THE JOURNAL OF FINANCE • VOL. LVIII, NO. 2 • APRIL 2003

How Sensitive Is Investment to Cash Flow When Financing Is Frictionless?

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ABSTRACT

I analyze the sensitivity of a firm's investment to its own cash flow in the benchmark case where financing is frictionless. This sensitivity has been proposed as a measure of financing constraints in earlier studies. I find that the investment–cash flow sensitivities that obtain in the frictionless benchmark are very similar, both in magnitude and in patterns they exhibit, to those observed in the data. In particular, the sensitivity is higher for firms with high growth rates and low dividend payout ratios. Tobin's q is shown to be a more noisy measure of near-term investment plans for these firms.

DOES A HIGH SENSITIVITY of a firm's investment to its own cash flow indicate that the firm is financially constrained? A large body of research, starting with Fazzari, Hubbard, and Petersen (1988) (hereafter FHP), suggests that firms facing financing constraints should exhibit high investment–cash flow sensitivities, reflecting the wedge between the costs of external and internal funds. Empirical findings seem to give support to this hypothesis, since firms classified as constrained on a priori criteria, such as size, dividend payout, or leverage, do have higher cash flow sensitivities, even after controlling for their investment opportunities by conditioning on Tobin's q. However, the reliability of these results critically depends on whether q is a proper control for the investment opportunity set. If q performs worse for certain classes of firms, higher sensitivities may obtain for these firms simply because cash flow reflects information about investment opportunities. This critique is not new. In his discussion of FHP, Poterba (1988) was the first to point out that measurement error in q may cloud the empirical results; similar concerns have been raised in subsequent studies as well. While the qualitative point is well known, its quantitative impact on the investment–cash flow sensitivity has not been analyzed. Can the link between cash flow and the investment opportunity set account for the observed magnitudes of the sensitivity? Or are these magnitudes too large to be explained without financing frictions?

*University of Texas at Austin. This paper is based on the first chapter of my dissertation at Carnegie Mellon University. I am indebted to my advisor, Rick Green, for suggesting to me to work on this topic, and my doctoral committee members, Burton Hollifield and Christine Parlour, for their constant guidance and support. I would like to thank Gian Luca Clementi, Chandra Das, Douglas Diamond (the editor), Mike Gallmeyer, Fatih Güvenen, Charles Himmelberg, Jacob Oded, Bryan Routledge, Toni Whited, and the participants at the 2001 WFA meetings and 2001 NBER Corporate Finance Workshop for many valuable comments. All remaining errors are mine.

This paper attempts to answer the above questions by analyzing the investment–cash flow sensitivities that obtain in the benchmark case with no financing constraints. I develop a model of firm growth and investment based on the standard neoclassical models of Lucas (1967), Treadway (1969), and Hayashi (1982). The new feature of the model is that younger firms face uncertainty about their growth prospects, and this uncertainty is resolved through time as cash flow realizations provide new information. The main results of the paper are as follows. First, investment is sensitive to cash flow for all firms, even after conditioning on Tobin's q. Second, and more importantly, the sensitivity is higher for young, small firms with high growth rates and low dividend payout ratios. In fact, the model successfully matches the sensitivities reported for these firms in empirical studies. Third, Tobin's q is shown to be a more noisy measure of the investment opportunity set for young firms with high growth rates. A substantial part of q for these firms represents the option value of long-term growth potential. Since this option value is not very informative about near-term investment plans, q performs poorly in controlling for current investment.

The model setup is the frictionless, neoclassical environment. Each firm is characterized by a production technology, where the profit rate is a function of the capital stock and current productivity. Productivity is composed of a permanent component (the *project quality*, dictating the long-run average firm size), and a mean-reverting transitory shock, but these two components are not separately observable. Each firm starts its life facing uncertainty about its project quality. The uncertainty is resolved in time as cash flow observations provide new information, as in Jovanovic (1982). Young firms are small, and they reach their steady-state average sizes after an extended growth period. Since firms are uncertain about their project qualities, cash flow shocks during this growth period are highly informative about long-run profitability. Hence, young firms revise their growth plans aggressively in response to cash flow shocks. This amplifies the link between cash flow and investment.

