

Homework Assignment 3

Solutions

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Problem 1

A company sets different prices for a particular stereo system in eight different regions of the country. The table below shows the numbers of units sold (in 1000s of units) and the corresponding prices (in hundreds of dollars).

Sales	420	380	350	400	440	380	450	420
Price	5.5	6.0	6.5	6.0	5.0	6.5	4.5	5.0

- (i) In Excel, regress sales on price and obtain the intercept and slope estimates.

SUMMARY OUTPUT

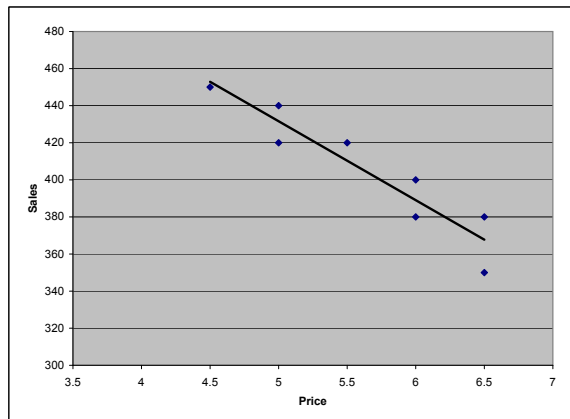
<i>Regression Statistics</i>	
Multiple R	0.937137027
R Square	0.878225806
Adjusted R Square	0.857930108
Standard Error	12.74227575
Observations	8

ANOVA

	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>Significance F</i>
Regression	1	7025.806452	7025.806452	43.27152318	0.000592135
Residual	6	974.1935484	162.3655914		
Total	7	8000			

	<i>Coefficients</i>	<i>Standard Error</i>	<i>t Stat</i>	<i>P-value</i>	<i>Lower 95%</i>	<i>Upper 95%</i>
Intercept	644.516129	36.68873299	17.56714055	2.18343E-06	554.7420336	734.2902244
X Variable 1	-42.58064516	6.473082556	-6.578109392	0.000592135	-58.41970755	-26.74158277

- (ii) Present a plot with the data and the regression line



- (iii) Based on this analysis, briefly describe your understanding of the relationship between sales and prices.

Problem 2: Match the Plots

Below (Figure 1) are 4 different scatter plots of an outcome variable y versus predictor x followed by 4 four regression output summaries labeled A, B, C and D. Match the outputs with the plots.

