point by comparing optimal screening intensity to optimal intensity in an alternative version of the model where the issuer does not observe screening intensity and therefore cannot respond to it.

We set $v_h = 1, v_\ell = -1, \phi = 0.1, \Delta = 0, \mu = 0.5, qk(m) = 0.25m^2, \lambda = 1$, and $c(\alpha) = 0.5\alpha^2$. Although the marginal cost of screening is finite for all $\alpha \leq 1$, the optimal value of α remains strictly below 1 for all parameter values we consider. Figure 1 shows the optimal values of α^*, $m^*(\alpha^*)$ and the error rate $\gamma(\alpha^*, m^*(\alpha^*))$ as η varies.

For many values of η, the CRA's payoff function Ψ has two local maxima, one with $m^* \in (0,1)$ and one with $m^* = 1$. At $\eta \approx 0.69$, the two local maxima are almost equal; for $\eta < 0.69$, the local maximum with $m^* \in (0,1)$ is the global maximum of Ψ, whereas for $\eta > 0.69$, the local maximum with $m^* = 1$ is the global maximum. As a result, at $\eta \approx 0.69$, there is a discontinuous jump in α^*. For values of η just below 0.69, m^* decreases with α, and the CRA chooses a relatively high



This figure shows the optimal screening intensity of the CRA (α^*) and manipulation intensity of the issuer (m^*), and the equilibrium error rate ($\gamma(\alpha^*, m^*(\alpha^*))$ as the prior project quality (η) varies. The parameters are $v_h = 1, v_\ell = -1, \phi = 0.1, \Delta = 0, \mu = 0.5, qk(m) = 0.25m^2, \lambda = 1$, and $c(\alpha) = 0.5\alpha^2$.

Figure 1: **Effect of varying prior project quality** η

level of screening intensity to discourage manipulation. Around $\eta = 0.69$, equilibrium manipulation intensity becomes sufficiently close to 1 that the CRA chooses to economize on screening intensity and allow manipulation intensity to reach its maximum. The equilibrium outcome thus has the flavor of an "arms race" (see, e.g., Baliga and Sjöström, 2004), with one optimum featuring low screening and low manipulation, and the other high screening and high manipulation.

Equilibrium error rate

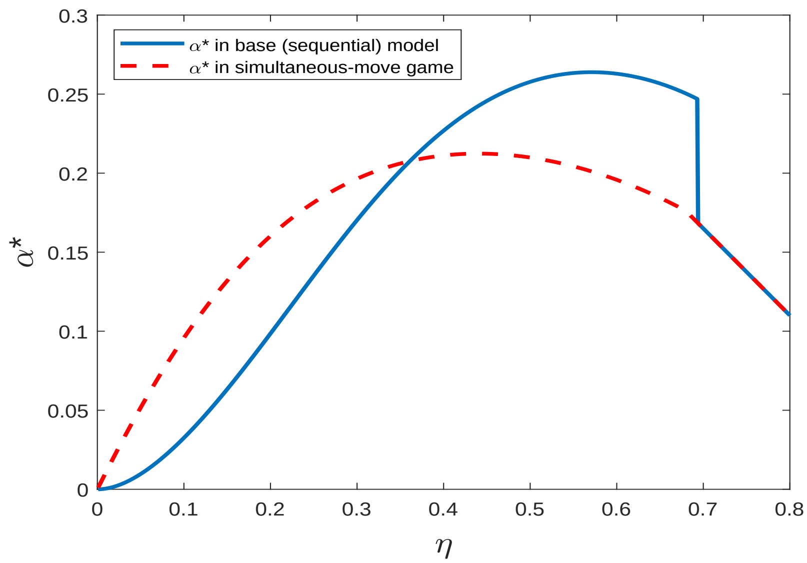
Consistent with the intuition of Proposition 4, for $\eta \in (0, 0.69)$, the optimal screening intensity α^* increases in η. In this region, the manipulation intensity m^* increases sharply in η, and the CRA responds by increasing its screening intensity even though the underlying project quality is higher. In fact, in the region $\eta \in (0, 0.35)$, the manipulation intensity increases in η so sharply that even after taking into account the greater screening intensity α^*, the overall error rate increases in η. That is, in equilibrium a better overall quality in the prior pool leads to more low-quality projects being financed. Note that this does not imply that the average quality of projects being financed declines, since there are now proportionately more high-quality projects in existence.

Note the difference between this result and the result in part (ii) of Proposition 3. In Proposition 3, α is held fixed. Here, we allow the CRA to choose α optimally. The intuition for this result is as follows. As part (ii) of Proposition 3 shows, the issuer's manipulation incentives increase sufficiently sharply with η when q is relatively small that the rating error rate increases as η increases (holding α fixed). As Proposition 4 shows, the CRA optimally increases its screening intensity as η increases to counter the effect of the increased manipulation intensity. However, the former effect dominates. As a result, the equilibrium error rate increases with a small increase in η as long as q is not too large.

Comparison between observed and unobserved α

To see how the CRA's screening intensity α affects the issuer, it is useful to consider an alternative variant of the model in which screening intensity α is unobserved. The issuer and investors both form beliefs about CRA screening intensity, and these beliefs must be correct in equilibrium. However, because α is unobserved, the issuer's manipulation intensity decision does not change as the actual value of α changes. The equilibrium in this game represents a fixed point, with the equilibrium value of m representing the issuer's best response given the CRA's equilibrium choice of α, and the equilibrium value of α representing the CRA's best response given the issuer's equilibrium choice of m. Figure 2 shows the equilibrium screening intensities in the base model and in the corresponding game with unobserved screening intensity.

As shown in the figure, when the prior project quality η is low (i.e., for $\eta < 0.35$), optimal screening is higher when α is unobserved than in the base model. Thus, observability of screening intensity in this case dampens the CRA's optimal screening intensity. This result parallels the finding of Goldman and Slezak (2006) that manipulation by the agent leads to flatter incentives, and is also similar to that in Frankel and Kartik (2022). For higher values of η (i.e., $\eta > 0.35$), the opposite happens — observability of screening intensity increases the CRA's optimal screening intensity. Since optimal screening intensity tends to be high when η is high, observability of screening intensity pulls the CRA's optimal screening intensity towards the extremes. The intuition follows



This figure shows the optimal screening intensity of the CRA (α^*) and manipulation intensity of the issuer (m^*), and the equilibrium error rate ($\gamma(\alpha^*, m^*(\alpha^*))$ as the prior project quality (η) varies. The parameters are $v_h = 1, v_\ell = -1, \phi = 0.1, \Delta = 0, \mu = 0.5, qk(m) = 0.25m^2, \lambda = 1$, and $c(\alpha) = 0.5\alpha^2$.

Figure 2: **Equilibrium screening intensity in sequential- and simultaneous-move games**

from Proposition 1, which shows that manipulation increases (decreases) with observed screening intensity when η is small (large). The CRA curbs screening intensity when η is small to avoid encouraging manipulation. In contrast, the CRA amplifies screening to discourage manipulation when η is large.

3 Two-Period Model

In the single-period model of the previous section, the equilibrium manipulation intensity m^* sometimes increases in the screening intensity α (Proposition 1), but the error rate $\gamma(\alpha, m^*(\alpha))$ is always at least weakly decreasing in α (Proposition 2). We now show that when the model is extended to two periods, the equilibrium error rate in period 1 may also *increase* with the screening intensity.

Consider a two-period version of the model described in Section 2. The sequence of events in each period is identical to the sequence of events in the single-period model. The game begins with the issuer drawing and observing its truthfulness type (O or T), which persists across periods. For $t = 1, 2$, let $v_t \in \{v_h, v_\ell\}$ denote project quality in period t, which is assumed to be independent across periods. Let α_t and m_t denote the CRA's period-t choice of screening intensity and the opportunistic issuer's period-t choice of manipulation intensity, respectively, and let $\tilde{m}_t$ denote investors' and the CRA's belief about the value of m_t. Let g_t denote the CRA's signal in period t and r_t its rating in period t. Finally, let μ_t denote investors' and the CRA's belief about the probability that the issuer's type is truthful entering period t, with μ_1 being a primitive of the model but μ_2 emerging endogenously.

All agents are risk-neutral and value period-2 cash flows using a common "discount" factor $\delta \geq 0$. We allow for the possibility that $\delta > 1$, in which case period-2 payoffs may be interpreted as a reduced-form way to capture the present value of all payoffs beyond period 1 in a game that extends for many periods. We assume that the CRA faces an adjustment cost to changing screening intensity from period 1 to period 2, $\beta(\alpha_2 - \alpha_1)^2$, where $\beta \geq 0$ is a constant. The adjustment cost captures the idea that the CRA has a standard screening process, and departing from that process is costly because it requires changing procedures and/or ratings models.