Given this theoretical setup, I calibrate the model parameters and simulate data to generate a panel of firms. FHP and subsequent studies assign firms into groups based on a priori likelihood of being financially constrained, and then analyze the investment–cash flow sensitivity for each group. Following FHP, I sort the model firms based on their dividend payout ratios.[1] For all dividend payout classes, investment is sensitive to cash flow, even though Tobin's q is included as a control variable in investment regressions. More importantly, the sensitivity is much higher for the low payout classes. These sensitivities are very close in magnitude to their counterparts reported in FHP. In the model, firms with high growth rates use their cash flow primarily for funding investment, and, hence, pay little or no dividends. High sensitivity of investment to cash flow is, in fact, a characteristic of such firms that possess significant growth opportunities. Low

[1] FHP suggest that low dividend payout firms are more likely to be financially constrained, as they are more dependent on external financing. See Hubbard (1998) for a survey of studies with alternative sample split criteria.

dividend payout proxies for growth; therefore, splits based on dividend payout result in differential investment–cash flow sensitivities across classes of firms.

Why is investment highly sensitive to cash flow for growth firms? One reason is that cash flow shocks within a year provide new information about project quality and trigger significant adjustments in investment, but beginning-of-the-year Tobin's q value, the control variable in investment regressions, fails to capture such new information. Interestingly, the sensitivity results survive even after this econometric issue is addressed by removing the surprise component of cash flow. Investment is sensitive to cash flow expectations at the beginning of the year, too, and even this sensitivity is substantially higher for growth firms. In other words, q is a more noisy measure of investment for these firms. Further analysis shows that the source of the noise in q is the value of long-term growth options introduced by project quality uncertainty. Part of q represents the option value of long-term growth potential, but this part is not very informative about near-term investment expectations. In effect, the option value adds noise to the part of q that reflects near-term investment plans. Cash flow is closely linked to current productivity, but not to the value of long-term growth options. Therefore, it serves as a useful instrument with respect to the noise in q.

Similar to this paper, Erickson and Whited (2000) argue that the neoclassical framework can account for the empirical cash flow sensitivity findings once Tobin's q is treated as a noisy proxy for marginal q. In their setup, the noise in Tobin's q is generic, whereas in this paper, the sources of such noise and their relative contributions to the investment–cash flow sensitivity are analyzed. In another related study, Gilchrist and Himmelberg (1995) construct estimates of marginal q using information in cash flow, and show that the constructed marginal q performs better than Tobin's q in explaining investment. Their result is in line with the finding in this paper that cash flow is informative about investment and is weakly related to the noise in q.

Kaplan and Zingales (1997, 2000) question the validity of the investment–cash flow sensitivity as a measure of financing constraints, though their line of argument is different. They show that the sensitivity is not necessarily higher for firms that are more constrained. However, unconstrained firms always exhibit zero sensitivity of investment to cash flow in their two-period setup. In contrast, the sensitivity is positive in the frictionless model of this paper, illustrating the alternative factors that may explain the empirical findings.

Concerned about a possible systematic relationship between firm sorting criteria (dividend payout, age, size, etc.) and the investment opportunity set, several studies have considered experiments that are more likely to isolate the financing role of cash flow. In Hoshi, Kashyap, and Scharfstein (1991), the sorting criterion is membership in a bank-centered industrial group, which is unlikely to be correlated with growth potential. Their result is that the investment–cash flow sensitivity is lower for member firms, which have easier access to financing due to their close ties to a major bank.[2] Lamont (1997) examines the response of invest-

[2] Diamond (1994), however, provides an alternative explanation for the Hoshi et al. (1991) result. In evaluating the control roles of public and bank debt, Diamond observes that the

ment by non-oil subsidiaries of oil companies to the 1986 oil shock and finds that these firms significantly decreased their investment. Finally, Whited (1992) and Bond and Meghir (1994) directly test the Euler equation of the dynamic optimization problem of the firm, and show that the Euler equation does not hold for highly levered firms, firms with no bond ratings (Whited), and firms with low dividend payout (Bond and Meghir).

The remainder of the paper is organized as follows. Section I describes the model and provides the characterization of the optimal investment rule. Section II discusses the calibration and the simulation methodology. Section III presents the results. Section IV concludes.

