Aggregate Asset-Pricing Implications of Human Capital Mobility in General Equilibrium*

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Abstract

We present a model in which aggregate mobility of labor affects risk and expected equity returns. Our setup is based on a multi-industry dynamic economy with production. We consider two different types of human capital. Generalist human capital can move between industries, while specialized human capital and physical capital cannot. The greater relative mobility of human capital relative to physical capital affects how aggregate risk in the economy is split between these two components of total wealth. We show that aggregate consumption and wealth increase when human capital is more mobile. However, at the same time, aggregate risk and the equity risk premium also increase under human capital mobility.

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1 Introduction

Human capital is the most important input for the production of goods and services in the economy and the main source of aggregate wealth.\(^1\) However, unlike physical capital, such as buildings or machines, human capital can literally walk away from the firm as managers and other employees switch employers. Observable flows of workers are significant and vary over time and across industries.\(^2\) This fact suggests that labor mobility is a promising macro variable for asset pricing and for the analysis of human capital. This paper extends the micro-founded labor mobility mechanism in Donangelo [2010] to shed light on the role of aggregate labor mobility as a macro-variable that predicts expected stock returns in the time-series.

We analyze the mechanism through which labor mobility affects equity risk and expected returns in a multi-industry dynamic general equilibrium economy. We endogenize aggregate labor mobility through heterogeneous types of human capital available in labor markets. In particular, individuals endowed with a “general” type of human capital have flexibility to move across industries, while individuals endowed with “specialized” human capital types have less flexibility. We show that, in this setting, aggregate labor mobility affects both conditional betas and the market risk premium.\(^3\)

Our model can be interpreted as a generalization of the “Two-Trees” model of Cochrane et al. [2008]. Whereas Cochrane et al. [2008] specify the dividend processes exogenously, our production-based model provides a mechanism through which labor mobility impacts the dividend processes of the two industries. Furthermore, our work builds upon a growing literature that explores the theoretical relationship between labor and stock returns (for instance, Danthine and Donaldson [1992], Danthine and Donaldson [2002], Boldrin et al. [2001], and Berk and Walden [2010])\(^4\) and that studies channels through which labor can affect stock returns. For instance, Merz and Yashiv [2007] focus on firms’ hiring decisions and Chen et al. [2010] consider labor unions. Donangelo [2010] constructs an empirical measure of labor mobility based on the occupational profile of workers across

\(^{1}\)See for example Jorgenson and Fraumeni [1989], Baxter and Jerman [1987], Lustig and Nieuwerburgh [2008], Lustig et al. [2009].

\(^{2}\)Kambourov and Manovskii [2008] show that realized worker mobility in the US has increased significantly since the late 1960s, even across broad industry classifications. Annual mobility across one-digit industries has increased from 7 percent in 1968 to 12 percent in 1997. Donangelo [2010] shows that industries differ significantly in the extent to which they rely in general versus industry-specific labor skills.

\(^{3}\)The conditional Capital Asset Pricing Model holds exactly with log utility and approximately with higher levels of relative risk aversion, as in Santos and Veronesi [2006].

\(^{4}\)In addition, various papers show empirical evidence of the impact of human capital on stock returns, such as Shiller [1995], Jagannathan and Wang [1996], Campbell [1996], Lustig and Nieuwerburgh [2008], and Eiling [2009].
industries. This measure is positively related to expected stock returns in the cross-section, which is in line with our model.

We show that aggregate labor mobility, i.e. the importance of generalist workers in the economy, increases the equity risk premium. Higher levels of aggregate labor mobility improve the allocation of resources in the economy and thereby increase consumption. In general, aggregate labor mobility makes the representative agent wealthier. However, at the same time, higher labor mobility also increases aggregate risk. The mechanism is as follows. First, the benefits from aggregate labor mobility are largest when productivity is different across industries. When one industry is much more productive than the other, workers switch to the more productive industry, thereby increasing production. As a result, shocks to the economy are magnified due to labor mobility.\(^5\) For instance, when generalist workers leave an industry after a negative shock, the productivity of capital in the industry is decreased even more. Thus, differences in productivity across industries become larger and the benefits from labor mobility are reduced. In other words, aggregate labor mobility creates a new source of risk; the risk of losing the benefits from aggregate labor mobility as the relative productivity between industries changes. In sum, when generalist mobile labor is a more important production factor, the representative agent becomes wealthier but also faces higher risk; the equity premium increases.

In our model, the risk-return profile of the generalist human capital type is close to that of the equity market portfolio, while the profile of specialist human capital is similar to that of equity in the worker’s industry. Therefore, stock markets are more effective for diversifying industry-specific labor risk for specialists than for more mobile generalists. All else equal, we should expect to see that mobile workers diversify less in the stock market while specialized workers choose to hold a large number of different stocks to diversify their risk.

Our specification of aggregate labor mobility is motivated by the traditional literature on human capital that distinguishes between general and specific labor skills. This literature points out how different types of labor skills, or human capital, affect labor mobility.\(^6\) Workers who possess specialized industry-specific skills are, in some sense, “locked” inside their industries of specialization.\(^7\)

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\(^5\)Note that in our model, all shocks are systematic.

\(^6\)See for example Becker [1964] and Castania and Helfat [2001]. Helwege [1992] and Neal [1995] are examples of empirical studies that suggest that sector-specific skills reduce labor mobility and thus increase labor risk.

\(^7\)We illustrate this with an example: if an investment banker were randomly chosen and asked to seek employment in another industry, she would probably receive lower compensation in her new job than in the financial industry.
In contrast, workers endowed with more general skills that are useful across industries have more inter-industry labor mobility.

We solve a special case of the economy with only two industries. Firms in each industry have identical constant-returns-to-scale production technologies that employ both industry-specific and general human capital as production factors, in addition to physical capital. There is perfect competition of firms and perfect labor markets within each industry. In particular, all workers can freely move across firms inside their industry. This simplification allows us to explore the labor mobility mechanism, while keeping the solution tractable. In this simple case, there are three types of workers, each endowed with a single type of human capital that determines their mobility: specialists who can only work in one of the two industries and generalists who possess skills to work in both industries.

The rest of the paper proceeds as follows. Section 2 discusses literature related to this work. Section 3 presents the economy and equilibrium and section 4 solves the model for the special case of two industries. Section 5 discusses the main results and section 6 concludes. The appendix contains proofs and further details on the derivation of our model.

2 Related literature

This paper is related to a growing literature that studies the dynamics of the interaction between labor and financial markets. This literature started with Danthine and Donaldson [1992], where labor market frictions expose workers to labor income shock oscillations and generate realistic cyclical variability in the share of wages to output. This seminal work, and virtually all the literature that follows it, relies on restrictions on the space spanned by available securities or restrictions on the access to financial markets of some of the agents to generate realistic results. Our main departure point and contribution is to replace such frictions by the assumption that workers have different degrees of mobility to transit across industries. This assumption generates interesting and novel results, even in a neo-classical framework with complete markets and full stock market participation.
Danthine and Donaldson [2002] is the first paper to discuss the dynamic asset pricing implications of labor induced amplification of shocks to capital. This work shows that the ratio of labor income to revenues over the business cycle acts as operating leverage that amplifies shocks to capital owners in bad times. The effect of their mechanism is similar to that of labor mobility in our model. Danthine et al. [2006] measure changes in labor operating leverage in the economy and conclude that it is of first order of importance for asset pricing, more so than aggregate shocks. Using their model, they show that time varying distributional risk is equivalent to habit formation.

A strand of the literature recognizes the economic importance of factor mobility for asset pricing. Boldrin et al. [2001] was the first asset pricing model to point out that limited inter-sectorial mobility of labor and capital is an important element that, in conjunction with habit formation, helps to explain the equity premium and risk-free rate puzzles. Most of this literature relies on significant labor adjustments costs faced by firms in order to generate implications for expected stock returns. We depart from this literature by focusing on mobility constraints for the supply side of labor instead of for the demand side of labor.

Our work is closest in spirit to Berk and Walden [2010] and Donangelo [2010]. Berk and Walden [2010] show that limited market participation naturally emerges from incomplete markets. In their model, agents with different degrees of mobility enter into binding long-term labor contracts. They find that flexible workers become the owners of equity and insure inflexible workers through these contracts. Under complete markets, we find that mobile workers have fewer incentives to diversify their risk in stock markets given that their human capital is closer to the fully diversified equity market portfolio.

Donangelo [2010] studies a similar relation between labor supply mobility and expected stock returns. In a partial equilibrium framework, the paper shows that the mobility of the firm’s workforce increases the volatility of its profits, equity volatility and expected returns. While our paper focuses on the asset pricing implications of aggregate labor mobility, the relation between industry-specific labor mobility and industry equity returns can also be seen in the context of our model.

The setup of our model is based on Palacios [2009]. That paper generates a counter-cyclical consumption to labor income ratio based on a Real Business Cycle model. The mechanism helps to explain the lower risk of aggregate wealth and human capital relative to equity. We extend Palacios [2009] to a multi-sector economy, allowing a fraction of the labor force to be mobile across industries.
3 A general equilibrium model with different degrees of labor mobility

We derive a dynamic general equilibrium model with multiple industries in which agents possess different types of human capital, represented by labor skills. The agents’ bundles of labor skills determine their labor mobility. We first discuss the general model setup with \( I \) industries and \( N \) types of human capital. In the following section we derive the solution of the model for the special case where \( I = 2 \) and \( N = 3 \). This keeps the solution to the model tractable, while showing the main mechanisms through which labor mobility and human capital affect risk and expected returns.