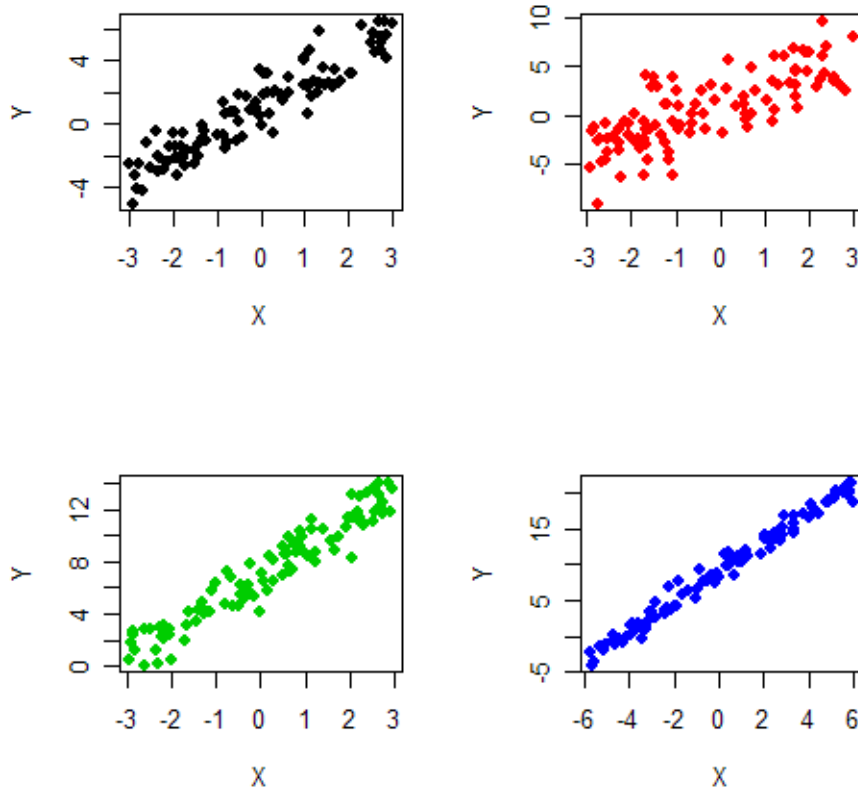


Figure 1: Scatter Plots

B-Black, C-Red, A-Green, D-Blue. Note: you can look at the R^2 , intercept and slope to decide how the outputs and plots match each other.

Regression A:

Coefficients:

	Estimate	Std. Error
(Intercept)	7.03747	0.12302
(Slope)	2.18658	0.07801

Residual standard error: 1.226

R-Squared: 0.8891

Regression B:

Coefficients:

	Estimate	Std. Error
(Intercept)	1.1491	0.1013
(Slope)	1.4896	0.0583

Residual standard error: 1.012

R-Squared: 0.8695

Regression C:

Coefficients:

	Estimate	Std. Error
(Intercept)	1.2486	0.2053
(Slope)	1.5659	0.1119

Residual standard error: 2.052

R-Squared: 0.6666

Regression D:

Coefficients:

	Estimate	Std. Error
(Intercept)	9.0225	0.0904
(Slope)	2.0718	0.0270

Residual standard error: 0.902

R-Squared: 0.9835

Problem 3

Suppose we are modeling house price as depending on house size. Price is measured in thousands of dollars and size is measured in thousands of square feet.

Suppose our model is:

$$P = 20 + 50s + \epsilon, \quad \epsilon \sim N(0, 15^2).$$

- (a) Given you know that a house has size $s = 1.6$, give a 95% predictive interval for the price of the house.

The point prediction is $\hat{P}_f = 20 + 50 \times 1.6 = 100$

The prediction interval is $[100 \pm 2 \times 15] = [70; 130]$

- (b) Given you know that a house has size $s = 2.2$, give a 95% predictive interval for the price.

The point prediction is $\hat{P}_f = 20 + 50 \times 2.2 = 130$

The prediction interval is $[130 \pm 2 \times 15] = [100; 160]$

- (c) In our model the slope is 50. What are the units of this number?

1,000\$ / 1,000 Sq. Feet = \$/Sq. Feet

- (d) What are the units of the intercept 20?

1,000\$ (same as P)

- (e) What are the units of the the error standard deviation 15?

1,000\$ (same as P)

- (f) Suppose we change the units of price to dollars and size to square feet
What would the values and units of the intercept, slope, and error standard deviation?

Intercept: 20,000 \$

Slope: 50 \$/Sq. Feet

error standard deviation: 15,000 \$

- (g) If we plug $s = 1.6$ into our model equation, P is a constant plus the normal random variables ϵ . Given $s = 1.6$, what is the distribution of P ?

When $s = 1.6$ the mean of house prices is $20 + 50 \times 1.6 = 100$. The error standard deviation is the same, 15. Therefore