I. The Model

The analysis is of partial equilibrium type, in that it focuses on a single firm operating in a risk-neutral economy with a constant discount rate. In Section III, where simulation evidence is presented, a large number of such firms are considered.

A. The Firm

The firm's operating cash flow is generated by a Cobb–Douglas profit function given by

$$F(K_t, \theta, z_t)i = e^{\theta + z_t} K_t^{\alpha}. \tag{1}$$

Here, $F(K_t, \theta, z_t)$ is the cash flow rate at time t, K_t is the capital stock, $\alpha < 1$ is the returns to scale parameter, θ is the project quality, and z_t is the transitory shock. The project quality θ is constant through time, and is drawn at $t = 0$ from a normal distribution with mean 0 and standard deviation σ_θ. The transitory shock z_t follows a mean reverting process given by

$$dz_t = -\rho z_t dt + \sigma_z d\omega_t, \tag{2}$$

where $\rho > 0$ is the mean reversion coefficient, σ_z is the instantaneous standard deviation of z, and w is a standard Brownian motion. The initial value of the transitory shock z_0 is drawn from the invariant distribution of z, which is normal with mean 0 and standard deviation $\sigma_z/\sqrt{2\rho}$. The values of θ and z_t are not separately observable to the firm. The firm can only observe the cash flow rate $F(K_t, \theta, z_t)$. Inverting (1), the firm therefore observes $c_t \equiv \theta + z_t$ at time t.

The capital adjustment cost rate is

$$\Psi(I_t, K_t) = \frac{\phi}{2}\left(\frac{I_t}{K_t} - \delta\right)^2 K_t, \tag{3}$$

ability of a bank to restructure debt more easily will attract those firms that are likely to default at a time when they have profitable investment opportunities, that is, firms with low sensitivities of investment to cash flow from existing operations. The resulting self-selection is another example of how the sorting criterion may be correlated with the characteristics of investment opportunities.

where ϕ is the cost parameter, δ is the depreciation rate of capital, and I_t is the investment rate at time t.

B. The Firm's Problem

The firm's objective is to maximize the expected discounted sum of future net cash flow. At time t, the firm has capital stock K_t, and has observed the cash flow history from time 0 to t. Since both θ and z_t are Gaussian, the information set of the firm at time t is summarized by three variables: (1) the conditional expectation of the project quality m_t^θ, (2) the conditional expectation of the transitory shock m_t^z, and (3) the common variance of the estimation error of these variables γ_t. The firm solves the following optimal control problem:[3]

$$V(K_t, m_t^\theta, m_t^z, \gamma_t) = \max_I E_t\left[\int_{s=t}^{\infty}[F(K_s, \theta, z_s) - I_s - \Psi(I_s, K_s)]e^{-r(s-t)}ds\right] \tag{4}$$

$$\text{s.t. } dK_s = I_s ds - \delta K_s ds, \tag{5}$$

$$dm_s^\theta = \frac{\gamma_s \rho}{\sigma_z} d\bar{w}_s, \tag{6}$$

$$dm_s^z = -\rho m_s^z ds + \frac{\sigma_z^2 - \gamma_s \rho}{\sigma_z} d\bar{w}_s, \tag{7}$$

$$d\gamma_s = -\left(\frac{\gamma_s \rho}{\sigma_z}\right)^2 ds, \tag{8}$$

where

$$dw_s = \frac{1}{\sigma_z}[dc_s + \rho m_s^z ds]. \tag{9}$$

The net cash flow of the firm at time s is the cash flow from operations $F(K_s, \theta, z_s)$, minus investment I_s, minus the capital adjustment cost $\Psi(I_s, K_s)$. In (4), the firm value V at time t is given by the time-t expected value of discounted future net cash flow generated by the optimal investment policy I. The discount rate is r. The optimization problem is subject to the law of motion of the capital stock, (5). Equations (6) to (8) describe the evolution of the information set of the firm.[4] Recall that the firm observes the cash flow rate, or equivalently $c_s = \theta + z_s$, at time s.

[3] Fleming and Rishel (1975) show that the original optimal control problem, which has an infinite dimensional state space generated by past cash flow realizations, can be written as in (4) to (8).