3.1 General model setup

3.1.1 Economic environment

The setting of the model is a competitive production economy with \( I \) different industries. Time is continuous and the time horizon is infinite. Agents are endowed with a bundle of \( N \) types of human capital skills that are used as input for different industries. We assume that agents can trade claims that span all possible outcomes of the economy. Hence, markets are complete.\(^{10}\) The economy has three markets: 1) a labor market for each type of human capital, 2) the market for goods produced in the economy, and 3) the financial market, where financial claims are traded. We are interested in pricing three types of claims: claims to different types of human capital, equity in different industries and the equity market portfolio, and an instantaneous risk free bond.

3.1.2 Agents

Each agent is endowed with an initial allocation of claims \( X_{j,0} \). The vector \( X_{j,ehc} \) contains the elements of \( X_j \) that are claims to actual production: equity and human capital. Both these claims have a net supply of one. \( X_{j,fin} \) contains the elements of \( X_j \) that are financial claims which are in zero net supply, such as a risk free bond and other contingent claims. Together, \( X_{j,ehc} \) and \( X_{j,fin} \) contain all the elements of \( X_j \). Each agent \( j \) is also endowed with a bundle of human capital skills \( S_j \), where

\(^{10}\)In this setup, human capital is considered a tradable asset. Several existing papers also treat human capital as a tradable asset (e.g. Jagannathan and Wang [1996] and Lustig and Nieuwerburgh [2008]), while other papers consider human capital to be nontradable (e.g. Mayers [1972]). In reality, human capital is arguably in between fully tradable and fully nontradable. The assumption of tradability and hence complete markets ensures the existence of a unique pricing kernel.
$S$ is a vector of length $N$. The $n^{th}$ element of vector $S_j$ corresponds to the units of human capital agent $j$ holds of skill $n$. A skill can be interpreted as an occupation, a certain type of education, the ability to perform a certain task, or all of these things combined. Without loss of generality, we normalize skills in the economy so that:

\[ \sum_{k=1}^{N} \int_j S_k dj = 1. \]  

(1)

3.1.3 Production

There are $I$ different industries in the economy. Each industry has a continuum of firms which produce one specific type of good. Production of each kind of good is accomplished by combining industry-specific physical capital and different types of human capital skills. We assume physical capital is immobile, so that the aggregate physical capital available to each industry cannot be changed. A possible interpretation is that industries are bundles of capital and occupations producing a certain good. The difference between industries lies in the different combination of production inputs required to produce goods. Production in industry $i$ will be given by:

\[ Y_i = Z_i \prod_{x=0}^{N} F_x^{\alpha_{x,i}}. \]  

(2)

This functional form corresponds to the Cobb-Douglas production function, where $Z_i$ is total factor productivity, $F_x$ is the input of production factor $x$, and $\alpha_{x,i}$ is the intensity of factor $x$ in the production function of industry $i$. Note that input $x = 0$ corresponds to industry-specific physical capital. To achieve constant returns of scale that allow us to aggregate firms into industry-representative firms, we add the condition $\sum_{x=0}^{I} \alpha_{x,i} = 1$.

3.1.4 Labor mobility

A production factor’s mobility depends on it being used in different industries with the same intensity $\alpha_{x,i}$. If an agent’s human capital consists mainly of skill $x$, and this skill is used as an input in many different industries, the agent’s labor mobility is relatively high. On the other hand, if skill $x$ is used only in one industry, the agent’s human capital is relatively immobile. In our setting, labor mobility can be measured similar to Donangelo [2010], as the Gini coefficient calculated over the $\alpha_{x,i}$ across
all $I$ industries. This provides a measure of mobility of human capital skill $x$. A Gini coefficient of 1 would imply that a given factor (human capital skill) is used equally in all industries, implying high levels of mobility for that factor. A Gini coefficient of 0 would imply that a given labor skill is only used in one industry; in this case there is no mobility. In our model, labor markets within industries are perfect, and therefore there is full mobility within each industry. Consequently, wages are set at the marginal product of labor in the industry.

Our definition of mobility stresses the use that different factors of production might have in different industries, rather than the possibility of physically moving the factor to meet some production need. We choose this specification because the largest fraction of production in the economy is human capital. Within any geographical market with multiple industries, the largest issue for an agent’s labor mobility will be whether she has the skills to participate in a given industry, rather than whether she can actually switch the location of where she goes to work. We recognize that geographical and legal frictions reduce mobility, but those are beyond the scope of this paper. Also, several labor economics papers show that occupation and industry tenure reduce labor mobility (see e.g. Jovanovic [1979], McLaughlin and Bils [2001], and Kambourov and Manovskii [2008]). In our model, an agent’s labor mobility is constant over time and determined entirely by her initial endowments of human capital skills. Making the agent’s labor mobility a function of her industry tenure would require an overlapping generations model, which is also beyond the scope of this paper.

### 3.1.5 Preferences

Agents consume a bundle of goods, ranking the utility according to the following relation:

$$U(C_1, C_2, ..., C_I) = \frac{1}{1-\gamma} \left( \sum_{i=1}^{I} \theta_i C_i^\rho \right)^{\frac{1-\gamma}{\rho}}.$$

(3)

Recall $I$ is the number of industries in the economy. $C_i$ is the agent’s consumption of good $i$. This specification assumes a Constant Elasticity of Substitution (CES) between different goods, with Constant Relative Risk Aversion (CRRA) preferences for aggregate bundles. The CES specification can also be viewed as the production function of a single consumption good using multiple intermediate goods as inputs. $\rho$ determines the substitutability across goods: a positive $\rho$ implies that the goods are substitutes, while a negative $\rho$ implies the goods are complements. When $\rho \to 0$, the CES aggre-
An agent’s lifelong utility is calculated as the standard discounted sum of each period’s utility:

$$LU = E_t \left[ \int_t^\infty \frac{e^{-\beta \tau}}{1 - \gamma} \left( \sum_{i=1}^I \theta_i C_{i,\tau}^{\rho,1-\gamma} \right) d\tau \right],$$  \hspace{1cm} (4)

where $\beta$ is the subjective discount rate. The assumption that all agents have identical preferences ensures the existence of an aggregate agent.

### 3.1.6 Uncertainty

Denote by $(\Omega, \mathcal{F}, \mathbb{P})$ a fixed complete probability state, and the stochastic process $(B_t)_{t \geq 0}$, a standard $I$-dimensional Brownian motion with respect to the filtration $(\mathcal{F}_t)$. The Brownian motion drives shocks to productivity in each industry $(Z_i)$ such that the dynamics of productivity are as follows:

$$dZ_{i,t} = \eta_i Z_{i,t} dt + \sigma_i Z_{i,t} dB_i, \text{ for } i \in \{1, ..., I\}. \hspace{1cm} (5)$$

We denote the instantaneous correlation between any two shocks by $\varphi_{i,j}$.

### 3.1.7 Firm’s maximization problem

The firm’s objective is to maximize the present value of dividends for shareholders. Capital is fixed in our model, which implies that there are no investments and no depreciation. Perfect competition implies that firms take the price for their output $P_i$ and the stochastic discount factor $M_t$ as given. The only decision left for a firm is the amount demanded of each type of human capital at any given period. This decision is denoted by the $N$-vector $L_i$. All revenues net of labor expenses are given back to shareholders as dividends $D_i$. For all firms in industry $i$, the optimization problem is as follows:

$$\max_{\{L_i\}_i} \left. E_t \left[ \int_t^\infty M_t D_{i,\tau} d\tau \right] \right|_{(6)}$$

subject to

$$D_{i,t} = P_{i,t} Y_{i,t} - W_{i,t} \cdot L_{i,t}, \quad \forall \ t \in [0, \infty), \hspace{1cm} (7)$$
where \( W_t \) is the \( N \)-dimensional vector of wages per unit of human capital.

### 3.1.8 Agent’s maximization problem

Agent \( j \)'s problem consists of maximizing her lifelong utility, taking prices and wages as given, subject to her budget constraint. The present value of her consumption is limited by the present value of her human and financial wealth. We assume the agent does not have any utility over leisure, and therefore optimally chooses to offer labor inelastically.\(^{11}\) The agent’s problem can be expressed as:

\[
\max_{\{C_t\}^\infty} \mathbb{E}_t \left[ \int_t^\infty e^{-\beta \tau} \left( \sum_{i=1}^I \theta_i C_{i,t}^0 \right)^{\frac{1-\gamma}{\rho}} d\tau \right] 
\]

\[
\text{s.t.} C_t' \cdot P_t = W_t' \cdot S_{j,t} + X_{j,t}' \cdot D_t, \quad \forall \; t \in [0, \infty).
\]

### 3.2 Definition of equilibrium

Goods, labor, and financial markets must clear in equilibrium. The definition that follows is standard:

**Definition 1.** In this economy, an equilibrium is defined as a stochastic path for the tuple:

\[
\{\{Z_{i,t}\}^I_{1}, \{L_{i,t}\}^N_{1}, \{P_{i,t}\}^I_{1}, \{W_{i,t}\}^N_{1}, \{Y_{i,t}\}^I_{1}, \{D_{i,t}\}^I_{1}, \{X_{j,t}\}_j, M_t\}^\infty_t
\]

such that, for every \( t \):

1. Given \( \{Z_{i,t}, P_{i,t}, \{W_{i,t}\}^N_{1}, M_t\} \), each firm chooses \( L_{i,t} \) to maximize the present value of dividends (Equation (6)).