There are four decision points in the game. In turn, these are: at time 1, (i) the CRA chooses the screening intensity α_1, and (ii) an opportunistic issuer with a low-quality period-1 project chooses the manipulation intensity m_1, and, at time 2, (iii) the CRA chooses the screening intensity α_2, and (iv) an opportunistic issuer with a low-quality period-2 project chooses the m_2. The CRA's choices are publicly observed, whereas the issuer's choices remain unobserved by the CRA and investors.

We start by characterizing the last choice, m_2, and then work backwards. Let $\gamma(\mu, \alpha, m)$ and $p(\mu, \alpha, m)$ denote functions mapping a generic prior μ over the issuer's type, CRA screening intensity α, and beliefs over manipulation intensity m to a rating error rate and the market price of a high-rated security, respectively. These functions are given by

$$\gamma(\mu, \alpha, m) = (1 - \eta)(1 - \mu)(1 - \alpha)m, \tag{10}$$

$$p(\mu, \alpha, m) = \frac{\eta v_h + \gamma(\mu, \alpha, m)v_\ell}{\eta + \gamma(\mu, \alpha, m)}. \tag{11}$$

There is no continuation value beyond period 2. Therefore, an opportunistic issuer with a low-quality period-2 project chooses m_2 to maximize the period-2 expected payoff given by

$$\pi_2 = (1-\alpha_2)\, m_2 \times (1-\phi)\, p(\mu_2, \alpha_2, \tilde{m}_2) - q\, k(m_2), \tag{12}$$

subject to $0 \leq m_2 \leq 1$. Let $m_2^*(\mu_2, \alpha_2)$ denote the solution to the opportunistic issuer's period-2 problem. In equilibrium, it must be that $\tilde{m}_2 = m_2^*(\mu_2, \alpha_2)$.

Prior to the opportunistic issuer's choice of m_2, the CRA chooses α_2, its observable period-2 screening intensity, to maximize

$$\begin{aligned}\psi_2 = {} & [\eta + \gamma(\mu_2, \alpha_2, m_2^*(\mu_2, \alpha_2))]\, \phi\, p(\mu_2, \alpha_2, m_2^*(\mu_2, \alpha_2)) \\ & - \gamma(\mu_2, \alpha_2, m_2^*(\mu_2, \alpha_2))\, \lambda - c(\alpha_2) - \beta(\alpha_2 - \alpha_1)^2. \end{aligned} \tag{13}$$

Let $\alpha_2^*(\mu_2)$ denote the solution to the CRA's period-2 problem.

Next, consider an opportunistic issuer's choice of period-1 manipulation intensity, m_1. When choosing m_1, the issuer must take into account the potential impact of this choice on investors' and the CRA's information sets entering period 2. Recall that the period-1 cash flow is revealed at the end of the period. Thus, there are three possible states at the beginning of period 2, each corresponding to a different value of μ_2. In state s ("success"), the issuer has received a high rating in period 1 but has been revealed to have low cash flow at the end of the period. This state occurs if and only if the issuer is opportunistic and has a low-quality period-1 project but manipulation succeeds. In state f ("failure"), the issuer has received a low rating in period 1 and therefore has not sold a security. This state occurs if project-1 project quality is low and the issuer either is truthful or is opportunistic but manipulation fails to induce a high rating. In state h ("high-quality"), the issuer has received a high rating in period 1 and cash flow is also high. This state occurs if and only if the period-1 project is high quality, independent of the issuer's type.

Note that the CRA does not directly observe the manipulation intensity in either period. Thus, if the CRA observes signal g_ℓ in the first period and then subsequently observes a low cash flow (which is always the case when the signal is g_ℓ), it does not know whether the issuer is truthful or is opportunistic but was unsuccessful at manipulating its signal. From the issuer's standpoint, successful manipulation in period 1 is a double-edged sword. While successful manipulation allows the issuer to sell a security and therefore increases its period 1 payoff, it also reveals the issuer to be opportunistic. This resulting reputational effect lowers the issuer's expected period-2 payoffs, as skeptical investors set a lower period-2 price, and the CRA potentially screens more intensely in period 2.

Let μ_j denote the posterior probability that the issuer is truthful in state $j = s, f, h$, given investors' and the CRA's updated beliefs after period 1. State s (high rating, low cash flow) reveals the issuer to be opportunistic. Thus, $\mu_s = 0$. State h reveals no information about the issuer's type, so $\mu_h = \mu_1$. Finally, state f can be reached by either type of issuer, but generally causes positive updating the probability that the issuer is truthful since an opportunistic type with a low-quality period-1 project sometimes succeeds in reaching state s instead of f. The posterior belief in state f is

$$\mu_f = \frac{\mu_1}{\mu_1 + (1-\mu_1)[1-(1-\alpha_1)\tilde{m}_1]} = \frac{\mu_1}{1-(1-\mu_1)(1-\alpha_1)\tilde{m}_1}. \tag{14}$$

Let $\alpha_j^* = \alpha_2^*(\mu_j)$ and $m_j^* = m_2^*(\mu_j, \alpha_j^*)$. Furthermore, let π_j denote an opportunistic issuer's discounted period-2 expected payoff in state $j = s, f, h$. Then,

$$\pi_j = \delta\,\left\{[\eta + (1-\eta)(1-\alpha_j^*)m_j^*]\,(1-\phi)\,p(\mu_j, \alpha_j^*, m_j^*) - (1-\eta)qk(m_j^*)\right\}, \tag{15}$$

where the term in the square brackets represents an opportunistic issuer's probability of receiving a high rating in period 2. Note that π_j is independent of the opportunistic issuer's actual manipulation intensity m_1 for $j = s, f, h$. However, π_f depends on $\tilde{m}_1$ (i.e., the belief of investors and the CRA about the manipulation intensity) through its dependence on μ_f. Similarly, it also depends on the screening intensity in period 1, α_1. We therefore write $\pi_f = \pi_f(\alpha_1, \tilde{m}_1)$.

An opportunistic issuer with a low-quality project in period 1 chooses m_1 to maximize

$$\Pi = (1-\alpha_1)\,m_1\,[(1-\phi)p(\mu_1, \alpha_1, \tilde{m}_1) + \pi_s] + [1-(1-\alpha_1)m_1]\,\pi_f(\alpha_1, \tilde{m}_1) - qk(m_1) \tag{16}$$

$$= \pi_f(\alpha_1, \tilde{m}_1) + (1-\alpha_1)m_1\,\{(1-\phi)p_1(\mu_1, \alpha_1, \tilde{m}_1) - [\pi_f(\alpha_1, \tilde{m}_1) - \pi_s]\} - qk(m_1). \tag{17}$$

The first term is independent of the issuer's choice m_1. Thus, recalling that $\Delta = \pi_f - \pi_s$ in the base model, the two-period expected payoff is exactly comparable to the single-period payoff defined in equation (2).

Let Π' denote the derivative of Π with respect to m_1. Then,

$$\Pi'(m_1) = (1-\alpha_1)\left\{(1-\phi)p(\mu_1, \alpha_1, \tilde{m}_1) - [\pi_f(\alpha_1, \tilde{m}_1) - \pi_s]\right\} - qk'(m_1). \tag{18}$$

Let $m_1^*(\alpha_1)$ denote the solution to the opportunistic issuer's period-1 problem. In equilibrium, $\tilde{m}_1 = m_1^*(\alpha_1)$ must hold, so that, if $m_1^*(\alpha_1) \in (0,1)$, it is given by the solution to $\Pi'(m_1^*(\alpha_1)) = 0$. The condition $\Pi'(m_1^*(\alpha_1)) = 0$ is similar to the equilibrium condition in the one-period model (see equation (6)), with the important difference that, in the two-period model, $\Delta = \pi_f - \pi_s$ is endogenous and depends (through π_f) on α_1 and the conjectured (and equilibrium) value of m_1.