[4] The initial values $m_0{}^\theta$ and $m_0{}^z$ are the corresponding expectations given the cash flow rate at time zero. For a proof that the conditional distributions of θ and z_t are Gaussian, and that (6) to (8) describe the laws of motion of the first two moments of these conditional distributions, see Liptser and Shiryayev (1977).

The unexpected component of the change in c_s, given by (9), provides *new information* to the firm. It is this new information content of cash flow that leads to revisions in the conditional expectations θ and z, hence, the laws of motion (6) and (7). Notice, from (8), that γ_t is decreasing in t. In words, the project quality estimate becomes more precise as the firm gets older.

The system (4) to (8) is a standard optimal control problem. The derivation of the Hamilton–Jacobi–Bellman (HJB) equation characterizing the optimal firm value is straightforward, and hence is omitted. The optimal investment rule is given by

$$\frac{I}{K} = \delta + \frac{V_K - 1}{\phi}. \tag{10}$$

Here, V_K, the partial derivative of V with respect to K, is the "marginal q" of the firm. As shown by Abel and Eberly (1994), marginal q is the present value of the stream of expected profit of an incremental unit of capital. The above equation indicates that the investment to capital ratio is a linear function of marginal q at all times. Intuitively, when marginal q exceeds one, investment is profitable; consequently the firm expands its capital stock by investing in excess of the depreciation rate. The opposite result holds when q is less than one. Higher values of the cost parameter ϕ dampen this adjustment process.

An analytical solution to V is difficult to obtain, because of the nonlinearities induced by the diminishing marginal product of capital and the mean reverting state m^z. Abel and Eberly (1996, 1997) are able to find closed form solutions, but only for cases where there is no mean-reverting technology shock and the production function is of constant returns to scale type. Given the difficulty with obtaining an analytical solution, I solve the problem (4) to (8) numerically.[5]

C. The Choice of Initial Capital Stock

I specify a simple technology to endow the firm with its initial capital stock. I assume that the firm solves the following problem when choosing the initial capital stock:

$$\max_{K_0} \; V(K_0, m_0^\theta, m_0^z, \gamma_0) - C_0 K_0, \tag{11}$$

where C_0 is the per unit cost of initial investment in capital goods. I assume that $C_0 > 1$, so that the initial investment is costly. In this case, the firm will start with a smaller capital stock (relative to the long-run average), so young firms will be smaller on average compared to mature firms.

[5] The numerical technique I use is polynomial approximation, as described in Judd (1999). Briefly, it involves writing V as a polynomial of its arguments in the HJB equation, and then solving for the polynomial coefficients that minimize an error criterion on a grid of points. The details of the numerical solution technique, as well as the derivation of the HJB equation, are available from the author upon request.

D. The Capital Structure of the Firm

Shareholders of the firm invest the necessary amount to purchase the initial capital stock. Therefore the capital structure is all-equity at time zero. Thereafter, the firm has access to a frictionless credit market, where it can borrow at the continuously compounded rate r if cash flow is not sufficient to finance new investment. Cash flow is primarily used for investment and paying back the creditors. The remaining amount, if any, is paid to shareholders as dividends.

II. Calibration and Simulation

A. Calibration

I set the discount rate $r = 0.05$, and the depreciation rate $\delta = 0.1$. Using COMPUSTAT data, Moyen (1999) estimates a value of 0.5866 for the persistence parameter of shocks to a Cobb–Douglas production function in a discrete-time setting. In the continuous-time setting here, her estimate approximately corresponds to $\rho = 0.5$, so I assume this value for ρ. I choose the remaining technology parameters so that the key characteristics of the model-generated mature firms match their counterparts in actual data. Mature firms in actual data have very high dividend payout ratios; therefore, they are unlikely to be financially constrained. Hence, the environment these firms operate in is closest to the frictionless setup of this paper. I set the returns to scale parameter $\alpha = 0.7$ to match the average cash flow–capital ratio of mature firms, which is 0.21 in FHP.