2. Given the processes for \( \{\{P_{i,t}\}^I_{1}, \{W_{i,t}\}^N_{1}, D_t, M_t\} \), the agent chooses \( \{C_{i,t}\}^I_t \) to maximize his expected lifelong utility, subject to the transversality condition (Equation (8)).

3. Goods markets clear: \( C_{i,t} = Y_{i,t} \).

4. Labor markets clear: \( L_{j,t} = S_j \).

\(^{11}\)This implies full employment in the economy.
5. Financial markets clear. This implies for aggregate equity and human capital shares \( \int_{j} X_{ehc,t} = 1 \) and for financial shares \( \int_{j} X_{fin,t} = 0 \).

4 Special case: Two industries and three types of human capital

Here we study a special case where the economy consists of two industries A and B and each agent is endowed with one of three types of human capital. In particular, each industry requires a unique industry-specific type of human capital skill as well as a general human capital skill.

4.1 Characterization of the two-sector economy

4.1.1 Preferences

In a two-good economy the representative agent’s utility simplifies to:

\[
U(C_A, C_B) = \frac{\left(\theta_A C_A^\rho + \theta_B C_B^\rho\right)^{\frac{1}{\gamma}}}{1 - \gamma}. \tag{10}
\]

The term \( \left(\theta_A C_A^\rho + \theta_B C_B^\rho\right)^{\frac{1}{\gamma}} \) can be interpreted as the agent’s instant utility from the consumption of a basket containing \( C_A \) and \( C_B \) of final goods A and B, respectively. \( \left(\theta_A C_A^\rho + \theta_B C_B^\rho\right)^{\frac{1}{\gamma}} \) can also be interpreted as the amount produced of a final good C from amounts \( C_A \) and \( C_B \) of intermediate goods A and B, respectively. Under the latter interpretation, the agent solely derives utility from consuming the final good C. Both interpretations lead to the exact same results.

4.1.2 Human capital skills of each type of agent

We assume that there are three types of agents, each endowed with a different labor skill. We label each type of agent as \( \{A, G, B\} \). The stock of labor skills of each type of agent is:

\[
S_A = \{1, 0, 0\} \tag{11a}
\]

\[
S_G = \{0, 1, 0\} \tag{11b}
\]

\[
S_B = \{0, 0, 1\}. \tag{11c}
\]
Hence, agents endowed with $S_A$ only have the skills to work in industry A, while agents endowed with specific skills $S_B$ can only work in industry B. Agents with general skills $S_G$ can work in both industries. We denote as $\xi_A$, $\xi_G$, and $\xi_B$, as the proportions of each type of agent in the total population, which are exogenously determined.

### 4.1.3 Production in industries A and B

Production in both industries simplifies to:

\[
Y_A = Z_A K_A^{1-\alpha_A} (L_A^{1-\psi_A} L_{G,A}^{\psi_A})^{\alpha_A} 
\]

\[
Y_B = Z_B K_B^{1-\alpha_B} (L_B^{1-\psi_B} L_{G,B}^{\psi_B})^{\alpha_B}. 
\]

The intensity of physical capital $K_i$ in the production function of industry $i$ is given by $(1 - \alpha_i)$. The intensity of general human capital input is given by $\psi_i \alpha_i$, and the intensity of industry-specific human capital input is given by $(1 - \psi_i) \alpha_i$. In the general model with $I$ industries, the Gini coefficient over the intensity of a certain type of human capital in the production process across all industries is a indication of its mobility. Similarly, in the case of two industries, the mobility of the generalist human capital depends on the difference between $\psi_A \alpha_A$ and $\psi_B \alpha_B$. If these two intensities are equal, the generalist human capital is fully mobile. The greater the difference, the lower the mobility. In the extreme, this type of human capital is only used in one industry, making it fully immobile specialist human capital.

### 4.2 Prices, wages and dividends in equilibrium

In this section we first derive equilibrium good prices and wages. We then use the dynamics of consumption and the stochastic discount factor to find the prices of claims to each type of human capital, equity prices in each industry, and the risk-free rate. We also price the equity market portfolio, which is a claim to the sum of the dividends in the two industries.

#### 4.2.1 Prices of goods and wages in equilibrium

We set the weighted production of goods A and B, $Y \equiv (\theta_A Y_A^\rho + \theta_B Y_B^\rho)^{\frac{1}{\rho}}$, as the numeraire in the economy. Non satiation and absence of investments implies that revenues equal consumption.
expenses:

\[ Y_A P_A + Y_B P_B = (\theta_A Y_A^\rho + \theta_B Y_B^\rho)^{\frac{1}{\rho}}. \]  

(13)

where \( P_A \) and \( P_B \) are the prices of goods A and B respectively. Taking partial derivatives with respect to \( Y_A \) and \( Y_B \) on both sides, leads to the market clearing conditions:

\[ P_A = \theta_A Y_A^{1-\rho} Y_A^{\rho - 1} \]  

(14a)

\[ P_B = \theta_B Y_B^{1-\rho} Y_B^{\rho - 1}. \]  

(14b)

Prices and wages are a function of variables known instantaneously. To save on notation, we omit the subscript \( t \) unless strictly necessary for clarity. Taking wages for each type of human capital and the price at which the firm can sell its products as given, the firm’s first order condition implies the following demands for labor, \( i \in \{ A, B \} \):

\[ L_{G,i} = \frac{Y_i P_i \psi_i \alpha_i}{w_G} \]  

(15)

\[ L_i = \frac{Y_i P_i (1 - \psi_i) \alpha_i}{w_i}, \]  

(16)

where \( L_i \) is the demand for human capital specific to industry \( i \), \( w_i \) is the wage rate for human capital specific to industry \( i \), and \( w_G \) is the wage rate for general human capital. Hence, the vector of wage rates per unit of labor takes the form \( W = \{ w_A, w_G, w_B \} \). Note that because labor markets clear and specific human capital is only employed in one industry, we have \( \xi_A = L_A \) and \( \xi_B = L_B \). This implies that expression (16) pins down wages for industry-specific human capital, which are proportional to the dividends in the industry. The following lemma summarizes the demand for general human capital in the two industries.

**Lemma 1.** Let \( \Phi = \{ K_A, K_B, \alpha_A, \alpha_B, \psi_A, \psi_B, \xi_G, \rho, \theta_A, \theta_B \} \). Then the equilibrium amount of generalist labor employed by industry A is \( L_{G,A} \equiv L_{G,A}(\Phi, Z_A, Z_B) \). \( L_{G,A}(\Phi, Z_A, Z_B) \) satisfies the following equation:

\[ 1 = \frac{\theta_A \psi_A \alpha_A}{\theta_B \psi_B \alpha_B} \left[ \frac{Z_A K_A^{1-\alpha_A} L_A^{(1-\psi_A)\alpha_A}}{Z_B K_B^{1-\alpha_B} L_B^{(1-\psi_B)\alpha_B}} \right]^{\rho} \frac{L_{G,A}^{\psi_A \alpha_A \rho - 1}}{(\xi_G - L_{G,A})^{\psi_B \alpha_B \rho - 1}}. \]

(17)

Proof: See Appendix.
Note that while an explicit solution for $L_{G,i}$ is not available, finding the solution numerically is trivial. Since every worker with general skills will earn the same wage, irrespectively of their industry of employment, market clearing implies that the wage of the general type of human capital is given by:

$$w_G = \frac{Y^{1-\rho}}{\xi_G} \left( Y_A^\rho \theta_A \psi_A \alpha_A + Y_B^\rho \theta_B \psi_B \alpha_B \right).$$

(18)

The next step in solving the model is finding the prices of claims to equity, each type of human capital, and the risk-free rate. First, we derive the dynamics of the stochastic discount factor.

### 4.3 Dynamics of consumption and the stochastic discount factor

At this point, it is convenient to change variables to simplify the solution of the model. We define $Z_T = Z_A + Z_B$ and $s = \frac{Z_A}{Z_A + Z_B}$. $Z_T$ is a measure of total productivity in the economy, while $s$ is a relative measure of productivity between the two technologies. An increase in $s$ implies that industry $A$ has become relatively more productive than industry $B$. Using these variables is helpful because they lend to interpretation of “aggregate shocks” (shocks to $Z_T$) and “industry shocks” (shocks to $s$). As shown below, all the asset pricing relationships depend on aggregate productivity and on the relative changes of productivity between industries. The function describing the amount of general labor in industry $A$ can then be expressed as $L_{G,A} \equiv L(\Psi, s)$.

The dynamics of the state variables $Z_T$ and $s$ are:

$$\frac{dZ_{T,t}}{Z_{T,t}} = (s_t \eta_A + (1 - s_t) \eta_B) dt + s_t \sigma_A dB_A + (1 - s_t) \sigma_B dB_B$$

(19)

$$\frac{ds_t}{s_t} = (1 - s_t) \left[ (\eta_A - \eta_B) - s_t \sigma_A^2 + (1 - s_t) \sigma_B^2 + (2s_t - 1) \sigma_A \sigma_B \varphi_{A,B} \right] dt$$

$$+ (1 - s_t) \left[ \sigma_A dB_A - \sigma_B dB_B \right],$$

(20)

where $\varphi_{A,B}$ is the correlation between the two Brownian motions $B_A$ and $B_B$. The dynamics of the productivity shares in the two industries $s_t$ are similar to the dynamics of dividend shares of the two

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12The expression below is obtained by setting $L_{G,A} + L_{G,A} = \xi_G$ and solving for $w_g$. 