Finally, let $\psi_j(\alpha_1)$ denote the CRA's discounted expected period-2 payoff as of the end of period 1 in state $j = s, f, h$. That is, $\psi_j(\alpha_j) = \delta\psi_2(\alpha_j, \mu_j)$. With probability η, the issuer has high cash flow in period 1, leading to state h. With probability $\gamma(\mu_1, \alpha_1, m_1^*(\alpha_1))$, an issuer with a low-quality project obtains a high rating, leading to state s. If neither state h nor state s obtains, we are in state f. Thus, the CRA chooses α_1 to maximize

$$\begin{aligned}\Psi = {} & [\eta + \gamma(\mu_1, \alpha_1, m_1^*(\alpha_1))]\,\phi\, p(\mu_1, \alpha_1, m_1^*(\alpha_1)) - \gamma(\mu_1, \alpha_1, m_1^*(\alpha_1))\,\lambda - c(\alpha_1) \\ & + \eta\,\psi_h(\alpha_1) + \gamma(\mu_1, \alpha_1, m_1^*(\alpha_1))\,\psi_s(\alpha_1) + [1 - \eta - \gamma(\mu_1, \alpha_1, m_1^*(\alpha_1))]\,\psi_f(\alpha_1).\end{aligned} \tag{19}$$

Let α_1^* denote the solution to the CRA's period-1 problem.

Note that the general two-period model is intractable. However, we are able to derive analytical results for the special case where infinite screening adjustment costs fix the CRA's screening intensity across periods (and therefore across period-2 states). We then consider the more general case with finite adjustment costs in a numerical example. Our main result in this section is that, in contrast to the single-period model, the error rate in period 1 can increase with the CRA's screening intensity, α_1.

3.1 Screening Intensity Fixed Across Periods

We first consider a special case in which the CRA's screening intensity is the same across both periods (i.e., the adjustment cost β is infinite). While this is an extreme case, a CRA typically relies on a common model when rating securities of a given type. Changing a ratings model may take considerable time and effort and potentially exposes the CRA to reputational risk if the unproven model proves faulty. In this case, more intense screening in period 1 causes the issuer to anticipate more intense screening in period 2 as well, which feeds back into optimal period-1 manipulation through its effect on the issuer's continuation payoffs.

For each state $j = s, f, h$, the analysis of the period-2 equilibrium is identical to the analysis of the issuer's continuation game equilibrium in the single-period setting in Section 2 with continuation payoffs set to 0 (i.e., $\Delta = 0$). We therefore focus our analysis on period 1. To obtain an analytic result, we set the issuer's manipulation cost parameter q to zero. In addition, we assume that the average project has weakly positive NPV. As we show in the proof of Proposition 5, these two assumptions together imply that the equilibrium manipulation intensity in period 2 is equal to one in all states, which simplifies the analysis. By continuity, these insights hold for small q as well.

Define $\gamma_1 = \gamma(\mu_1, \alpha_1, m_1^*)$ to be the induced error rate in the first period when, following a given α_1, a perfect Bayesian equilibrium is played in the rest of the game. That is, the opportunistic issuer chooses an optimal first-period manipulation intensity m_1^*, and, for each state $j = s, f, h$ at time 2, the CRA chooses an optimal screening intensity α_j^* followed by the issuer choosing an optimal manipulation intensity m_j^*.

Proposition 5. *Suppose that (i) $\beta = \infty$ (so that the CRA's screening intensity is fixed across periods; i.e., $\alpha_1 = \alpha_2$), (ii) the expected NPV of the issuer's project is nonnegative (i.e., $\eta v_h + (1-\eta)v_\ell \geq 0$), and (iii) the issuer's manipulation cost is zero (i.e., $q = 0$). Then, there exist thresholds $\bar{\delta}$ and $\underline{\delta}$, with $0 < \underline{\delta} < \bar{\delta}$, such that, for all $\delta \in (\underline{\delta}, \bar{\delta})$, the period-1 rating error rate γ_1 strictly increases in the CRA's screening intensity α_1.*

In the proof of the proposition, we show that when $\delta \in (\underline{\delta}, \bar{\delta})$, it follows that $m_1^* \in (0, 1)$. When $\delta \leq \underline{\delta}$, $m_1^* = 1$, and when $\delta \geq \bar{\delta}$, $m_1^* = 0$. In other words, under the conditions of the proposition, whenever m_1^* is in the interior, the first-period error rate strictly increases in the screening intensity.

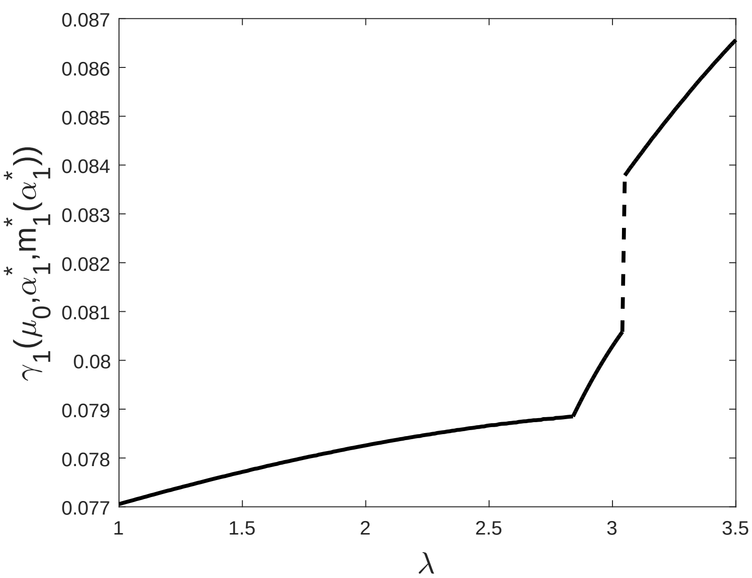
Our two-period result contrasts with the single-period case exhibited in part (i) of Proposition 2, where the equilibrium error rate is invariant to the screening intensity α when $q = 0$ and the manipulation intensity is between zero and one. As discussed after Proposition 2, in the single-period case when $q = 0$ and $m^* \in (0, 1)$, the equilibrium manipulation intensity is determined by the equation $p^*(\alpha, m^*(\alpha)) = \frac{\Delta}{1-\phi}$. Therefore, the price of the high-rated security must remain constant when the screening intensity α changes by a small amount. As a result, the error rate γ must also remain constant.

In the two-period case, when there is a cost to adjusting the screening intensity across periods, a change in the first-period screening intensity α_1 has an additional effect: the payoff difference between the failure and success states, Δ, also changes with α_1. In particular, an increase in α_1 leads to the opportunistic issuer anticipating greater screening intensity in period 2 as well. More intense screening in period 2 reduces the benefit of arriving in period 2 with a favorable reputation (i.e., higher μ_2) since the opportunistic issuer is more likely to be screened out when it has a low-quality project in period 2. As $\mu_f > \mu_s$, this effect reduces the payoff difference Δ. The net result is that the opportunistic issuer's incentives to manipulate in period 1 are strengthened, relative to the single-period model. This added effect shifts the outcome from one where the error rate γ is invariant to a small change in screening intensity to one where the error rate in period 1 increases with screening intensity.

3.2 Comparative Statics

We illustrate the comparative statics of the two-period model through an example. We have already shown (Example 1) that, in the single-period model, the optimal screening intensity α^* and the error rate $\gamma(\alpha^*, m^*(\alpha^*))$ may both increase as the prior quality of the project improves (i.e., as η increases). We now show that, in the two-period model, the equilibrium error rate in the first period, $\gamma_1^* = \gamma_1(\mu_1, \alpha_1^*, m_1^*(\alpha_1))$, may also increase with λ, the penalty the CRA suffers when it erroneously confers a high rating on a low-quality project. A necessary condition for our two-period result is that $\gamma_1(\mu_1, \alpha_1, m^*(\alpha_1))$ must be increasing in α_1, so the example also illustrates that the conditions in Proposition 5 are not particularly restrictive — even after relaxing these conditions, there exist parameter values such that γ_1 is increasing in α_1.

In the one-period model, an increase in λ leads to the CRA screening more intensely (i.e., α^* increases), and by Proposition 2, the result is fewer errors by the CRA. In contrast, in the two-period model, the first-period equilibrium error rate may instead increase with λ, as shown in Figure 3. In this example, we set $v_h = 1, v_\ell = 0, \eta = 0.5, \phi = 0.1, \mu_1 = 0.7, \delta = 2$, the manipulation cost to $qk(m) = 0.15m^2$, the CRA's screening cost to $c(\alpha) = \alpha^2$, and $\beta = 1$ (so that the CRA's adjustment cost in period 2 is $(\alpha_2 - \alpha_1)^2$). At each value of α_1, we choose the lowest manipulation intensity m_1^* that represents an equilibrium.[14] We vary λ between 1 and 3.5. As seen from the figure, throughout this region, γ_1^* is increasing in λ, with a discontinuous jump at $\lambda \approx 3.05$. This jump occurs because, when λ is just below 3.05, period-1 manipulation is close to 1 and increasing in λ. For slightly higher values of λ, the CRA economizes on screening intensity and allows period-1 manipulation intensity to reach 1.