To calibrate σ_z, I use the time-series variation in CF/K. However, there is a continuum of $\{\sigma_z, \phi\}$ pairs that deliver a given level of variability in CF/K. Intuitively, an increase in σ_z makes cash flow more variable, but a smaller ϕ allows for faster capital adjustment and leads to smaller variability in CF/K. In choosing the $\{\sigma_z, \phi\}$ pair within this continuum, I match the investment–cash flow sensitivity of Class 3 firms of FHP, which is 0.23. The parameters that deliver this value (along with $std(CF/K) = 0.06$ from FHP) for the model-generated mature firms are $\phi = 4$ and $\sigma_z = 0.32$. The fact that these parameters are chosen to match an investment–cash flow sensitivity may raise concerns about overfitting. Notice, however, that only information about Class 3 firms of FHP is used in this parameterization. As mentioned above, these firms constitute a natural control group for this study. The main focus of the paper is on young, growing firms, and the current parameterization does not make use of any information on such firms.

I set $C_0 = 3.9$ based on a comparison of mean and median q values of mature and young firms.[6] Given these parameter values, there is a one-to-one correspondence between σ_θ and the variance of the initial quality estimate γ_0. As $\sigma_\theta \to \infty$, γ_0 converges to a positive constant equal to $\sigma_z^2/2\rho$. For the specific parameterization

[6] In FHP, the ratio of mean q values of Class 1 (growth firms) to Class 3 (mature firms) is 2.375 (3.8/1.6). The same ratio for medians is 1.6 (1.6/1). Setting $C_0 = 3.9$ results in approximately the same values for these ratios.

Table I
Parameter Values

Parameter		Value
Discount rate	γ	0.05
Depreciation rate	δ	0.1
Returns to scale	α	0.7
Mean reversion	ρ	0.5
Adjustment cost	ϕ	4
Variability of shock	σ_z	0.32
Initial investment cost	C_0	3.9
Initial uncertainty	γ_0	0.1

here, this bound is 0.1024. I choose a value of $\gamma_0 = 0.1$, implying a high degree of initial uncertainty. The corresponding value for σ_θ is 2.0656.[7]

Table I summarizes the choice of parameter values.

B. Simulation Procedure

The simulations are carried out to generate a data set similar to that of FHP. FHP use the annual Value Line database, using observations on manufacturing firms from 1969 to 1984. They form three classes of firms based on dividend payout. Class 1 firms have a ratio of dividends to income less than 0.1 for at least 10 of the 15 years in the data. For Class 2 firms, the ratio is less than 0.2 but more than 0.1 for at least 10 years. Class 3 includes all other firms.

I specify a slightly different sorting criterion than the one in FHP, since model firms initiate dividends at a rather high rate. I assign firms that pay no dividends for at least 10 out of 15 years to Class 1. Firms that pay no dividends for at least 5 but at most 9 years are assigned to Class 2. All other firms are assigned to Class 3.

The artificial data set is constructed as follows. For each firm, I simulate data for 100 years.[8] Then I choose a random year j between 1 and 86, inclusive. The data of the firm between years j and $j+14$ are extracted, and the firm is assigned to one of the three classes described above based on its dividend payout in these 15 years. I continue this procedure until each class has 3,000 firms.

III. Results

A. Summary Statistics

Table II presents the summary statistics of the model-generated data. The corresponding values from FHP are replicated in brackets for comparison. Recall

[7] The results are not very sensitive to the specific value of $\sigma_\theta = 2.0656$ that characterizes the initial uncertainty. For example, $\sigma_\theta = 1$ leads to results that are very similar to those that obtain with $\sigma_\theta = 2.0656$.

[8] In generating data, I discretize time by dividing a year into 40 equal periods, so that $\Delta t = 0.025$, and assume that the optimal policy at the beginning of a period is followed for Δt units of time.

Table II

Summary Statistics of the Model-Generated Data

The table reports the summary statistics of the model-generated data set. The same statistics from FHP are provided in brackets.