---
assets in the model of Cochrane et al. [2008]. The drift of $ds_t$ equals zero if $s_t = 0, 1$ or $\varpi$, where

$$\varpi = \frac{(\eta_A - \eta_B) + \sigma_B^2 - \varphi_{A,B}\sigma_A\sigma_B}{\sigma_A^2\sigma_B^2 - 2\varphi_{A,B}\sigma_A\sigma_B}. \quad (21)$$

If $0 < \varpi < 1$, the drift is positive for $s_t \in (0, \varpi]$ and negative for $s_t \in (\varpi, 1)$. The diffusion of $ds_t$ is the largest when $s_t = 0.5$, so the productivity share process is most volatile if productivity is the same in the two industries. This share process $s_t$ will be nonstationary, as ultimately one of the two Brownian motions will dominate. This less desirable feature is common for this type of share process (see also Cochrane et al. [2008]).

We first derive the dynamics of consumption. Denote the equilibrium value of a consumption bundle by $CO(Z_t, s_t)$ as:

$$CO(Z_{T,t}, s_t) = \left(\theta AC_A^p + \theta BC_B^p\right)^\frac{1}{p}. \quad (22)$$

The utility the aggregate agent derives from the equilibrium mix of goods produced in the economy can be rewritten as:

$$U = \frac{CO(Z_{T,t}, s_t)^{1-\gamma}}{1-\gamma}. \quad (23)$$

We start by showing that the optimal bundle of the representative agent is a function only of $Z_T$ and $s$. The following lemma formalizes the result:

**Lemma 2.** Given $\Phi$, $Z_{T,t}$ and $s_t$, the equilibrium consumption bundle consumed by agents in the economy is:

$$CO(Z_{T,t}, s_t) = Z_{T,t}co(s_t). \quad (24)$$

The first derivative and second derivative of $co(s_t)$ are given by:

$$co'(s_t) = co(s_t)F(s_t) \quad (25)$$

$$co''(s_t) = co(s_t)(G(s_t) + F(s_t)^2), \quad (26)$$
where
\[ F(s_t) = \frac{f(s_t) - 1}{(1 - s_t)} \]  
(27)

\[ G(s_t) = \frac{1}{s_t^2(1 - s_t)^2} \left[ -f(s_t) \left[ (1 - 2s_t) - g(s_t)(1 - f(s_t)) \right] - s_t^2 \right], \]  
(28)

and
\[ f(s_t) = \frac{P_A Y_A}{P_A Y_A + P_B Y_B}. \]  
(29)

\[ g(s_t) = \left( \frac{1 - \alpha_A \psi_A}{\rho - \alpha_A \psi_A} + \frac{Y_A^p \alpha_A \xi_G}{Y_A^p \alpha_A \psi_A + Y_B^p \psi_B \alpha_B} (\alpha_A \phi_A - \alpha_B \phi_B) \right)^{-1}. \]  
(30)

Proof: See Appendix.

The production share ratio can be interpreted as the ratio of total dividends paid out in the two industries, consisting of dividends paid to equity holders and dividends paid to workers (i.e. wages). This allows us to link our consumption dynamics to those in the two-asset model of Cochrane et al. [2008]. Similar to our model, their consumption volatility depends on the relative dividend share of the two assets. However, Cochrane et al. [2008] model the dividend processes exogenously, while our production-based model endogenizes dividend dynamics. In particular, we show a mechanism through which labor mobility affects dividends.

By Itô’s Lemma, the dynamics of consumption \( CO(Z_t, s_t) \) are as follows:

\[ \frac{dCO_t}{CO_t} = \mu_C dt + \sigma_{C,A} dB_A + \sigma_{C,B} dB_B, \]  
(31)

where
\[ \mu_C = s_t \eta_A + (1 - s_t) \eta_B + s_t (f(s_t) - 1)(\eta_A - \eta_B) \]
\[ + \frac{1}{2} \left[ -f(s_t) \left[ (1 - 2s_t) - g(s_t)(1 - f(s_t)) \right] - s_t^2 \right] (\sigma_A^2 + \sigma_B^2 - 2\sigma_A \sigma_B \varphi_{A,B}) \]  
(32)

\[ \sigma_{C,A} = \sigma_A f(s_t) \]  
(33)

\[ \sigma_{C,B} = \sigma_B (1 - f(s_t)). \]  
(34)

Note that conveniently, the dynamics of consumption do not depend on total productivity \( Z_T \), but they only depend on the relative productivity share \( s_t \). When relative productivity share \( s_t \) goes to zero or one, consumption dynamics are the same as in the single industry benchmark case: mean
consumption growth equals the mean productivity growth $\eta_i$ of the productive industry $i$ and consumption volatility equals its diffusion $\sigma_i$. When both industries are productive, mean consumption growth also depends on the mean of the difference in productivity growth of the two industries, $(\eta_A - \eta_B)$, and its variance, $\text{Var} \left( \frac{dZ_A}{Z_A} - \frac{dZ_B}{Z_B} \right)$. Consumption volatility depends on both $\sigma_A$ and $\sigma_B$, weighted by $f(s_t)$, which in turn is affected by labor mobility.

Having determined the utility from the basket of goods and its dynamics, we can find the dynamics of marginal utility and the stochastic discount factor. Marginal utility, measured in terms of the optimal basket of goods, will be given by:

$$CO(Z_{T,t}, s_t)^{-\gamma} = Z_{T,t}^{-\gamma}co(s_t)^{-\gamma}. \quad (35)$$

Thus, the stochastic discount factor equals:

$$M(t, Z_{T,t}, s_t) = e^{-\beta t}Z_{T,t}^{-\gamma}co(s_t)^{-\gamma}, \quad (36)$$

where $\beta$ is the subjective time discount rate. Since we know the dynamics of $Z_T$ and $s$, we can derive the dynamics of the stochastic discount factor:

$$\frac{dM_t}{M_t} = \mu_M dt + \sigma_{M,A} dB_A + \sigma_{M,B} dB_B, \quad (37)$$

where

$$\mu_M = -\beta - \gamma \mu_C + \gamma (1 + \gamma) \frac{\Omega_C^2}{2} \quad (38)$$

$$\Omega_C^2 = \sigma_{C,A}^2 + \sigma_{C,B}^2 + 2\varphi_{A,B} \sigma_{C,A} \sigma_{C,B} \quad (39)$$

$$\sigma_{M,A} = -\gamma \sigma_{C,A} \quad (40)$$

$$\sigma_{M,B} = -\gamma \sigma_{C,B} \quad (41)$$

Market completeness implies that the instantaneous risk-free rate equals the negative of the drift of the stochastic discount factor: $r_{f,t} = -\mu_{M,t}$. The expression for $r_{f,t}$ is the same as in the standard case with only one sector in the economy. The risk free rate is determined by the standard subjective discount rate, expected consumption growth and a precautionary savings term, which depends on the
variance of consumption growth. However, while in the standard case $r_f$ is constant over time, in our two-sector economy it varies over time; $\mu_C$ and $\Omega_C$ both depend on the relative share of productivity in each industry, $s_t$.

In equilibrium, the risk premium on a claim equals the negative of the covariance between the SDF and the returns on the claim: $-\sigma_M \sigma_{Cl}$. In our model, labor mobility affects both $\sigma_M$ and $\sigma_{Cl}$. The results section of the paper discusses how labor mobility affects $\sigma_{Cl}$; here we first focus on $\sigma_M$.

As consumption volatility is affected labor mobility through $f(s_t)$, so is the diffusion of the stochastic discount factor (SDF):

\[
\begin{align*}
\sigma_{M,A} &= -\gamma \sigma_A f(s_t) \quad (42a) \\
\sigma_{M,B} &= -\gamma \sigma_B (1 - f(s_t)), \quad (42b)
\end{align*}
\]

This expression shows that volatility of the SDF (and hence the market price of risk) varies over time. In the special case where the economy is characterized by identical production functions, the production share $f(s_t)$ simplifies to:

\[
f(s_t) = 1 - \frac{1}{1 + \left(\frac{s_t}{1-s_t}\right)^{\frac{1}{\Psi}}} \quad (43)
\]

where $\Psi = \alpha_A \psi_A = \alpha_B \psi_B$. This case explicates the intuition for the effect of labor mobility on the time variation in the market price of risk, which is the main point of departure from an endowment economy. Higher values of $\Psi$ increase the importance of mobile workers have in the economy. As the productivity share $s_t$ varies stochastically over time, generalist workers move between industries A and B, thereby affecting dividends. This suggests that a measure of the dispersion of generalist workers across the economy may help forecast future market risk premia.

### 4.4 Value of equity and human capital

The next step in the analysis is to derive the value of claims in this economy. There are multiple ways to solve the problem. The easiest is to use the result that in a complete market the discounted process, including dividends, for any claim in the economy is a martingale. The economy can be characterized by $Z_T$ and $s$, so the solution to the value of any claim only depends on $Z_T$ and $s$. Furthermore,
given the structure of the economy and our use of CRRA utility, we can guess that the value of claim $C_l$ is of the form $C_l(Z_t, s) = Z_T cl(s)$. Remember that we are interested in valuing three types of claims: equity in industries A and B and the equity market portfolio, general and industry-specific human capital and an instantaneous risk free bond. In the previous section we derived $r_{f,t}$. In this section we value equity and human capital. The following proposition characterizes the solution to value claims in this economy, allowing us to estimate expected returns, volatilities and betas.