This figure shows how the equilibrium first period error rate γ_1 varies as the penalty on the CRA for rating errors (λ) increases. For each value of λ, we compute an equilibrium of the two-period game and the resulting error rate. The parameters are $v_h = 1, v_\ell = 0, \eta = 0.5, \phi = 0.1, \mu_1 = 0.7, \delta = 2$, $qk(m) = 0.15m^2$, $c(\alpha) = \alpha^2$, and $\beta = 1$.

Figure 3: **Effect of increased CRA's error penalty on first-period equilibrium error rate**

[14] In the two-period model, given an α_1, there may be multiple equilibria in the remainder of the game. In particular, there may be multiple values of $m_1^*(\alpha_1)$ that each represent an equilibrium.

4 Implications

Our results yield practical implications for researchers studying credit ratings (and other certification settings), for rating agencies (and other certification providers), and for regulators and policymakers. For researchers, our results produce distinct testable implications. For example, our results in Proposition 1 suggest that manipulation may increase (decrease) in response to a positive (negative) shock to screening, say due to a court ruling affecting rating agency culpability for rating errors.[15] Such a response would be difficult to square with other ratings models. Alp (2013) presents evidence that rating standards may indeed change meaningfully over time. In addition, our results in Proposition 5 suggest that ratings accuracy may worsen (improve) in the short run after a positive (negative) shock to screening, which, again, would be difficult to square with other models.

For ratings agencies, our results in Proposition 1 suggest the importance of considering how issuers are likely to respond to changes in screening policies. If conditions generally support accurate ratings (few low-quality projects, high cost of manipulation), then tougher screening policies may have two benefits – making it harder for issuers to mislead and discouraging them from doing so. On the other hand, if conditions generally make accurate ratings challenging (many low-quality projects, low cost of manipulation), then tougher screening policies may exacerbate incentives to mislead, at least partly undermining the benefit of making it harder for issuers to mislead. Both the direct and incentive effects of screening should factor into screening policy decisions. Our results in Proposition 4 also suggest somewhat counterintuitively that a rating agency may want to consider increasing diligence in response to an overall improvement in the distribution of asset quality in the economy, for example during a period of economic expansion.

For regulators and policymakers, our results offer two sets of policy implications. First, our results in Proposition 5 and depicted in Figure 3 suggest that efforts to encourage greater CRA diligence (a higher λ in the model) may appear ineffective or counterproductive in the short run, even if they are effective at improving diligence and making ratings more informative in the long

[15]Several court cases over the years involve questions of First Amendment protections for rating agencies. See, for example, https://www.reuters.com/article/idUSN1E7AO0H7.

run. It is noteworthy that Dimitrov et al. (2015) find that the quality of credit ratings may have worsened rather than improved following the passage of the Dodd-Frank Act. Second, our results in Proposition 3 suggest that increasing disclosure requirements to weed out low-quality projects before they even reach the potential funding stage, which can be interpreted as increasing η, may encourage issuers who meet the requirements to further mislead rating agencies, undermining the effectiveness of the requirements.

5 Conclusion

Our analysis highlights the importance of accounting for the possibility that an issuer manipulates the information on which a CRA relies when rating a security in evaluating the effectiveness of CRA policies. It suggests that issuer manipulation can either dampen or magnify the effect of more diligent CRA screening, making it difficult to infer the effectiveness of policies that encourage more diligent screening from observed rates of rating errors. Indeed, because of the dynamic reputational effects illustrated in the two-period version of the model, more diligent screening may actually result in more rating errors in the short run. Our analysis is also relevant for other settings in which a certification seeker can manipulate the information on which a certifier relies to assess certification-seeker quality.

While our model explicitly focuses on the interaction between a CRA and an issuer, its implications apply to other settings as well. For example, more intense emissions standards screening for automobiles may lead to greater incentives to cheat in order to pass tests if car buyers are willing to pay a premium for low-emission cars.[16] Our model also suggests that efforts to curb cheating by students may incentivize more cheating since a high grade would represent a stronger signal of student quality in such a setting.[17]

[16]While it is difficult to link one specific instance of such cheating to changes in screening standards, it is worth noting that the Volkswagen emissions scandal took place during an environment in which the demand for low-emissions cars was increasing significantly.

[17]Consider, for example, the cheating incident at Harvard in 2012; see `https://www.nytimes.com/2012/08/31/education/harvard-says-125-students-may-have-cheated-on-exam.html`.

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Appendix: Proofs

Proof of Lemma 1. (i) As shown in equation (2), the payoff of the opportunistic issuer with a low-quality project is

$$\Pi(m) = (1-\alpha)m[(1-\phi)p(\alpha,\tilde{m}) - \Delta] - qk(m), \tag{20}$$

where $\tilde{m}$ is the market's belief about the extent of manipulation. The derivative of Π with respect to m is

$$\Pi'(m) = (1-\alpha)[(1-\phi)p(\alpha,\tilde{m}) - \Delta] - qk'(m). \tag{21}$$

In equilibrium, the market's belief must be correct, that is, $\tilde{m} = m^*(\alpha)$. Making this substitution, we have

$$\Pi'(m^*(\alpha)) = (1-\alpha)\left[(1-\phi)p(\alpha, m^*(\alpha)) - \Delta\right] - qk'(m^*(\alpha)). \tag{22}$$

From equation (4), it follows that $p(\alpha,m)$ is strictly decreasing in m. Since $k''(m) \geq 0$, we therefore have that $\Pi''(m) < 0$. Further, noting that $p(\alpha,0) = v_h > \frac{\Delta}{1-\phi}$ (Assumption 1) and that $k'(0) = 0$, it follows that $\Pi'(0) > 0$.

Observe that $\Pi'(1) = (1-\alpha)[(1-\phi)p(\alpha,1) - \Delta] - qk'(1)$. There are now two cases to consider:

(a) $\Pi'(1) < 0$. Then, by the intermediate value theorem, there exists a unique equilibrium intensity $m^*(\alpha) \in (0,1)$ such that $\Pi'(m^*(\alpha)) = 0$.

(b) $\Pi'(1) \geq 0$. In this case, it must be that the equilibrium manipulation intensity is $m^*(\alpha) = 1$.

Now, the condition $\Pi'(1) \geq 0$ is equivalent to $q \leq \underline{q}$, where

$$\underline{q} = \frac{(1-\alpha)\left[(1-\phi)p(\alpha,1) - \Delta\right]}{k'(1)} = \frac{(1-\alpha)\left[(1-\phi)\frac{\eta v_h + (1-\mu)(1-\eta)(1-\alpha)v_\ell}{\eta+(1-\mu)(1-\eta)(1-\alpha)} - \Delta\right]}{k'(1)}. \tag{23}$$

(ii) From part (i), when $q > \underline{q}$, we have $m^*(\alpha) \in (0,1)$, so that $\Pi'(m^*(\alpha)) = (1-\alpha)[(1-\phi)\,p(\alpha, m^*(\alpha)) - \Delta] - qk'(m^*(\alpha)) = 0$. Observe that the expression $\Pi'(m^*(\alpha))$ is strictly decreasing in both $m^*(\alpha)$ and q. Therefore, if q increases by a small amount, $m^*(\alpha)$ must decrease to restore

$\Pi'(m^*(\alpha)) = 0$. That is, $m^*(\alpha)$ is strictly decreasing in q. $\square$

Proof of Lemma 2. Given α, the equilibrium condition whenever $m^*(\alpha) \in (0, 1)$ is

$$(1-\alpha)[(1-\phi)p(\alpha, m^*(\alpha)) - \Delta] - qk'(m^*(\alpha)) = 0. \tag{24}$$

Applying the implicit function theorem, we have

$$\frac{dm^*}{d\alpha} = \frac{-[(1-\phi)p(\alpha, m^*(\alpha)) - \Delta] + (1-\alpha)(1-\phi)\frac{\partial p}{\partial \alpha}}{-(1-\alpha)(1-\phi)\frac{\partial p}{\partial m} + qk''(m^*(\alpha))} \tag{25}$$

From equation (4), we have

$$\frac{\partial p}{\partial m} = \frac{dp}{d\gamma}\frac{\partial \gamma}{\partial m} = -\frac{\eta(v_h - v_\ell)}{(\eta+\gamma)^2}(1-\mu)(1-\eta)(1-\alpha) < 0. \tag{26}$$