Statistic	Class 1	Class 2	Class 3
Average retention ratio	0.95	0.80	0.51
	[0.94]	[0.83]	[0.58]
Percent of years with positive dividends	0.15	0.54	0.99
	[0.33]	[0.83]	[0.98]
Average sales growth	10.61	7.81	2.75
	[13.7]	[8.7]	[4.6]
Average I/K	0.24	0.17	0.11
	[0.26]	[0.18]	[0.12]
Average CF/K	0.28	0.26	0.22
	[0.30]	[0.26]	[0.21]
Average of firm standard deviations of I/K	0.12	0.05	0.02
	[0.17]	[0.09]	[0.06]
Average of firm standard deviations of CF/K	0.11	0.08	0.06
	[0.20]	[0.09]	[0.06]
Average q value	5.78	3.62	2.41
	[3.8]	[2.4]	[1.6]
Median q value	3.94	3.25	2.34
	[1.6]	[1.4]	[1.0]
Average debt–capital stock ratio	0.48	0.18	0
	[0.57]	[0.52]	[0.33]

that only information about FHP's Class 3 firms was used in choosing the parameters of the model. Therefore, the match of the statistics for Classes 1 and 2 is an indicator of whether the model replicates the major aspects of firm growth in actual data. Three important statistics in this regard are the average retention ratio, the average investment–capital ratio, and the average cash flow–capital ratio.[9] Table II shows that these characteristics in the model-generated data are very similar to their counterparts in actual data. Therefore the model successfully captures the average patterns in growth dynamics of real-world firms. On the other hand, the model falls short of generating the observed time-series variation in investment and cash flow. Notice that the average firm standard deviations of both investment and cash flow in Class 1 are below those reported in FHP.

B. Regression Results

In this section, I report the results of regressions of the form

$$\frac{I_{i,t}}{K_{i,t}} = c_i + c_1 \frac{CF_{i,t}}{K_{i,t}} + c_2 q_{i,t} + \varepsilon_{i,t}. \tag{12}$$

[9] In the rest of the analysis, cash flow is defined as income after interest expense.

Table III

The Investment–Cash Flow Sensitivities in the Frictionless Benchmark

The left-hand-side panel of the table reports the coefficients of the regression

$$\frac{I_{i,t}}{K_{i,t}} = c_i + c_1 \frac{CF_{i,t}}{K_{i,t}} + c_2 q_{i,t} + \varepsilon_{i,t}$$

for each dividend payout class, and for different time intervals. The three classes of firms are generated by the simulation procedure described in Section II.B, using the parameter values in Table I. The data cover 15 years for each firm. The top panel reports the results of the regression using observations from years 1 to 6. The middle panel contains the results for years 1 to 10. The bottom panel reports the results for the full 15-year sample. The right-hand-side panel replicates the corresponding results in FHP for comparison. Above, $I_{i,t}$ is the investment of firm i in year t, $CF_{i,t}$ is the cash flow in the same year, $K_{i,t}$ is the capital stock at the beginning of year t, and $q_{i,t}$ is the beginning-of-the-year firm value divided by $K_{i,t}$. The fixed firm effect c_i, the investment–cash flow sensitivity c_1, and the investment-q sensitivity c_2 are the coefficients to be estimated (only c_1 and c_2 are reported below), and $\varepsilon_{i,t}$ is the error term. Also reported are the adjusted R_2 values.

	Model			FHP		
	Class 1	Class 2	Class 3	Class 1	Class 2	Class 3
Years 1–6						
$q_{i,t}$	0.0127	0.0208	0.0177	– 0.0010	0.0072	0.0014
$CF_{i,t}/K_{i,t}$	0.6587	0.3751	0.2860	0.670	0.349	0.254
R^2	0.96	0.91	0.96	0.55	0.19	0.13
Years 1–10						
$q_{i,t}$	0.0152	0.0270	0.0236	0.0002	0.0060	0.0020
$CF_{i,t}/K_{i,t}$	0.5863	0.3217	0.2706	0.540	0.313	0.185
R^2	0.96	0.91	0.97	0.47	0.20	0.14
Years 1–15						
$q_{i,t}$	0.0177	0.0304	0.0274	0.0008	0.0046	0.0020
$CF_{i,t}/K_{i,t}$	0.5109	0.3082	0.2600	0.461	0.363	0.230
R^2	0.95	0.94	0.97	0.46	0.28	0.19

Above, $I_{i,t}$ is the investment of firm i in year t, $CF_{i,t}$ is the cash flow in the same year, $K_{i,t}$ is the capital stock at the beginning year t, and $q_{i,t}$ is the beginning-of-the-year Tobin's q, defined as the total market value of the firm normalized by $K_{i,t}$. The fixed firm effect c_i, the investment–cash flow sensitivity c_1, and the investment–q sensitivity c_2 are coefficients to be estimated, and $\varepsilon_{i,t}$ is the error term.