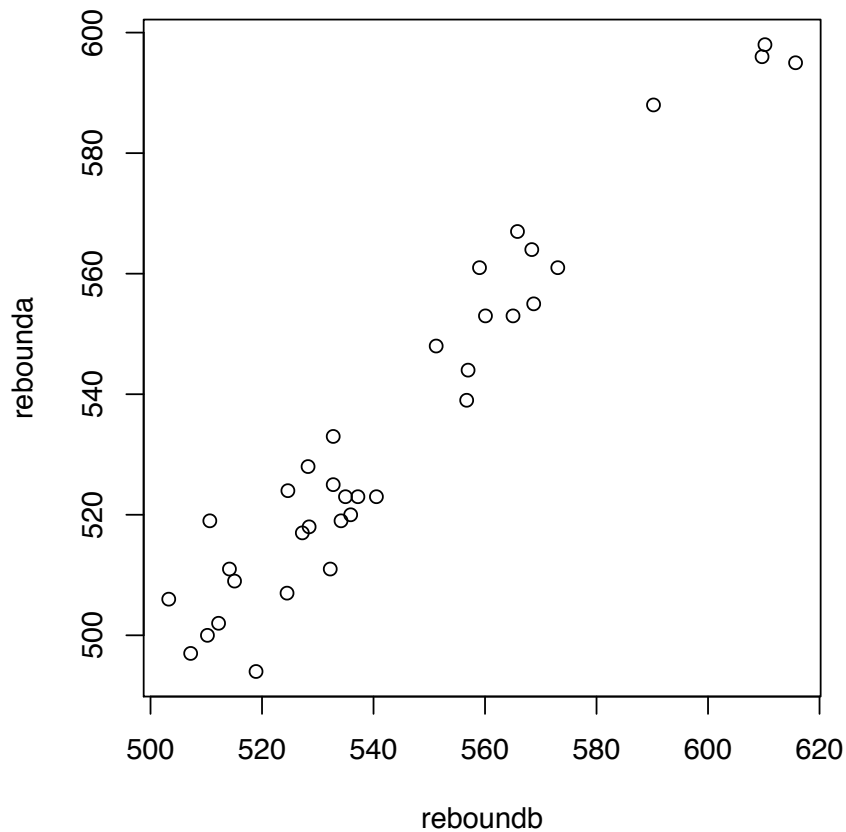
$$P|s = 1.6 \sim N(100, 15^2)$$

Problem 4: The Shock Absorber Data

Regression Statistics	
Multiple R	0.9666
R Square	0.9344
Adjusted R Squar	0.9324
Standard Error	7.6697
Observations	35.0000

ANOVA					
	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>Significance F</i>
Regression	1.0000	27635.7568	27635.7568	469.7986	0.0000
Residual	33.0000	1941.2146	58.8247		
Total	34.0000	29576.9714			

	<i>Coefficients</i>	<i>Standard Error</i>	<i>t Stat</i>	<i>P-value</i>	<i>Lower 95%</i>	<i>Upper 95%</i>
Intercept	18.2259	23.8852	0.7631	0.4508	-30.3690	66.8208
reboundb	0.9495	0.0438	21.6748	0.0000	0.8603	1.0386



(a)

We are trying to determine whether or not the before measurement is predictive of the after measurement. Therefore the dependent variable (Y) should be the after measurement and the explanatory variable (X) the before measurement.

(b) From the output above, we see that the 95% confidence interval for the slope is $[0.8603; 1.0386]$.

- (c) Is zero a plausible value for β_0 ? By looking at the 95% confidence interval we see that yes, it is. We can also conclude the same thing by looking at the $t - stat = 0.7631$ meaning that the distance between the estimate $b_0 = 18.22$ and the proposed value $\beta_0^o = 0$ is only 0.7631 standard deviations... ie, not that far. The conclusion is that, with the information in hand, we CAN'T reject the hypothesis that $\beta_0 = 0$!
- (d) What line would represent "equality" between the before measurement and the after measurement? That would be a line with intercept equal to zero and slope equal to one.
- Test whether the intercept is equal to the value proposed by the shock maker.
Is $\beta_0 = 0$ a plausible value for the intercept. Again as in item (c) yes!
- Test whether the slope is equal to the value proposed by the shock maker.
Is $\beta_1 = 1$ a plausible value for the slope. By inspecting the 95% confidence interval we see that yes, $\beta_1 = 1$ is a plausible hypothesis.
- (f) Suppose the before measurement is 550.
What is the plug-in predictive interval given $x\text{-before}=550$. The plug-in predictive interval is $18.22 + 0.949 \times 550 \pm 2 \times 7.67 = [524.83; 555.51]$
What does this interval suggest about the shock maker's claim? It looks like the shock maker is correct, ie., with $x\text{-before}=550$ we can predict with 95% probability that the after measurement will be within acceptable bounds.