**Proposition 1.** If the function $cl(Z_{T,t}, s_t)$ exists and is twice continuously differentiable, then the equilibrium arbitrage-free price of a claim in this economy is:

1. $C_l(Z_{T,t}, s_t) = Z_{T,t} cl(s_t)$
2. $cl(s_t)$ solves the following ODE:

$$0 = dc_l(s) + cl(s) A_1(s) + cl'(s) A_2(s) + cl''(s) A_3(s),$$

(44)

where

$$A_1(s) = -\beta + (1 - \gamma) \left( s \eta_A + (1 - s) \eta_B - \gamma \frac{s^2}{2} \right)$$

$$+ (1 - \gamma) \gamma \left( \frac{\Omega^2_s}{2s(1-s)} - s \left( \frac{s^2}{2} - \sigma^2_B \right) \right)$$

$$+ (1 + \gamma) \gamma \frac{\Omega^2_s}{2} F(s_t)^2$$

$$- \gamma s(1-s) \left( \eta_A - \eta_B \gamma \left( \frac{\sigma^2_A}{2} - \frac{\sigma^2_B}{2} - \left( \frac{1}{2} - s \right) \frac{\Omega^2_s}{s(1-s)^2} \right) \right)$$

$$= \frac{\gamma s^2}{2} G(s_t)$$

(45a)

$$A_2(s) = s(1-s) \left( \left[ \eta_A - \eta_B \right] - \gamma \left( \frac{\sigma^2_A}{2} - \frac{\sigma^2_B}{2} - \Omega^2_s s^2(1-s)^2 \left( \frac{1}{2} - s \right) + \frac{\Omega^2_s}{s(1-s)} F(s_t) \right) \right)$$

(45b)

$$A_3(s) = \frac{\Omega^2_s}{2},$$

(45c)

with $cl(0) = \frac{dC_l(0)}{r_{f,0} + \gamma \sigma^2_B - \eta_B}$ and $cl(1) = \frac{dC_l(1)}{r_{f,1} + \gamma \sigma^2_A - \eta_A}$ as boundary conditions.

**Proof:** See Appendix.

The boundary conditions are given by the value of a claim in a one-industry economy. When $s = 0$, industry B dominates the economy, and thus aggregate volatility and growth is given by the growth rate and volatility of industry B’s productivity growth. When $s = 1$, industry A dominates the economy, and its volatility and growth rate equal the economy’s volatility and growth rate.

As long as we can express dividends as $D_{C_l} = Z_T d_{C_l}$, this ODE does not depend on $Z_T$, but
only on the state variable s. \( d_{CI} \) is interpreted as the dividends of a claim, normalized by aggregate productivity. In order to price claims in our economy, we need to calculate the normalized dividend of each claim and solve this ODE for \( c_1(s) \). Therefore, the final step is to find these normalized dividends. The second proposition characterizes the dividend process for each of the claims we are interested in.

**Proposition 2.** Let \( \pi(s) = \frac{\theta_A}{\theta_B} \left( \frac{Y_A}{Y_B} \right)^\rho \). The dividend paid to owners of shares of equity in each industry, owners of claims to the wages of specialized human capital in each industry, and owners of claims to the wages of generalized human capital, normalized by \( Z_T \), is given by:

1. **Equity of industry A**

   \[
   d_A = (1 - \alpha_A) \frac{co(s)}{1 + \pi(s)} \tag{46}
   \]

2. **Equity of industry B**

   \[
   d_B = (1 - \alpha_B) \frac{co(s)}{1 + \pi(s)} \tag{47}
   \]

3. **Specialized human capital of industry A**

   \[
   w_{AL} = (1 - \psi_{g,A}) \alpha_A \frac{co(s)}{1 + \pi(s)} \tag{48}
   \]

4. **Specialized human capital of industry B**

   \[
   w_{BL} = (1 - \psi_{g,B}) \alpha_B \frac{co(s)}{1 + \pi(s)} \tag{49}
   \]

5. **Generalist human capital:**

   \[
   w_{GL} = \alpha_A \psi_{g,A} \frac{co(s)}{1 + \pi(s)} + \alpha_B \psi_{g,B} \frac{co(s)}{1 + \pi(s)} \tag{50}
   \]

6. The equity market portfolio is a claim on the sum of the dividends in the two industries. The normalized dividends for the equity market portfolio follow as the sum of \( d_A \) and \( d_B \).

7. The aggregate wealth portfolio is a claim on the sum of the dividends of each industry and aggregate wages paid to all types of workers.

Proof: See appendix.

The expressions for the dividends paid to each claim in the economy highlight the sources of risk for the owner of each asset. All claimants are exposed in the same way to aggregate productivity shocks. However, each claim has a different exposure to industry shocks, as the dividends for each
claim depend differently on $s$. Whether such exposure results in higher or lower expected returns depends on the parameters of the model. We study those in the next section.

5 Analysis of results

Our numerical results convey the intuition behind the model and show how labor mobility affects human capital and expected stock returns. In this section we first explain our parameter choice and then we present the results.

5.1 Parameter choice

Since Mehra and Prescott [1985] we know that a parsimonious CRRA-utility framework does not match all the asset pricing moments while providing smooth consumption and a low risk-free rate. Rather than making the model less parsimonious by adding frictions or changing agents’ preferences, we focus on the mechanism through which labor mobility affects risk for workers and shareholders. The goal of our numerical analysis is not to match the equity premium and the risk-free rate, but to show the relative movement of risk premia in the economy.

For the utility function, we assume the agent places equal weight on each of the two goods ($\theta_A = \theta_B = .5$) and that the goods are perfect substitutes (elasticity of substitution $\rho = 1$). This parameter choice stresses the impact of labor mobility in the presence of decreasing returns to labor, while shutting down the effect of less than perfect substitution. We use a subjective discount rate $\beta = .1\%$ with the objective of obtaining a small risk-free rate. As for the coefficient of risk aversion, we use a conservative value of $\gamma = 2$. We could increase this parameter to achieve higher risk premia, but at the cost of also increasing the risk-free rate. Whereas the magnitude of the moments changes as $\gamma$ changes, the direction of the results does not change.

We have more flexibility when choosing the parameters of the production functions. We assume symmetric labor intensities across industries $\alpha_A = \alpha_B = .64$, implying that workers receive 64% of production in the economy.\footnote{See for example Palacios [2007] for empirical estimates of labor intensity in the economy.} We normalize capital in each industry to one and use two settings for $\psi$. In the first one, we solve for $\psi = 0$, corresponding to the benchmark “no labor mobility” case. For the second setting of $\psi$ we use .5. This setting imples that half of the wages in the economy are paid
to generalist workers. In the benchmark case, we assume independent and identically distributed technology shocks across industries, with expected growth rate $\eta$ of 2%, and volatility $\sigma$ of 10%. A growth rate of 2% implies a long-term consumption growth rate of 2%, while a volatility of 10% allows us to achieve unlevered volatilities of 10% for equity. This choice implies a high volatility for consumption as well, a known drawback of the neo-classical framework. The appendix discusses the numerical solution method used.

5.2 Results

We begin exploring the dynamics of the economy. After analyzing dividends and the values of the different claims, we examine their return volatilities, risk premia and conditional betas. Finally, we show how labor mobility affects returns by changing the importance of generalist labor in the economy.

5.2.1 Dividends and asset prices

Figure 1 shows the value of a consumption bundle as well as the instantaneous payoff (i.e., the dividend) for equity and general mobile human capital, all scaled by total productivity $Z_T$, for an economy with no labor mobility. Dividends to human capital specialized in each of the two industries are not reported in figure 1 since they are simply scaled dividends to equity of the respective industries. The figure shows that total consumption remains constant as the relative productivity of each industry changes. Any gains in productivity due to one industry becoming more productive is captured by total productivity and as a result consumption per unit of total productivity remains constant. Dividend payments per unit of total productivity grow linearly from zero—when the industry is so unproductive that its dividends are swamped by those produced by the other—to a maximum value, corresponding to the dividend per unit of productivity for that industry. Generalist human capital has a constant dividend of zero, since no such workers exist in the economy.

Figure 2 shows the impact that labor mobility has on the dividends paid by different claims. Panel A at the top of the figure contrasts consumption with and without mobility. With labor mobility, consumption per unit of total productivity increases when one industry becomes relatively more productive than the other. The reason is that after one industry becomes relatively more productive its marginal product of labor increases, which opens the opportunity for capital being
better used when some workers switch from the relatively unproductive industry. The result is a more efficient economy. Panel B at the bottom of the figure shows that the economy-wide gains are not evenly distributed among the different stakeholders. In particular, the dividend paid by equity can be smaller or larger in the presence of labor mobility depending on whether the industry is relatively productive or not. Labor mobility amplifies the impact of becoming relatively more productive, so that in the presence of labor mobility the relatively productive industry gets two extra dividends: one from being relatively more productive, and the other from being able to use a larger workforce. When the industry is relatively *unproductive* labor mobility acts as an extra punishment. Not only is it able to pay a smaller dividend, its workers leave, further reducing its productive capacity.

To better understand what drives the increase in consumption as $s$ gets farther from .5, figure 3 shows the fraction of mobile employees who work in industry A for different values of the relative productivity of industry A. As industry A becomes relatively more productive, the fraction of mobile employees who work in it increases. This movement amplifies the increase in capital’s productivity, as we can see in figure 4. Figure 4 plots the ratio between production and $Z_A K_A$ (or $Z_B K_B$ in the case of industry B), which is a normalization of capital’s productivity. As the figure shows, a relative increase in $Z_A$ does not change the ratio between production and $Z_A K_A$ when the labor force is immobile. In contrast, with a mobile labor force, a relative increase in $Z_A$ triggers an additional increase in capital’s productivity. The extra increase comes from the new workers that enter the industry and this is the mechanism through which labor mobility affects asset prices.