The right-hand-side panel in Table III replicates Table 4 of FHP, where they report their results for the above regression. Table 4 of FHP constitutes their main evidence of financing constraints. It shows that the investment–cash flow sensitivity is positive for all firm classes, and is a lot larger for Class 1. FHP also estimate the same regression for the earlier periods of the data, namely the first 6 and the first 10 years. The result is that the cash flow sensitivity for Class 3 does not significantly change as one considers the earlier periods, but increases substantially for Class 1. FHP interpret these results as evidence of severe financing constraints. The reasoning is as follows. Tobin's q included in the regression accounts for the investment opportunity set of the firm; therefore, a significant estimate of the cash flow coefficient must reflect the effect of financing frictions. Firms in Class 1, which are typically small and young, face more stringent constraints due to asymmetric information problems; hence, they exhibit high sensitivity of investment to cash flow. FHP argue that the higher sensitivities that obtain for the earlier periods strengthen this view, since Class 1 firms are younger and less recognized in the earlier part of the sample.

The left-hand-side panel in Table III reporting the results for the model-generated data shows that the FHP story is not necessarily true. There are no financing constraints in the current model; firms have access to a frictionless credit market. Yet the resulting cash flow sensitivity patterns and magnitudes are quite similar to the ones in FHP. Investment is highly sensitive to cash flow for all firm classes and all periods. The full-sample sensitivities for classes 1 through 3 are 0.5109, 0.3082, and 0.2600, respectively. These are very similar to 0.461, 0.363, and 0.230 reported by FHP. Also, the sensitivity for Class 1 increases substantially as one considers the earlier periods. When the regressions are run for the first six years of the sample, the cash flow sensitivity for Class 1 rises to 0.6587, whereas the same coefficient for Class 3 is only 0.2860.

Overall, the results indicate that the observed investment–cash flow sensitivities are not anomalies in a frictionless market. Investment is sensitive to cash flow in the benchmark case without financing constraints, and the sensitivity is higher for low dividend payout, high-growth firms. In fact, the model is able to match the observed magnitudes of the investment–cash flow sensitivity for these firms quite successfully.

Why do low dividend payout firms have higher investment–cash flow sensitivities? Dividend payout per se has no effect on a firm's investment decisions, since Miller–Modigliani perfect market conditions are satisfied in the model. It is rather the case that dividend payout is correlated with firm age, and younger firms exhibit higher investment–cash flow sensitivities. Recall that in the model, growth in the early years is financed by issuing debt. Only after the debt is paid back does the firm start paying dividends. Therefore, younger firms tend to pay little or no dividends. Regression results based on an age sort (unreported) indicate that the investment–cash flow sensitivity is monotonically declining in age. Hence, sorting firms on the basis of dividend payout results in differential investment–cash flow sensitivities across firm classes.[10]

A remark about firm age is in order. In the model, age coincides with the maturity of the only project of the firm. Real world firms invest in multiple projects that arrive sequentially in time. Hence, an old but small firm may as well discover

[10] It should be noted that the specific capital structure policy model firms follow is a conservative one in terms of generating the high investment–cash flow sensitivity of low dividend payout firms. If external financing needs were satisfied in part by issuing equity, or if the firms had targeted a positive debt–equity ratio, firms would have less debt to run down, and, hence, would start paying dividends earlier. In that case, the typical firm in the low-payout class would be even younger, and the investment–cash flow sensitivity of the low-payout class would be even higher.

a profitable project and exhibit high growth. Clearly, a high investment–cash flow sensitivity reflects the fact that the growth project is the main source of cash flow in the model; otherwise, firm age is not the directly relevant factor. Therefore, the results should be interpreted within a more general context where young firms are those that have recently discovered major growth opportunities relative to the size of their existing operations.