Problem 5

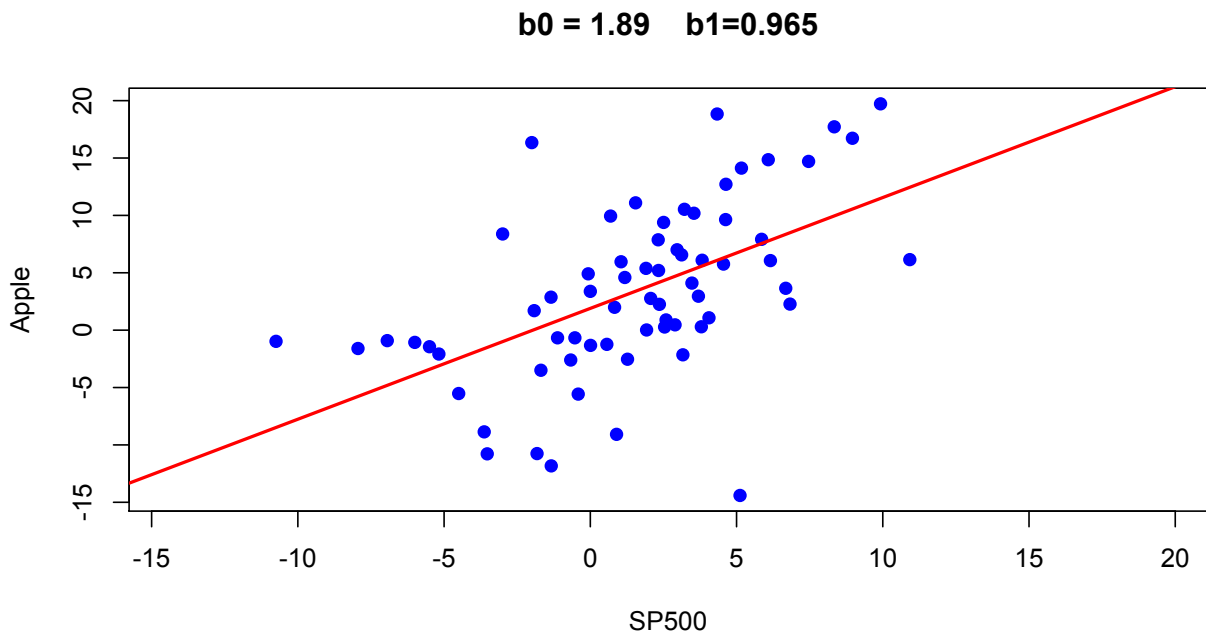
The data file for this question is available in the course website.

Consider the regression model

$$Apple_t = \alpha + \beta SP500_t + \epsilon_t \quad \epsilon_t \sim N(0, \sigma^2)$$

where $Apple_t$ represents the return on Apple Computers in month t and $SP500_t$ represents the return on the S&P 500 in month t .

- (a) What is the interpretation of β in terms of a measure of risk of the stock? β is a measure of the risk of the stock relative to the market. If the return on the market goes up (or down) by 1% then we expect the return on the stock to go up (or down), on average, by $\beta\%$. We also call β a measure of systematic risk, i.e., the part of the variation of the stock associated with the market (the economy).
- (b) What is the interpretation of α ? α is the average return the stock gets regardless of the market behavior... when the market is not moving the expected value of the return for the stock is α . We tend to think of this as the average return you tend to get on top of the market for the stock.
- (c) Plot $Apple$ against $SP500$. What graphical evidence is there of a relationship between $Apple$ and $SP500$? Does the relationship appear to be linear? Why or why not?



Yes, by looking at the plot it appears that $Apple$ and $SP500$ could be linearly related...

- (d) Estimate β . What does this estimate tell you about the risk of Apple?

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Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)   1.8967      0.8306   2.284  0.0257 *
SP500          0.9659      0.1857   5.203 2.15e-06 ***
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Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1 1

Residual standard error: 6.392 on 65 degrees of freedom
Multiple R-squared:  0.294, Adjusted R-squared:  0.2831
F-statistic: 27.07 on 1 and 65 DF,  p-value: 2.146e-06

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The estimate of β is 0.965. This implies that *Apple* is less risky relative to the market as a 1% change in the market returns would result on a 0.96% change, on average, in Apple's return.