Given that consumption is valley-shaped (as seen in figure 1), marginal utility is relatively high when $s$ gets close to .5. Therefore, claims with high payoffs in states when $s$ is close to .5 act as hedges and command smaller risk premia, while claims with payoffs in states where one industry is much more productive than the other ($s$ close to 0 or 1) will be relatively riskier and have higher risk premia. Inspection of figure 1 reveals that neither equity nor human capital act as perfect hedges. Equity (and industry-specific human capital) pays most in times of low marginal utility (when the industry associated with equity is relatively productive) but its dividend decreases for all other states. On the other hand, generalist human capital pays well regardless of which industry is relatively productive, making it a better but not perfect hedge, since its maximum payoff coincides with the point in which marginal utility is lowest (close to $s = 0$ or $s = 1$). We can therefore expect a complex relationship between relative productivity, equity risk, and human capital risk.
Figure 5 shows the value of generalist human capital, equity in industries A and B, and aggregate wealth. Unsurprisingly, equity is very valuable when the industry associated with it is relatively productive, while generalist human capital is valuable regardless of the state of the economy. As would also be expected, the relationship between relative productivity and the value of mobile human capital is non-monotonic. Starting from $s = 0$, the value of mobile human capital falls and then rises as the value of its dividend falls and rises after $s = .5$. The value of aggregate wealth exhibits the same behavior as a function of $s$. Given our choice of parameters, the economy is symmetric and aggregate wealth is a constant multiple of generalist human capital. As a result, aggregate wealth changes non-monotonically as the relative productivity of each industry changes.

The most striking difference between each claim’s value is that generalist human capital provides a hedge against movements in relative productivity, while the value of equity (and immobile human capital) in each industry collapses as that industry becomes relatively unproductive. In the special case when both industries have identical characteristics, a mobile worker is perfectly hedged against economic fluctuations and does not need to diversify labor risk in financial markets.

5.2.2 Volatilities, risk premia and betas

Now that we have found the value for each claim in the economy, we can characterize their risk and return characteristics. Without loss of generality, we focus on three claims: a claim to dividends from industry A, which we refer simply as “equity” from now on, a claim to the aggregate wages paid to mobile workers, which we call generalist human capital, and a claim to the sum of dividends in the two industries, which is the equity market portfolio. Note that in our model, specialist wages are scaled dividends of equity in the industry. Therefore, the risk premium on specialist human capital is the same as the risk premium on equity in the industry.

In our model the consumption CAPM holds. However, except for the special cases when $\gamma = 1$, the conditional CAPM with respect to the total wealth portfolio does not hold. Nevertheless, small values of relative risk-aversion ($\gamma = 2$) produce very high correlations between consumption and aggregate wealth, implying that the conditional CAPM holds approximately, as in Santos and Veronesi [2006]. Therefore, in addition to analyzing volatilities and risk premia, we also consider conditional betas.\(^{14}\)

\(^{14}\)Note that in a symmetric economy (i.e. all parameters are identical for industries A and B), the equity market portfolio is a constant fraction of the aggregate wealth portfolio. Consequently, in a symmetric economy the beta measured with respect to the market or with consumption is the same.
However, we emphasize that our main conclusions are based on risk premia and volatilities, which
are the quantities that follow from our model directly and do not depend on whether the conditional
CAPM holds or not. We use the conditional betas to better understand the behavior of risk premia.

We begin by analyzing the behavior of consumption volatility and the risk-free rate. Figure 6
shows that consumption volatility is minimized when $s = .5$. In other words, when productivity is
equal in both industries, consumption is well diversified and is least risky. This decreases the precau-
tionary savings motive, and consequently we see that the instantaneous risk free rate is maximized
at $s = .5$. The parameter values lead to a reasonable risk-free rate (the largest value is about 3%)
and equity volatility, but unsurprisingly, too high levels of consumption volatility.

Figure 6 also shows the effect of labor mobility on consumption volatility and the risk-free rate.
When one industry is much more productive than the other ($s = \{0.1\}$), or when both industries
are equally productive ($s = .5$), labor mobility has no effect. Elsewhere labor mobility increases
consumption’s volatility, and as a consequence reduces the risk-free rate. Labor mobility increases
consumption’s volatility because workers switching from one industry to another amplify the positive
(negative) industry shocks by making capital even more (less) productive.

To explore the impact that labor mobility has on asset prices, we first present the volatility, risk
premium and beta for different claims in the presence of labor mobility. Figures 7, 8, and 9 present
the volatility, risk premium, and beta for each claim. First, figure 7 shows that the volatility
of the returns on general human capital reaches a minimum at $s = .5$; here the generalist worker
is best diversified. Equity volatility depends on the relative productivity of the industry ($s$), the
sensitivity of the value of the claim to changes in $s$, and the volatility of $s$ itself (see equation (65)
in the appendix). We can see that as industry A becomes relatively more productive, its equity
return volatility increases initially. However, at some point, around $s = .75$, the volatility reaches a
maximum and then decreases. This fall in volatility as $s_t$ increases follows from the volatility of $s_t$
going to zero as $s_t$ approaches one. The pattern for equity in industry B follows by symmetry.

Figure 8 shows that the overall patterns in the risk premia closely follow the patterns in volatilities.
The risk premium for generalist human capital is lowest when $s = .5$ and increases as either one of the
two industries becomes relatively more productive. On the other hand, the risk premium for equity
in industry A increases as the industry becomes relatively more productive. In this case, industry

\[15^{15}\text{The conditional beta should not be confused with the subjective discount rate, with symbol } \beta.\]
A becomes a larger part of the economy and its equity is more highly correlated with aggregate consumption, thereby increasing its risk premium. When $s$ is close to .9, the risk premium decreases slightly, which is due to the decrease in equity return volatility.

The sensitivity of equity to productivity shocks stems from the magnifying effect that labor movement has on capital’s productivity. The flip side of this observation is that generalist human capital is less exposed to shocks to relative productivities, i.e. shocks to $s$. Comparing the risk premium of specialist human capital (which is the same as that of industry equity) to that of generalist human capital, leads the following observation. When one industry is relatively more productive than the other, the risk premium of generalist human capital is smaller than the risk premium of specialized workers in that industry. However, when an industry is relatively unproductive, specialist human capital’s exposure to relative productivity shocks is smaller than that of generalist human capital, and as a result, generalist human capital’s risk premium is higher than that of the specialist in the industry in that particular region of $s$.

Figure 9 shows the conditional betas of equity and generalist human capital. In this symmetric economy, generalist human capital is a constant share of aggregate wealth and therefore has a conditional beta of one. When $s = .5$, both industries have the same productivity. Given that we also assume that all other parameters are the same for both industries, they must both have a beta of one. As industry A becomes relatively more important (i.e. $s > .5$), its systematic risk increases and therefore its beta goes up. Since the two industry betas must aggregate to one in this symmetric economy, the beta of industry B decreases.

### 5.2.3 The relationship between labor mobility, volatility, risk and expected excess returns

Now that we understand how volatilities, risk premia and betas depend on the relative productivity share $s$, we take a closer look at the role of labor mobility. Labor mobility depends on the relative importance of generalist human capital in the production processes of the two industries, measured by intensity $\psi_i$. We vary the level of $\psi$ from 0 (no labor mobility) to .5 (generalist human capital represents 50% of the wages paid in the economy).

We start by analyzing the impact of labor mobility on aggregate risk. Figure 10 shows the risk-premium, volatility, and beta of the market with and without labor mobility. Panel A in the top
of the figure shows that the risk-premium of the market is not affected by labor mobility when one industry dominates the other \((s = 0, 1)\) or when both industries are equally productive \((s = .5)\). Elsewhere, labor mobility increases increases the market’s risk premium. This result follows from studying the impact that labor mobility has on the market price of risk and on the market’s volatility. The market price of risk \((\gamma \sigma_c)\) changes as the volatility of consumption changes. As we saw in figure ??, labor mobility increases consumption’s volatility when one industry is relatively more productive than the other. Thus, the market price of risk is higher in those same instances. As for the market’s volatility, panel B in the middle of figure 10 shows the effect that labor mobility has on it. Labor mobility does not have an impact on the market’s volatility in either of the cases described above \((s = \{0, .5, 1\})\), but increases it elsewhere. This result is also driven by the effect that labor mobility has on consumption’s volatility. In the symmetric economy that we study here, the dividends paid by the market are proportional to consumption, and thus their volatility increases when consumption’s volatility increases.

Next, figure 11 plots the risk premium, volatility and beta for equity for different levels of labor mobility. Panel A in the top of the figure shows that labor mobility does not have an unambiguous impact on industry A’s risk premium. Labor mobility increases the riskiness of industry A when it is relatively more productive than industry B. On the other hand, labor mobility decreases industry A’s riskiness when it is relatively less productive.