In addition, $k''(m) \geq 0$. Thus, the denominator of $\frac{dm^*}{d\alpha}$ in equation (25) is strictly positive. Hence, the sign of $\frac{dm^*}{d\alpha}$ is equal to the sign of the numerator, or the sign of

$$w = (1-\alpha)\frac{\partial p}{\partial \alpha} + \frac{\Delta}{1-\phi} - p(\alpha, m^*(\alpha)). \tag{27}$$

From equation (4), it follows that

$$\frac{\partial p}{\partial \alpha} = \frac{dp}{d\gamma}\frac{\partial \gamma}{\partial \alpha} = \frac{\eta(v_h - v_\ell)}{(\eta+\gamma)^2}(1-\mu)(1-\eta)m = \frac{\eta(v_h - v_\ell)}{(\eta+\gamma)^2}\frac{\gamma}{1-\alpha}. \tag{28}$$

Hence,

$$w = \frac{\eta(v_h - v_\ell)\gamma}{(\eta+\gamma)^2} + \frac{\Delta}{1-\phi} - \frac{\eta v_h + \gamma v_\ell}{\eta+\gamma} = \frac{1}{\eta+\gamma}\left(\frac{\eta(v_h - v_\ell)\gamma}{\eta+\gamma} + \frac{\Delta(\eta+\gamma)}{1-\phi} - (\eta v_h + \gamma v_\ell)\right). \tag{29}$$

Therefore, $\frac{dm^*}{d\alpha} > 0$ if and only if

$$\frac{\eta(v_h - v_\ell)\gamma}{\eta+\gamma} > \eta\left(v_h - \frac{\Delta}{1-\phi}\right) - \gamma\left(\frac{\Delta}{1-\phi} - v_\ell\right). \tag{30}$$

Note that the left-hand side of this inequality is increasing in γ, whereas the right-hand side is decreasing in γ (because $v_\ell < \frac{\Delta}{1-\phi}$ by Assumption 1). Furthermore, the inequality does not hold if $\gamma = 0$. Thus, there exists a unique threshold error rate $\bar{\gamma} > 0$ such that $\frac{dm^*}{d\alpha} > 0$ if and only if $\gamma > \bar{\gamma}$ (and $m^* \in (0,1)$). This threshold $\bar{\gamma}$ is given by the unique positive root of the equation

$$\eta(v_h - v_\ell)\gamma = \eta^2 \left(v_h - \frac{\Delta}{1-\phi}\right) - \eta\gamma \left(\frac{\Delta}{1-\phi} - v_\ell\right) + \eta\gamma \left(v_h - \frac{\Delta}{1-\phi}\right) - \gamma^2 \left(\frac{\Delta}{1-\phi} - v_\ell\right), \quad (31)$$

or, equivalently, of the equation

$$\gamma^2 + 2\eta\gamma - \eta^2 \frac{v_h - \frac{\Delta}{1-\phi}}{\frac{\Delta}{1-\phi} - v_\ell} = 0, \quad (32)$$

and hence is given by

$$\bar{\gamma} = \eta \left(\sqrt{1 + \frac{v_h - \frac{\Delta}{1-\phi}}{\frac{\Delta}{1-\phi} - v_\ell}} - 1 \right). \quad (33)$$

Assumption 1 ensures that both the numerator and the denominator of the second term under the square root sign are positive, so that the term under the square root is greater than 1, which implies that $\bar{\gamma} > 0$. $\square$

Proof of Proposition 1. From Lemma 2, two conditions are necessary and sufficient for $\frac{dm^*}{d\alpha} > 0$. First, it must be that $\gamma > \bar{\gamma}$, and second, it must be that $m^*(\alpha) \in (0,1)$.

Consider first the condition that $\gamma > \bar{\gamma}$ or, equivalently, that $(1-\mu)(1-\eta)(1-\alpha)m^*(\alpha) > \bar{\gamma}$. The maximum value of $(1-\alpha)m^*(\alpha)$ is 1 (when $\alpha = 0$ and $m^*(\alpha) = 1$), so a necessary condition for $\gamma > \bar{\gamma}$ is that $(1-\eta)(1-\mu) > \bar{\gamma}$ or, equivalently, that

$$\eta < \frac{1-\mu}{\sqrt{1 + \frac{v_h - \frac{\Delta}{1-\phi}}{\frac{\Delta}{1-\phi} - v_\ell}} - \mu} \equiv \bar{\eta}. \quad (34)$$

Clearly, $\bar{\eta} \in (0,1)$.

Now suppose that $\eta < \bar{\eta}$. Then, $\gamma > \bar{\gamma}$ requires that $(1-\alpha)m^*(\alpha) > \frac{\bar{\gamma}}{(1-\mu)(1-\eta)}$. As the maximal value of $m^*(\alpha)$ is 1, it must therefore be that $\alpha < 1 - \frac{\bar{\gamma}}{(1-\mu)(1-\eta)} \equiv \bar{\alpha}(\eta)$. Observe that, by construction, $\eta < \bar{\eta}$ implies that $(1-\eta)(1-\mu) > \bar{\gamma}$, so that $\bar{\alpha}(\eta) > 0$.

Next, suppose that $\eta < \bar{\eta}$ and $\alpha < \bar{\alpha}(\eta)$. Then, $\gamma > \bar{\gamma}$ if and only if $m^*(\alpha) > \frac{\bar{\gamma}}{(1-\mu)(1-\eta)(1-\alpha)} \equiv$

$\bar{m}(\eta, \alpha)$. Observe that $\bar{m}(\eta, \alpha) = \frac{1-\bar{\alpha}(\eta)}{1-\alpha} < 1$ when $\alpha < \bar{\alpha}(\eta)$. Recall that $m^*(\alpha)$ is unique given α, and that $m^*(\alpha)$ is strictly decreasing in q. Therefore, the condition $m^*(\alpha) > \bar{m}(\eta, \alpha)$ is equivalent to

$$(1-\alpha)[(1-\phi)p(\alpha, \bar{m}(\eta, \alpha)) - \Delta] > qk'(\bar{m}(\eta, \alpha)) \tag{35}$$

or

$$q < \frac{(1-\alpha)[(1-\phi)p(\alpha, \bar{m}(\eta, \alpha)) - \Delta]}{k'(\bar{m}(\eta, \alpha))} \equiv \bar{q}(\eta, \alpha). \tag{36}$$

As mentioned above, if $\eta < \bar{\eta}$ and $\alpha < \bar{\alpha}(\eta)$, then $\bar{m}(\eta, \alpha) < 1$. Given that $m^*(\alpha)$ is strictly decreasing in q, it is now immediate that $\underline{q}$, the value of q at which $m^*(\alpha) = 1$ (defined in the proof of Lemma 1), is strictly less than $\bar{q}$. Furthermore, if $q = 0$, it follows from the equilibrium condition in equation (5) that $p(\alpha, m^*(\alpha)) = \frac{\Delta}{1-\phi}$ or, equivalently, that $\gamma = \eta \left(v_h - \frac{\Delta}{1-\phi}\right) / \left(\frac{\Delta}{1-\phi} - v_\ell\right)$. This error rate clearly exceeds the threshold $\bar{\gamma}$ defined in equation (33), which implies that $\bar{q}(\eta, \alpha) > 0$. Thus, there exist thresholds $\bar{\eta} > 0$, $\bar{\alpha}(\eta) > 0$, and $\bar{q}(\eta, \alpha) > 0$ such that, if $\eta < \bar{\eta}$, $\alpha < \bar{\alpha}(\eta)$, and $q \in (\underline{q}, \bar{q}(\eta, \alpha))$, then $m^*(\alpha) \in (0, 1)$ and $\gamma > \bar{\gamma}$. This concludes the "if" part of the proof.

For the "only if" part, observe that if any of the following three conditions are met, then $\gamma \leq \bar{\gamma}$: (i) $\eta \geq \bar{\eta}$, (ii) $\alpha \geq \bar{\alpha}(\eta)$, or (iii) $m^*(\alpha) \leq \bar{m}(\eta, \alpha)$. The third condition corresponds to $q \geq \bar{q}(\eta, \alpha)$. □

Proof of Proposition 2. (i) Suppose that $q = 0$. Then, the equilibrium condition for an interior value of m^* in equation (24) reduces to $p(\alpha, m^*) = \frac{\Delta}{1-\phi}$, which implies that $m^* = \frac{\eta}{(1-\alpha)(1-\mu)(1-\eta)} \times \frac{v_h - \frac{\Delta}{1-\phi}}{\frac{\Delta}{1-\phi} - v_\ell}$. It follows that $m^* > 0$, and that when $\alpha < 1 - \frac{\eta}{(1-\mu)(1-\eta)} \left(\frac{v_h - \frac{\Delta}{1-\phi}}{\frac{\Delta}{1-\phi} - v_\ell}\right)$, we have $m^* \in (0, 1)$.