- (e) Is the estimate of β obtained in part (c) the actual value of β ? Why or why not? No, it is an estimate and indeed our best guess about the "true" β .
- (f) Now consider the regression models

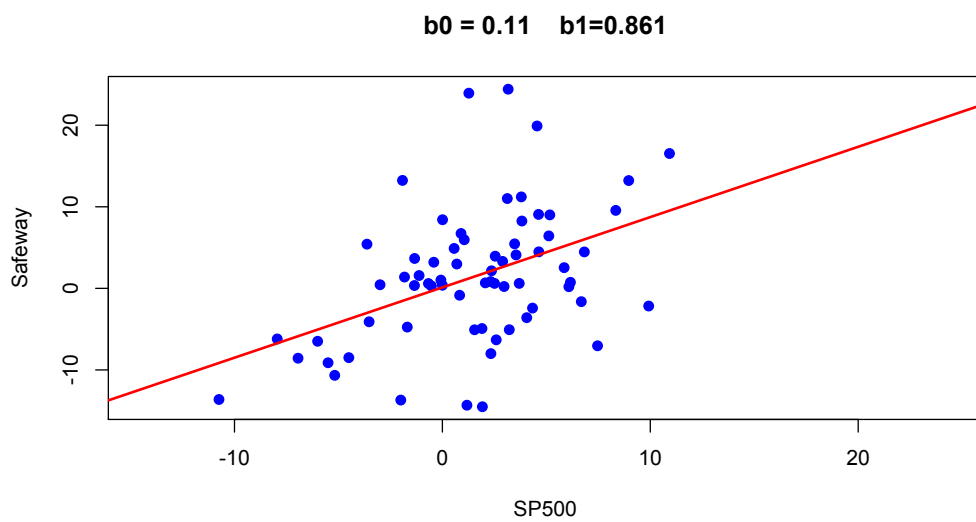
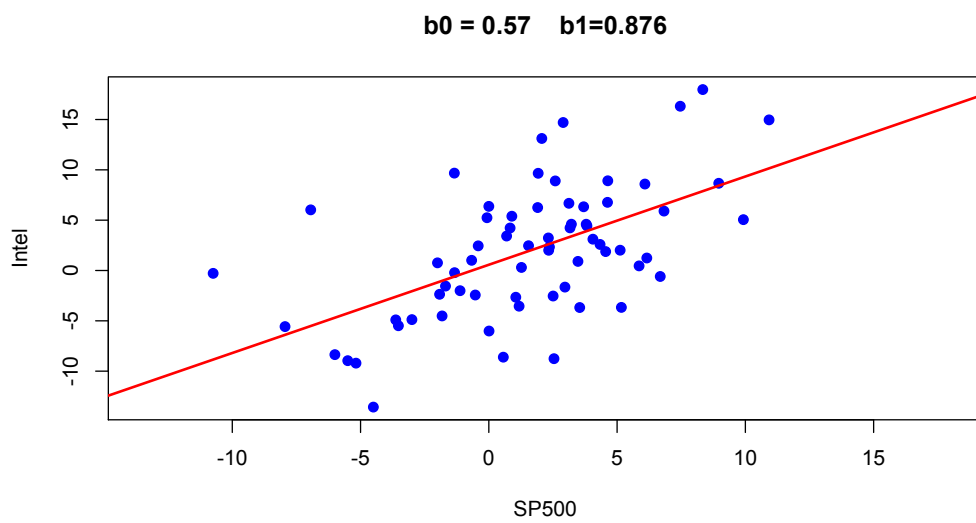
$$Intel_t = \alpha + \beta SP500_t + \epsilon_t \quad \epsilon_t \sim N(0, \sigma^2)$$

where $Intel_t$ represents the return on Intel in month t , and

$$Safeway_t = \alpha + \beta SP500_t + \epsilon_t \quad \epsilon_t \sim N(0, \sigma^2)$$

where $Safeway_t$ represents the return on Safeway in month t . How does the beta risk of the three companies compare? What do their α 's tell you?

The market risk of Safeway is the estimated to be the smallest with Intel as the second riskier and Apple as the stock with the highest market risk. (see results below). These are based on our estimates of the β 's for each stock.



Problem 6: Presidential Election
(Imagine this is October 2012!)