To further understand the effect of labor mobility on the risk premium on equity in industry A, it is useful to consider its effect on volatility and the conditional beta. Panel B in the middle of figure 11 contrasts the volatility of industry A with and without labor mobility. One can see that labor mobility has an asymmetric effect on volatility. Volatility with labor mobility is always larger, but the effect is much larger when industry A is relatively more productive \((S > .5)\). This would suggest that industry A’s equity premium would be unconditionally larger with labor mobility. The reason why this is not the case can be found studying the conditional beta. Panel C in the lower part of figure 11 shows that industry A’s conditional beta is somewhat larger when industry A is relatively more productive, but much lower when it is relatively less productive.

The previous result suggests that the effect of labor mobility on the risk premium is due to an “operating leverage” effect, related to volatility, and a “systematic effect”, related to equity’s correlation with the market. Volatility is driving industry A’s higher risk premium when it is more
productive, whereas a smaller correlation with the market is driving a lower risk premium when industry A is relatively less productive. Donangelo [2010] builds a model that relates labor mobility through “operating leverage”. The results presented here also link the effect of labor mobility with changes in the systematic risk of industry A. In conclusion, labor mobility affects the risk-premium simultaneously through its volatility and its correlation with the market.

5.2.4 Discussion

Our results highlight the effect of labor mobility on equity and human capital risk and returns. When the importance of generalist labor in the economy increases, we observe two distinct effects. First, both the market portfolio and generalist human capital become riskier and earn higher risk premia. Second, industry-specific human capital and equity also become riskier and earn higher risk premia, but only when the industry is relatively more productive. Furthermore, labor mobility affects the cross-section of equity returns. When an industry relies more on mobile labor, its equity becomes riskier and carries higher expected excess returns. This section discusses some of the implications of our findings.

First, the result that the market and generalist human capital become riskier as generalist labor becomes more important in the economy might seem paradoxical, as it would suggest that labor mobility made the representative agent worse off. Some reflection reveals that this is not the case. When labor mobility increases, the representative agent is better off, as the production frontier of the economy expands. An increase in mobility makes him wealthier. However, the benefits of mobility are sensitive to the relative productivity of the industries in the economy. The benefits are largest when one industry is relatively more productive than the other, and smallest when neither industry is much more productive than the other. Thus, labor mobility creates a new source of risk: the risk of losing the benefits of labor mobility as the relative productivity between industries change. After an increase in labor mobility the representative agent is wealthier, but faces more risk; the equity premium goes up.

Next, our results show that the equity risk premium –both for each industry and the market– varies over time as the productivity share $s_t$ changes. Generalist workers move between industries of employment in response to changes in relative productivities, thereby affecting dividends. This suggests that we can relate equity risk premia to the importance of generalist workers in the economy
and to the dispersion of these workers across industries.

Lastly, this model can be extended to the study the portfolio decisions of agents with different types of human capital. Generalists are mainly exposed to market risk, while specialists are more exposed to their own industry risk. In the special case when the industries are identical—the main focus of our numerical analysis—the incentives to diversify away labor risk in financial markets is maximum for specialists.

6 Conclusion

We extend Donangelo [2010] into a general equilibrium model by incorporating labor mobility in a multi-industry dynamic economy with production. This paper shows that aggregate labor mobility impacts risk and expected returns on equity and human capital. Even when shocks in different industries are uncorrelated, aggregate labor mobility induces systematic variations in firms’ profits, affecting their risk and expected returns. Each industry uses industry-specific labor as well as general labor as inputs for production, in addition to physical capital. Individuals are endowed with different types of human capital skills. Those with industry-specific labor skills are fully immobile and can only work in one of the industries, whereas those with general skills are mobile as they can work in different industries.

We show that aggregate labor mobility affects the time-series of human capital and equity returns. Aggregate labor mobility affects the time series variation in the stochastic discount factor. These finding suggest that a measure of labor aggregate mobility is a promising new macroeconomic variable for asset pricing. Finally, in our model, the risk and return profile of generalist human capital is closer to that of the market portfolio, while that of specialist human capital is closer equity in the industry. Mobile human capital is intrinsically more diversified, suggesting that stock markets are relatively less attractive for diversification purposes to generalists than to specialists.
References


A Proofs

A.1 Proof of Lemma 1

To find the general labor demanded in industries A and B, we proceed as follows. The consumer’s budget constraint, taking prices and wages as given, implies:

\[
\frac{\theta_A}{\theta_B} \left( \frac{C_A}{C_B} \right)^{\rho} = \frac{P_A C_A}{P_B C_B}.
\]

(51)

In equilibrium, goods markets clear \((C_i = Y_i)\), leading to:

\[
\frac{\theta_A}{\theta_B} \left( \frac{Y_A}{Y_B} \right)^{\rho} = \frac{P_A Y_A}{P_B Y_B}.
\]

(52)

Combining expression (15) for each industry, and expression (52), we obtain the following relationship between the general labor demanded in each industry and production in each industry:

\[
1 = \frac{\theta_A \psi_A \alpha_A}{\theta_B \psi_B \alpha_B} \left[ \frac{Z_A K_A^{1-\alpha_A} L_A^{\psi_A \alpha_A} (1-\psi_A) \alpha_A}{Z_B K_B^{1-\alpha_B} L_B^{(1-\psi_B) \alpha_B}} \right]^\rho \frac{L_G^A \alpha_{\rho-1}}{L_G^B \alpha_{\rho-1}}.
\]

(53)

Finally, the labor market clearing condition for general human capital implies \(L_{G,A} + L_{G,B} = \xi_G\). Hence:

\[
1 = \frac{\theta_A \psi_A \alpha_A}{\theta_B \psi_B \alpha_B} \left[ \frac{s K_A^{1-\alpha_A} L_A^{(1-\psi_A) \alpha_A}}{(1-s) K_B^{1-\alpha_B} L_B^{(1-\psi_B) \alpha_B}} \right]^\rho \frac{L_G^A \alpha_{\rho-1}}{(\xi_G - L_{G,A}) \psi_B \alpha_{\rho-1}}.
\]

(54)

A.2 Proof of Lemma 2

First, using the definition of \(s\), equation (54) can be expressed as:

\[
1 = \frac{\theta_A \psi_{g,A} \alpha_A}{\theta_B \psi_{g,B} \alpha_B} \left[ \frac{s K_A^{1-\alpha_A} L_A^{(1-\psi_{g,A}) \alpha_A}}{(1-s) K_B^{1-\alpha_B} L_B^{(1-\psi_{g,B}) \alpha_B}} \right]^\rho \frac{L_G^A \alpha_{\rho-1}}{(\xi_g - L_{g,A}) \psi_{g,B} \alpha_{\rho-1}}.
\]

(55)

Consequently, the general labor demanded in each industry, which follows from this expression, only depends state variable \(s\) and the model parameters \(\Phi\).

Next, we derive the consumption of a basket of goods from the two industries, denoted by

\[16\]This follows from the agent’s first order conditions when choosing between goods. Aggregate consumption must be financed through the wages and dividends that agents receive. The Lagrangian for the representative agent is:

\[
1 - \gamma (\theta_A C_A^\rho + \theta_B C_B^\rho)^{1/\gamma} + \lambda (Bud - P_A C_A - P_B C_B) \text{ where } Bud = W^r \cdot S_j + X_j \cdot D \text{ and } \lambda \text{ is the Lagrange multiplier.} 
\]
We can write the dynamics of a claim $Cl$. Because goods markets clear we have $C_t = Y_t$.

\[
CO(Z_t, s_t) = \left[ \theta_A Y^\rho_A + \theta_B Y^\rho_B \right]^{1/\rho} = \left[ \theta_A \left( Z_A K^{1-\alpha_A} L_A^{(1-\psi_A)\alpha_A} L_{G,A}(s)^{\psi_A\alpha_A} \right)^\rho + \theta_B \left( Z_B K^{1-\alpha_B} L_B^{(1-\psi_B)\alpha_B} (\xi_G - L_{G,A}(s))^{\psi_B\alpha_B} \right)^\rho \right]^{1/\rho}
\]

\[
= Z_T \cdot \left[ \theta_A \left( s K^{1-\alpha_A} L_A^{(1-\psi_A)\alpha_A} L_{G,A}(s)^{\psi_A\alpha_A} \right)^\rho + \theta_B \left( (1-s) K^{1-\alpha_B} L_B^{(1-\psi_B)\alpha_B} (\xi_G - L_{G,A}(s))^{\psi_B\alpha_B} \right)^\rho \right]^{1/\rho}
\]

\[
= Z_T \cdot co(s).
\]

Note that even though we have not derived explicitly the function $co(\Phi, s)$, equations (55) and (56) determine it.

We can find the expression for $L'_{G,A}(s)$ by multiplying both sides of expression (55) by $w_B$ and taking the first derivative of both sides. This leads to the following expression for $L'_{G,A}(s)$:

\[
L'_{G,A}(s) = \frac{\rho L_{G,A}(s)(\xi_G - L_{G,A}(s))}{(1-s)(\xi_G (1-\rho \alpha_A \psi_A) + \rho L_{G,A}(s)(\alpha_A \psi_A - \alpha_B \psi_B))}.
\]

The first and second derivatives of $co(s)$ follow:

\[
co'(s) = co(s) F(s),
\]

\[
co''(s) = co(s) \left[ G(s) + F(s)^2 \right],
\]

where

\[
F(s) = \frac{1}{s(1-s)} \left( 1 + \frac{\psi_A \alpha_A (\xi_G - L_{G,A}(s))}{\alpha_B \psi_B L_{G,A}(s)} \right) - \frac{1}{1-s}
\]

\[
G(s) = \frac{1}{s(1-s)} \left( 1 - \frac{\psi_A \alpha_A (\xi_G - L_{G,A}(s))}{\alpha_B \psi_B L_{G,A}(s)} \right)
\]

\[
\left( \frac{1-2s}{s(1-s)} - \frac{\rho \xi_G (1-\rho \alpha_A \psi_A) + \rho L_{G,A}(s)(\alpha_A \psi_A - \alpha_B \psi_B)}{(1-s) (\xi_G (1-\rho \alpha_A \psi_A) + \rho L_{G,A}(s)(\alpha_A \psi_A - \alpha_B \psi_B))} \right) - \frac{s}{(1-s)}
\]

Functions $F(s)$ and $G(s)$ can be rewritten as in Lemma 2.