From the equilibrium condition, using the expression for $p(\alpha, m)$ in equation (4), we can explicitly solve for the error rate:

$$\gamma(\alpha, m^*(\alpha)) = \eta \frac{v_h - \frac{\Delta}{1-\phi}}{\frac{\Delta}{1-\phi} - v_\ell}. \tag{37}$$

It is immediate that $\gamma(\alpha, m^*(\alpha))$ is invariant to changes in α in this case.

(ii) Suppose that $m^*(\alpha) = 1$ and consider a small increase in α. Then, either $m^*(\alpha)$ stays at 1, or it decreases and becomes strictly less than 1. In either case, it is immediate that $\gamma(\alpha, m^*(\alpha))$ strictly decreases in α.

Next, suppose that $q > 0$ and $m^*(\alpha) \in (0,1)$. Suppose also that $\frac{d\gamma}{d\alpha} \geq 0$. In this case, $m^*(\alpha)$ is the unique solution to equation (6). Totally differentiating equation (6) and rearranging, we have:

$$(1-\alpha)(1-\phi)\,\frac{dp}{d\gamma}\,\frac{d\gamma}{d\alpha} = \frac{q}{1-\alpha}\,k'(m^*(\alpha)) + qk''(m^*(\alpha))\,\frac{dm^*}{d\alpha}. \tag{38}$$

Since $\frac{dp}{d\gamma} < 0$ and $\frac{d\gamma}{d\alpha} \geq 0$, the term on the left-hand side is weakly negative. The first term on the right-hand side is positive since $k'(m) > 0$ for $m > 0$. Thus, a necessary condition for this equality to hold is that the second term on the right-hand side be negative. Noting that $qk''(m) \geq 0$, we must have $\frac{dm^*}{d\alpha} < 0$. However, in this case, $\frac{d\gamma}{d\alpha} = \frac{\partial\gamma}{\partial\alpha} + \frac{\partial\gamma}{\partial m}\frac{dm^*}{d\alpha} < 0$ because $\frac{\partial\gamma}{\partial\alpha} < 0$ and $\frac{\partial\gamma}{\partial m} > 0$, contradicting the assumption that $\frac{d\gamma}{d\alpha} \geq 0$. $\square$

Proof of Proposition 3. Suppose that $m^*(\alpha) \in (0,1)$. Then, we can write the equilibrium condition in equation (5) in terms of the error rate γ as $f(\gamma) = 0$, where

$$f(\gamma) = (1-\alpha)\left[(1-\phi)\,\frac{\eta v_h + \gamma v_\ell}{\eta+\gamma} - \Delta\right] - qk'\left(\frac{\gamma}{(1-\mu)(1-\eta)(1-\alpha)}\right). \tag{39}$$

From equation (39), it follows that

$$f_\gamma = -(1-\alpha)(1-\phi)\,\frac{\eta(v_h - v_\ell)}{(\eta+\gamma)^2} - qk''\left(\frac{\gamma}{(1-\mu)(1-\eta)(1-\alpha)}\right)\frac{1}{(1-\mu)(1-\eta)(1-\alpha)}, \tag{40}$$

where f_x denotes the partial derivative of f with respect to x. Since $k''(\cdot) \geq 0$, clearly $f_\gamma < 0$.

(i) The result that the error rate strictly decreases in the CRA's commission rate ϕ follows immediately from the Implicit Function Theorem because

$$f_\phi = -(1-\alpha)\,\frac{\eta v_h + \gamma v_\ell}{\eta+\gamma} < 0. \tag{41}$$

Thus, $\frac{d\gamma}{d\phi} = -\frac{f_\phi}{f_\gamma} < 0$.

(ii) As $f_\gamma < 0$, it follows from the Implicit Function Theorem that $\frac{d\gamma}{d\eta} = -\frac{f_\eta}{f_\gamma} > 0$ if and only if

$f_\eta > 0$. From equation (39), we have

$$f_\eta = (1-\alpha)(1-\phi)\,\frac{\gamma(v_h - v_\ell)}{(\eta+\gamma)^2} - qk''\left(\frac{\gamma}{(1-\mu)(1-\eta)(1-\alpha)}\right)\frac{\gamma}{(1-\mu)(1-\eta)^2(1-\alpha)}. \tag{42}$$

Since $\gamma \leq (1-\mu)(1-\eta)(1-\alpha)$, f_η is strictly positive if

$$q < (1-\alpha)(1-\phi)\,\frac{v_h - v_\ell}{[\eta + (1-\mu)(1-\eta)(1-\alpha)]^2}\,\frac{(1-\mu)(1-\eta)^2(1-\alpha)}{\bar{k}_2} \equiv \bar{q}_\eta, \tag{43}$$

where $\bar{k}_2 = \max_{m\in[0,1]} k''(m) < \infty$. □

Proof of Lemma 3. Observe that the CRA's payoff function Ψ in equation (8) is continuous in α. As α must lie between 0 and $\alpha_{\max}$ and Ψ is bounded above by $\Lambda + \phi v_h$, it follows from Weierstrass' Theorem that a maximum α^* exists. Further, as shown in Lemma 1, given α, there is a unique equilibrium m^* in the issuer's continuation game. Thus, a pure strategy equilibrium of the overall game (α^*, m^*) exists.

Next, we show that there exists a $\underline{c}$ such that, if $c''(\alpha) > \underline{c}$ and $m^* \in (0,1)$, then $\Psi(\alpha)$ is strictly concave at that value of α. It is immediate that $\Psi'(0) > 0$ and $\Psi'(\alpha_{\max}) < 0$. Since $\Psi'(\alpha)$ is a continuous function on $(0, \alpha_{\max})$, this implies that there exists an $\alpha^* \in (0, \alpha_{\max})$ such that $\Psi'(\alpha^*) = 0$. The screening intensity α^* is uniquely determined by the first-order condition $\Psi'(\alpha^*) = 0$ if $\Psi''(\alpha) < 0$ for all $\alpha \in (0, \alpha_{\max})$, where

$$\Psi''(\alpha) = -(\lambda - \phi v_\ell)\,\frac{d^2\gamma}{d\alpha^2} - c''(\alpha). \tag{44}$$

The error rate γ is implicitly defined by the condition $f(\gamma) = 0$, where $f(\gamma)$ is defined in equation (39). From the Implicit Function Theorem, we have

$$\frac{d^2\gamma}{d\alpha^2} = -\frac{1}{f_\gamma^3}\left(f_{\alpha\alpha}\, f_\gamma^2 - 2f_{\alpha\gamma}\, f_\alpha\, f_\gamma + f_{\gamma\gamma}\, f_\alpha^2\right), \tag{45}$$

where, as before, f_x denotes the partial derivative of f with respect to x. The condition $\Psi''(\alpha) < 0$

is thus equivalent to

$$c''(\alpha) > (\lambda - \phi v_\ell)\left(\frac{f_{\alpha\alpha}}{f_\gamma} - \frac{2f_{\alpha\gamma}\,f_\alpha}{f_\gamma^2} + \frac{f_{\gamma\gamma}\,f_\alpha^2}{f_\gamma^3}\right). \tag{46}$$

From the definition of $f(\gamma)$ in equation (39), it immediately follows that

$$f_\alpha = -\left[(1-\phi)\,\frac{\eta v_h + \gamma v_\ell}{\eta+\gamma} - \Delta\right] - qk''(m^*)\,\frac{\gamma}{(1-\eta)(1-\mu)(1-\alpha)^2}, \tag{47}$$

$$f_\gamma = -(1-\alpha)(1-\phi)\,\frac{\eta(v_h - v_\ell)}{(\eta+\gamma)^2} - qk''(m^*)\,\frac{1}{(1-\eta)(1-\mu)(1-\alpha)}, \tag{48}$$

$$f_{\alpha\alpha} = -qk''(m^*)\,\frac{2\gamma}{(1-\eta)(1-\mu)(1-\alpha)^3} - qk'''(m^*)\left(\frac{\gamma}{(1-\eta)(1-\mu)(1-\alpha)^2}\right)^2, \tag{49}$$

$$f_{\gamma\gamma} = 2(1-\alpha)(1-\phi)\,\frac{\eta(v_h - v_\ell)}{(\eta+\gamma)^3} - qk'''(m^*)\left(\frac{1}{(1-\eta)(1-\mu)(1-\alpha)}\right)^2, \tag{50}$$

$$f_{\alpha\gamma} = (1-\phi)\,\frac{\eta(v_h - v_\ell)}{(\eta+\gamma)^2} - qk''(m^*)\,\frac{1}{(1-\eta)(1-\mu)(1-\alpha)^2} - qk'''(m^*)\,\frac{\gamma}{(1-\eta)^2(1-\mu)^2(1-\alpha)^3}, \tag{51}$$

where $m^* = \frac{\gamma}{(1-\eta)(1-\mu)(1-\alpha)}$.