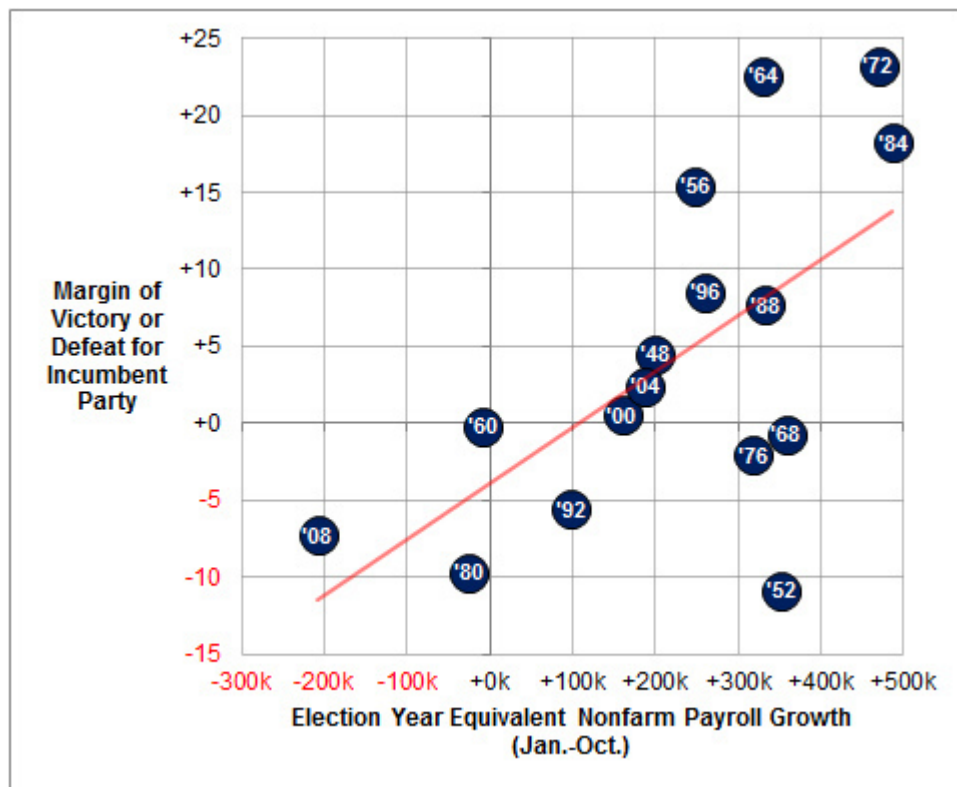
FiveThirtyEight

Nate Silver's Political Calculus



This question is based on an analysis presented in the New York Times blog FiveThirtyEight by Nate Silver...

According to the blog post, the most accurate economic indicator to predict the results of the election for an incumbent president is the election year “equivalent non-farm payroll growth” (in number of jobs from Jan to Oct). In the blog they provided a picture summarizing the relationship between this economic indicator and the election results. The analysis was based on the last 16 elections where a sitting president was seeking re-election. The picture is displayed below:



This week, The Department of Labor released the most recent employment numbers which says that the economy has added, on average, 100,000 non-farm jobs since January 2012.

Now, I am a gambling person and I need your help... Currently, on intrade.com, I can buy or sell a future contract on Obama's re-election for \$64.7. This contract pays \$100 if Obama wins (see below).



1. Based on the model presented in the New York Times blog and the numbers released by the Department of Labor how should I bet on the Intrade website (i.e., should I buy or sell the future contract)? Why?

The model from the NYT blog predicts that Obama has a 50% chance of winning given the 100k jobs information. The futures market are trading a contract that pays 100 if Obama wins at 64.7. So, you sell short!

Basically you are getting payed 64.7 for a lottery where you think there is only a 50 expected value. If you dont believe me, think about it this way: how much are you willing the pay me to flip a coin such that if it lands heads I will give 100? If you say more than 50 please come to my office and lets play this game!! :)

2. Do you think the answer in the question above is a good idea? Why or why not?

The NYT blog model only takes into account one factor (the best available economic indicator). The market is probably looking at every other aspect of the election... social issues, the "47%" comment, Romney's hair, etc... A better regression model would have additional X variables to account for other important factors. The jobs variable is an important factor but doesn't tell all the story.