### A.3 Proof of Proposition 1

We can write the dynamics of a claim $Cl(Z_T, s) = Z_T cd(s)$ as follows:

\[
\frac{dCl_t}{Cl_t} = \mu_{Cl} dt + \sigma_{Cl,A} dB_A + \sigma_{Cl,B} dB_B,
\]
where

\[ \mu_{CI} = s\eta_A + (1 - s)\eta_B + s(1 - s)(\eta_A - \eta_B)\frac{cl'(s)}{cl(s)} + \frac{\Omega^2}{2} \frac{cl''(s)}{cl(s)} \]  

(65)

\[ \sigma_{CI,A} = s\sigma_A \left( 1 + (1 - s)\frac{cl'(s)}{cl(s)} \right) \]  

(66)

\[ \sigma_{CI,B} = (1 - s)\sigma_B \left( 1 - s\frac{cl'(s)}{cl(s)} \right) \]  

(67)

Denote the diffusion of a claim as \( \sigma_{CI} = \{\sigma_{CI,A}, \sigma_{CI,B}\} \).

Using Itô’s Lemma, we can derive the dynamics of the discounted value of a claim, i.e. \( M(t, Z_T, s)Cl(Z_T, s) \). We are mainly interested in the drift of this process, which equals:

\[ \mu_{CI} - r_f - \gamma (\sigma_{CI}\sigma_C + \varphi_{A,B} (\sigma_{CI,A}\sigma_{C,B} + \sigma_{CI,B}\sigma_{C,A})) \].  

(68)

In a complete market, this drift equals zero. An alternative expression for the drift of the discounted process of the value of a claim comes from the definition of the value of a claim:

\[ Cl_t = E_t \int_t^\infty D_{Cl,\tau} M_\tau d\tau, \]  

(69)

where \( D_{Cl,t} \) is the dividend of the claim (i.e. for equity this is the dividend paid out by the firm, and for human capital is the total amount of wages paid). Applying Itô to this integral leads to a drift of \( \beta - \frac{D_{Cl}}{Z_T cl(s)} \). Next, we equate this to the first expression of the drift (68), which also equals zero. Substitution of all the terms lead to the ODE in the proposition.

### A.4 Proof of Proposition 2

The goods market-clearing condition implies that the value of the basket of goods produced equals the value of production:

\[ P_A Y_A + P_B Y_B = Z_T \cdot co(\Psi, s). \]  

(70)

Using equation (52), the relationship between the value of production in each industry can be written as:

\[ P_B Y_B \pi(\Psi, s) = P_A Y_A. \]  

(71)

Solving for the value of production of industry B we find:

\[ P_B Y_B (1 + \pi(\Psi, s)) = Z_T \cdot co(\Psi, s). \]  

(72)

The dividend paid to owners of shares in industry B is \( (1 - \alpha_B)P_B Y_B \). This comes from the FOC of the manager’s decision of how much labor to hire given the wage rate. Therefore, the dividend of equity in industry B equals

\[ D_B = Z_T (1 - \alpha_B) \frac{co(\Psi, s)}{1 + \pi(\Psi, s)}. \]  

(73)

Conveniently, the value of the dividend normalized by \( Z_T, d_B \), is only a function of \( s \). Similar reasoning produces expressions for the dividends of all the other claims in the economy.
B Numerical solution method

We solve the ODE for each claim using the boundary conditions and an initial guess for the derivative of the function when \( s = 0 \) and \( s = 1 \). The coefficient multiplying the derivative (\( A_2(s) \)) of the function changes sign in the interval (0,1), which makes the solution unstable using standard numerical techniques. To solve the problem, we find the value of \( s \) in the interval for which \( A_2(s) = 0 \). Denote this point as \( s^* \). We proceed to guess the derivative of \( cl(s) \) when \( s = 0 \) (the value of \( cl(0) \) is given by the boundary conditions) and find \( cl(s^*_+) \) and \( cl'(s^*_+) \). (\( s^*_- \) denotes the solution approaching from the left). We repeat starting from \( s = 1 \) and moving backwards on \( s \) and find \( cl(s^*_+) \) and \( cl'(s^*_+) \). We iterate our guesses until \( cl(s^*_-) = cl(s^*_+ \) and \( cl'(s^*_-) = cl'(s^*_+) \). Once we have solved for the ODE, we can calculate all the relevant variables in the model.
Figure 1: Dividends for different claims as a function of relative productivity (without labor mobility)

Results with the following parameters: $\psi_A = \psi_B = .5, K_A = K_B = 1, \alpha_A = \alpha_B = 0.64, \eta_A = \eta_B = 0.02, \sigma_A = \sigma_B = 0.1, \varphi_{A,B} = 0, \gamma = 2, \rho = 1, \beta = .001.$
Results with the following parameters: $\psi_A = \psi_B = .5$, $K_A = K_B = 1$, $\alpha_A = \alpha_B = 0.64$, $\eta_A = \eta_B = 0.02$, $\sigma_A = \sigma_B = 0.1$, $\varphi_{A,B} = 0$, $\gamma = 2$, $\rho = 1$, $\beta = .001$.

Figure 2: Dividends for consumption and industry A (with and without labor mobility)
Results with the following parameters: $\psi_A = \psi_B = .5$, $K_A = K_B = 1$, $\alpha_A = \alpha_B = 0.64$, $\eta_A = \eta_B = 0.02$, $\sigma_A = \sigma_B = 0.1$, $\varphi_{A,B} = 0$, $\gamma = 2$, $\rho = 1$, $\beta = .001$.

Figure 3: General labor employed in industry A as a function of relative productivity
Results with the following parameters: $\psi_A = \psi_B = 0.5$, $K_A = K_B = 1$, $\alpha_A = \alpha_B = 0.64$, $\eta_A = \eta_B = 0.02$, $\sigma_A = \sigma_B = 0.1$, $\varphi_{A,B} = 0$, $\gamma = 2$, $\rho = 1$, $\beta = 0.001$.

Figure 4: Productivity per unit of physical capital scaled by total productivity
Results with the following parameters: $\psi_A = \psi_B = .5$, $K_A = K_B = 1$, $\alpha_A = \alpha_B = 0.64$, $\eta_A = \eta_B = 0.02$, $\sigma_A = \sigma_B = 0.1$, $\varphi_{A,B} = 0$, $\gamma = 2$, $\rho = 1$, $\beta = .001$.

Figure 5: Value of equity, generalist human capital and aggregate wealth as a function of relative productivity
Results with the following parameters: $\psi_A = \psi_B = .5$, $K_A = K_B = 1$, $\alpha_A = \alpha_B = 0.64$, $\eta_A = \eta_B = 0.02$, $\sigma_A = \sigma_B = 0.1$, $\varphi_{A,B} = 0$, $\gamma = 2$, $\rho = 1$, $\beta = .001$.

Figure 6: Consumption volatility and the risk-free rate
Results with the following parameters: $\psi_A = \psi_B = .5$, $K_A = K_B = 1$, $\alpha_A = \alpha_B = 0.64$, $\eta_A = \eta_B = 0.02$, $\sigma_A = \sigma_B = 0.1$, $\varphi_{A,B} = 0$, $\gamma = 2$, $\rho = 1$, $\beta = .001$.

Figure 7: Volatility of equity and generalist human capital
Results with the following parameters: $\psi_A = \psi_B = .5$, $K_A = K_B = 1$, $\alpha_A = \alpha_B = 0.64$, $\eta_A = \eta_B = 0.02$, $\sigma_A = \sigma_B = 0.1$, $\phi_{A,B} = 0$, $\gamma = 2$, $\rho = 1$, $\beta = .001$.

Figure 8: Risk premium of equity and generalist human capital
Results with the following parameters: $\psi_A = \psi_B = .5$, $K_A = K_B = 1$, $\alpha_A = \alpha_B = 0.64$, $\eta_A = \eta_B = 0.02$, $\sigma_A = \sigma_B = 0.1$, $\varphi_{A,B} = 0$, $\gamma = 2$, $\rho = 1$, $\beta = .001$.

Figure 9: Beta of equity and generalist human capital
Assumes $\psi_A = \psi_B$. Results with the following parameters: $K_A = K_B = 1$, $\alpha_A = \alpha_B = 0.64$, $\eta_A = \eta_B = 0.02$, $\sigma_A = \sigma_B = 0.1$, $\varphi_{A,B} = 0$, $\gamma = 2$, $\rho = 1$, $\beta = .001$.

Figure 10: Impact of labor mobility on the market
Results with the following parameters: $\psi_B = 0.8$, $K_A = K_B = 1$, $\alpha_A = \alpha_B = 0.64$, $\eta_A = \eta_B = 0.02$, $\sigma_A = \sigma_B = 0.1$, $\varphi_{A,B} = 0$, $\gamma = 2$, $\rho = 1$, $\beta = 0.001$.

Figure 11: Impact of labor mobility on industry A