We first derive an upper bound for the term $f_{\alpha\alpha}/f_\gamma$ in equation (46). Let $\underline{k}_2 = \min_{m\in[0,1]} k''(m) > 0$. Since $\alpha \in (0, \alpha_{\max})$ and $m^* \in (0,1)$, we have

$$f_\gamma < -(1-\alpha_{\max})(1-\phi)\,\frac{\eta(v_h - v_\ell)}{[\eta + (1-\eta)(1-\mu)]^2} - \frac{q\underline{k}_2}{(1-\eta)(1-\mu)} \equiv \bar{f}_\gamma. \tag{52}$$

Similarly, let $\bar{k}_2 = \max_{m\in[0,1]} k''(m) > 0$ and $\bar{k}_3 = \max\{\max_{m\in[0,1]} k'''(m), 0\} \geq 0$. A lower bound for $f_{\alpha\alpha}$ is then given by

$$f_{\alpha\alpha} > -\frac{q\left(2\bar{k}_2 + \bar{k}_3\right)}{(1-\alpha_{\max})^2} \equiv \underline{f}_{\alpha\alpha}. \tag{53}$$

Since $\bar{f}_\gamma < 0$ and $\underline{f}_{\alpha\alpha} < 0$, the term $f_{\alpha\alpha}/f_\gamma$ is bounded from above by $\underline{f}_{\alpha\alpha}/\bar{f}_\gamma > 0$.

Next, consider the term $f_{\alpha\gamma}\,f_\alpha/f_\gamma^2$. Clearly, $\underline{f}_\alpha < f_\alpha < 0$, where

$$\underline{f}_\alpha = -\left[(1-\phi)\,v_h - \Delta\right] - \frac{q\bar{k}_2}{1-\alpha_{\max}}. \tag{54}$$

Furthermore,

$$f_{\alpha\gamma} < (1-\phi)\,\frac{v_h - v_\ell}{\eta} - \frac{q\underline{k}_3}{(1-\eta)(1-\mu)(1-\alpha_{\max})^2} \equiv \bar{f}_{\alpha\gamma}, \tag{55}$$

where $\underline{k}_3 = \min\{\min_{m\in[0,1]} k'''(m), 0\} \leq 0$. Since $f_\gamma < \bar{f}_\gamma < 0$, $\underline{f}_\alpha < f_\alpha < 0$, and $f_{\alpha\gamma} < \bar{f}_{\alpha\gamma}$ (> 0), the term $-2f_{\alpha\gamma}\, f_\alpha/f_\gamma^2$ is bounded from above by $-2\bar{f}_{\alpha\gamma}\,\underline{f}_\alpha/\bar{f}_\gamma^2 > 0$.

Finally, consider the term $f_{\gamma\gamma}\, f_\alpha^2/f_\gamma^3$. Clearly,

$$f_{\gamma\gamma} > -\frac{q\bar{k}_3}{[(1-\eta)(1-\mu)(1-\alpha_{\max})]^2} \equiv \underline{f}_{\gamma\gamma}. \tag{56}$$

Since $f_\gamma < \bar{f}_\gamma < 0$, $\underline{f}_\alpha < f_\alpha < 0$, and $f_{\gamma\gamma} > \underline{f}_{\gamma\gamma}$ (≤ 0), the term $f_{\gamma\gamma}\, f_\alpha^2/f_\gamma^3$ is bounded from above by $\underline{f}_{\gamma\gamma}\,\underline{f}_\alpha^2/\bar{f}_\gamma^3 \geq 0$.

Putting everything together, we thus have

$$(\lambda - \phi v_\ell)\left(\frac{f_{\alpha\alpha}}{f_\gamma} - \frac{2f_{\alpha\gamma}\, f_\alpha}{f_\gamma^2} + \frac{f_{\gamma\gamma}\, f_\alpha^2}{f_\gamma^3}\right) < (\lambda - \phi v_\ell)\left(\frac{\underline{f}_{\alpha\alpha}}{\bar{f}_\gamma} - \frac{2\bar{f}_{\alpha\gamma}\,\underline{f}_\alpha}{\bar{f}_\gamma^2} + \frac{\underline{f}_{\gamma\gamma}\,\underline{f}_\alpha^2}{\bar{f}_\gamma^3}\right). \tag{57}$$

A sufficient condition for the objective function $\Psi(\alpha)$ to be strictly concave at α is therefore given by

$$c''(\alpha) > \underline{c} \equiv (\lambda - \phi v_\ell)\left(\frac{\underline{f}_{\alpha\alpha}}{\bar{f}_\gamma} - \frac{2\bar{f}_{\alpha\gamma}\,\underline{f}_\alpha}{\bar{f}_\gamma^2} + \frac{\underline{f}_{\gamma\gamma}\,\underline{f}_\alpha^2}{\bar{f}_\gamma^3}\right). \tag{58}$$

Thus, if $c''(\alpha) > \underline{c}$ for all $\alpha \in [0, \alpha_{\max}]$, strict concavity of Ψ implies that, in the region of α over which $m^* \in (0,1)$, there can be at most one maximum α^*, and hence at most one equilibrium of the overall game.

Next, consider the region of α over which $m^* = 1$. Then,

$$\Psi(\alpha) = \Lambda + \phi\eta v_h - (\lambda - \phi v_\ell)(1-\eta)(1-\mu)(1-\alpha) - c(\alpha), \tag{59}$$

which is immediately seen to be strictly concave in α. Thus, fixing $m^* = 1$, there can be at most one value of α at which Ψ is maximized, and hence at most one equilibrium of the overall game. □

Proof of Proposition 4. The optimal screening intensity α^* is determined by the first-order condition $\Psi'(\alpha) = 0$. Using equation (9), we can write this condition as

$$-(\lambda - \phi v_\ell)\,\frac{d\gamma}{d\alpha} - c'(\alpha) = 0. \tag{60}$$

(i) From the Implicit Function Theorem, we have $\frac{d\alpha}{d\phi} = -\frac{\frac{\partial \Psi'}{\partial \phi}}{\Psi''(\alpha)}$. Now, $\Psi''(\alpha) < 0$ at the optimal value α^*, as this is just the second-order condition for optimality. Further, $\frac{\partial \Psi'}{\partial \phi} = v_\ell \frac{d\gamma}{d\alpha}$.

Now, suppose that $q > 0$. Then, from part (ii) of Proposition 2, we have $\frac{d\gamma}{d\alpha} < 0$. Therefore, if $v_\ell < 0$, it follows that $\frac{d\alpha}{d\phi} > 0$. Conversely, if $q = 0$, then $\frac{d\gamma}{d\alpha} = 0$ (Proposition 2, part (i)), so $\frac{\partial \Psi'}{\partial \phi} = 0$. Similarly, if $v_\ell = 0$, then $\frac{\partial \Psi'}{\partial \phi} = 0$.

(ii) From the Implicit Function Theorem, we have $\frac{d\alpha}{d\eta} = -\frac{\frac{\partial \Psi'}{\partial \eta}}{\Psi''(\alpha)}$. As commented above, $\Psi''(\alpha) < 0$, so the sign of $\frac{d\alpha}{d\eta}$ is equal to the sign of $\frac{\partial \Psi'}{\partial \eta}$. From equation (9), we have

$$\frac{\partial \Psi'}{\partial \eta} = -(\lambda - \phi v_\ell)\,\frac{\partial}{\partial \eta}\left(\frac{d\gamma}{d\alpha}\right). \tag{61}$$

Thus, $\frac{\partial \Psi'}{\partial \eta} > 0$ if and only if $\frac{\partial}{\partial \eta}\left(\frac{d\gamma}{d\alpha}\right) < 0$.

If $m^* \in (0,1)$, the equilibrium condition in equation (6) can be written in terms of the error rate γ as

$$(1-\alpha)\left[(1-\phi)\,\frac{\eta v_h + \gamma v_\ell}{\eta + \gamma} - \Delta\right] - qk'\left(\frac{\gamma}{(1-\eta)(1-\mu)(1-\alpha)}\right) = 0. \tag{62}$$

From the Implicit Function Theorem, it hence follows that

$$\frac{d\gamma}{d\alpha} = -\frac{(1-\phi)\,\frac{\eta v_h + \gamma v_\ell}{\eta+\gamma} - \Delta + qk''\left(\frac{\gamma}{(1-\eta)(1-\mu)(1-\alpha)}\right)\frac{\gamma}{(1-\eta)(1-\mu)(1-\alpha)^2}}{(1-\alpha)(1-\phi)\,\frac{\eta(v_h - v_\ell)}{(\eta+\gamma)^2} + qk''\left(\frac{\gamma}{(1-\eta)(1-\mu)(1-\alpha)}\right)\frac{1}{(1-\eta)(1-\mu)(1-\alpha)}} \tag{63}$$

$$= -\frac{(\eta+\gamma)\left[\eta v_h + \gamma v_\ell - (\eta+\gamma)\frac{\Delta}{1-\phi}\right] + qk''\left(\frac{\gamma}{(1-\eta)(1-\mu)(1-\alpha)}\right)\frac{\gamma(\eta+\gamma)^2}{(1-\phi)(1-\eta)(1-\mu)(1-\alpha)^2}}{(1-\alpha)\eta(v_h - v_\ell) + qk''\left(\frac{\gamma}{(1-\eta)(1-\mu)(1-\alpha)}\right)\frac{(\eta+\gamma)^2}{(1-\phi)(1-\eta)(1-\mu)(1-\alpha)}}. \tag{64}$$

In the limit as $q \to 0$, we have

$$\frac{\partial}{\partial \eta}\left(\frac{d\gamma}{d\alpha}\right) = \frac{\partial}{\partial \eta}\left(-\frac{(\eta+\gamma)\left(v_h - \frac{\Delta}{1-\phi}\right) - \left(1 + \frac{\gamma}{\eta}\right)\gamma\left(\frac{\Delta}{1-\phi} - v_\ell\right)}{(1-\alpha)(v_h - v_\ell)}\right) \tag{65}$$

$$= -\frac{v_h - \frac{\Delta}{1-\phi} + \left(\frac{\gamma}{\eta}\right)^2 \left(\frac{\Delta}{1-\phi} - v_\ell\right)}{(1-\alpha)(v_h - v_\ell)}, \tag{66}$$

which is strictly negative because, by assumption, $v_\ell < \frac{\Delta}{1-\phi} < v_h$. Since $\frac{d\gamma}{d\alpha}$ is a continuous function of q, this implies that, if $\underline{q} \leq 0$ (and hence $m^* \in (0,1)$ for any $q \geq 0$), there exists a threshold $\hat{q} > 0$ such that, for all $q < \hat{q}$, $\frac{\partial \Psi'}{\partial \eta} > 0$ and hence $\frac{d\alpha}{d\eta} > 0$. $\square$

Proof of Proposition 5. When $\beta = \infty$, the CRA's screening intensity is the same across both periods. Denote this screening intensity by α. By assumption, $\eta v_h + (1-\eta)v_\ell \geq 0$. Because $\alpha \leq \alpha_{\max} < 1$, in period 2 we have $\gamma_j = (1-\mu_j)(1-\eta)(1-\alpha)m_j < 1-\eta$ for each state $j = s, f, h$. Thus, for each state $p_j > 0$, where p_j is the price of the high-rated security in state j. In period 2, the continuation payoff Δ_2 is zero, so $(1-\phi)p_j > \Delta_2$, and our analysis of the single-period model when $q = 0$ implies that the equilibrium manipulation intensity is $m_j^* = 1$ for $j = s, f, h$.

Now, consider the difference in the opportunistic issuer's continuation payoffs in period 1, $\Delta(\delta) = \pi_f - \pi_s$. In state j in period 2, this issuer obtains a high rating with probability $\eta + (1 - \eta)(1-\alpha)$ (because $m_j^* = 1$). The price of the security in state j is $p_j = p(\mu_j, \alpha, 1)$. Thus,

$$\Delta(\delta) = \delta(1-\phi)[\eta + (1-\eta)(1-\alpha)]\left[p(\mu_f, \alpha, 1) - p(0, \alpha, 1)\right]. \tag{67}$$

If $q = 0$ and $m_1^* \in (0,1)$, the equilibrium condition in the two-period model in equation (18) reduces to

$$(1-\phi)p(\mu_1, \alpha, m_1^*) = \Delta(\delta). \tag{68}$$

Since $p(\mu, \alpha, m)$ is strictly increasing in μ and $\mu_f > \mu_s = 0$, we have $p(\mu_f, \alpha, 1) > p(0, \alpha, 1)$. Thus, $\Delta(\delta) > 0$ for any $\delta > 0$. Hence, there exists a $\underline{\delta} > 0$ such that $(1-\phi)p(\mu_1, \alpha, 1) = \Delta(\underline{\delta})$ and a $\bar{\delta} > 0$ such that $(1-\phi)p(\mu_1, \alpha, 0) = \Delta(\bar{\delta})$. That is, $m_1^* = 1$ when $\delta = \underline{\delta}$ and $m_1^* = 0$ when $\delta = \bar{\delta}$. Furthermore, the first-period price $p(\mu_1, \alpha, m_1^*)$ decreases in m_1^*, the second-period price in state f, $p(\mu_f, \alpha, 1)$, increases in m_1^* (because μ_f increases in m_1^*), and the second-period price in state s, $p(0, \alpha, 1)$, is invariant to m_1^*. Hence, it follows that $\underline{\delta} < \bar{\delta}$ and that $m_1^* \in (0,1)$ for $\delta \in (\underline{\delta}, \bar{\delta})$.

Now, suppose that $\delta \in (\underline{\delta}, \bar{\delta})$ so that the manipulation intensity m_1^* satisfies the equilibrium condition in equation (68). Let $\gamma_1 = \gamma(\mu_1, \alpha, m_1^*) = (1-\mu_1)(1-\eta)(1-\alpha)m_1^*$ denote the first-period error rate. From equation (11), we know that the first-period price $p(\mu_1, \alpha, m_1^*)$ can be written in terms of γ_1. Let $w(\alpha, \gamma_1 \mid \mu_1) = (1-\phi)p(\alpha, \gamma_1 \mid \mu_1) - \Delta(\delta)$. Then, equation (68) may be written as $w(\alpha, \gamma_1 \mid \mu_1) = 0$. By the implicit function theorem,

$$\frac{d\gamma_1}{d\alpha} = -\frac{\partial w/\partial \alpha}{\partial w/\partial \gamma_1}. \tag{69}$$

Now, an increase in γ_1 reduces p, the first-period price. Further, it leads to an increase in μ_f, the posterior probability of the truthful issuer in state f, and hence an increase in the second-period price in state f, $p(\mu_f, \alpha, 1)$. That is, it increases $\Delta(\delta)$. Thus, $\frac{\partial w}{\partial \gamma_1} < 0$. It therefore follows that $\frac{d\gamma_1}{d\alpha} > 0$ if and only if $\frac{\partial w}{\partial \alpha} > 0$.

Since the first-period price $p(\mu_1, \alpha, m_1^*)$ depends on α only through its effect on γ_1 (i.e., $\frac{dp}{d\alpha} = 0$ for γ_1 fixed), $\frac{\partial w}{\partial \alpha}$ has the opposite sign to $\frac{\partial \Delta}{\partial \alpha}$. From the definition of $\Delta(\delta)$ in equation (67), we have

$$\Delta(\delta) = \delta(1-\phi)[\eta + (1-\eta)(1-\alpha)] \left(\frac{\eta v_h + (1-\mu_f)(1-\eta)(1-\alpha)v_\ell}{\eta + (1-\mu_f)(1-\eta)(1-\alpha)} - \frac{\eta v_h + (1-\eta)(1-\alpha)v_\ell}{\eta + (1-\eta)(1-\alpha)} \right) \tag{70}$$

$$= \delta(1-\phi) \frac{\mu_f \eta (1-\eta)(1-\alpha)(v_h - v_\ell)}{\eta + (1-\mu_f)(1-\eta)(1-\alpha)} \tag{71}$$

$$= \delta(1-\phi) \frac{\mu_f \eta (v_h - v_\ell)}{1 - \mu_f + \frac{\eta}{(1-\mu_f)(1-\eta)(1-\alpha)}}. \tag{72}$$

Since $\mu_f = \frac{\mu_1}{1 - \frac{\gamma_1}{1-\eta}}$ does not depend on α for a given γ_1, it follows immediately that $\frac{\partial \Delta}{\partial \alpha} < 0$. Hence, $\frac{d\gamma_1}{d\alpha} > 0$. $\square$