C. The Information Content of Cash Flow

The results of the previous subsection indicate that cash flow is highly informative about investment opportunities for growth firms. In this subsection, I examine the nature of the information reflected by cash flow. In (12), cash flow and investment are aggregates of flow variables that are realized within each year, whereas Tobin's q is measured at the beginning of the year. Therefore, investment may be sensitive to cash flow both because cash flow shocks within a year provide new information, and because cash flow reflects information that is already known at the beginning of the year but is not captured by Tobin's q. To evaluate these two possibilities, I decompose cash flow into its expected and surprise components, and estimate the sensitivity of investment to each component separately.[11] Note that both q and the expected cash flow are in the information set of the firm at the beginning of each year, whereas the surprise component of cash flow is realized within the year.

Panel A of Table IV reports the sensitivity of investment to cash flow surprises. The sensitivity is positive for all three classes, and it is considerably higher for Class 1. These results are not surprising. Cash flow shocks provide new information about investment opportunities, and firms respond by adjusting their capital stocks within the year. The response is stronger for young firms, as cash flow shocks are informative not only about current productivity but also about long-run growth prospects for these firms.

Panel B of Table IV shows the results of the regression where the independent variables are Tobin's q and the expected cash flow. For all three firm classes, investment is sensitive to cash flow expectations, indicating that Tobin's q is a noisy measure of the investment opportunity set. More importantly, this sensitivity is a lot higher for Class 1. In fact, the difference between the sensitivities of Class 1 and Class 3 remains as large as in Table III after the surprise component of cash flow is removed.[12] Hence, the high investment–cash flow sensitivities of young firms in part reflect the poor proxy quality of Tobin's q.

[11] Specifically, I calculate the expected value of year-t cash flow given the information set of the firm at the beginning of year t. There is no closed form solution for this expectation, so I calculate it numerically through simulations.

[12] FHP run a similar regression, where they instrument cash flow with lagged variables. Unfortunately they do not report the results of that regression; they just mention that the resulting difference between the investment–cash flow sensitivities of Class 1 and Class 3 firms is as large as the difference that obtains in the original (uninstrumented) regression. Since the instrumental variables regression removes the new information content of cash flow, FHP interpret their finding as further evidence of financing constraints. Panel B of Table IV shows that the same result obtains in the frictionless benchmark.

Table IV

Information Content of Cash Flow

The table reports the coefficients of the regression

$$\frac{I_{i,t}}{K_{i,t}} = c_i + c_1 X_{i,t} + c_2 q_{i,t} + \varepsilon_{i,t}$$

for each dividend payout class. In Panel A, $X_{i,t}$ is the surprise component of cash flow for firm i in year t normalized by the beginning of the year capital stock, $[CF_{i,t} - ECF_{i,t}]/K_{i,t}$. In Panel B, $X_{i,t}$ is the expected component of cash flow for year t at the beginning of the year normalized by capital stock, $ECF_{i,t}/K_{i,t}$. In both panels, $q_{i,t}$ is the beginning-of-the-year Tobin's q value. In Panel C, $X_{i,t} \equiv ECF_{i,t}/K_{i,t}$, but $q_{i,t}$ is replaced by $q^*_{i,t}$, which is the q value of an otherwise identical firm for which $\gamma_t = 0$.

	Class 1	Class 2	Class 3
		Panel A: Cash Flow Surprises	
$q_{i,t}$	0.0262	0.0433	0.0808
$[CF_{i,t} - ECF_{i,t}]/K_{i,t}$	0.5821	0.4141	0.3014
R^2	0.9247	0.9016	0.9350
		Panel B: Expected Cash Flow	
$q_{i,t}$	0.0203	0.0345	0.0500
$ECF_{i,t}/K_{i,t}$	0.3418	0.1889	0.1386
R^2	0.8968	0.8045	0.7108
		Panel C: Noise-free q	
$q^*_{i,t}$	0.0326	0.0448	0.0567
$ECF_{i,t}/K_{i,t}$	0.2035	0.1351	0.1197
R^2	0.9114	0.8200	0.7110

Why is Tobin's q a more noisy measure of investment for young firms? Recall that a young firm faces project quality uncertainty. The fact that the uncertainty will be resolved over time creates implicit growth options. The option value stems from the upside growth potential that will be realized if the actual project quality turns out to be substantially higher than the current estimate—an event that is more likely if the current estimate is very imprecise. Notice that this option value is not very informative about near-term investment plans; rather, it relates to the resolution of uncertainty in the long-run, and, hence, reflects long-term growth expectations. But being a part of total firm value, the option value directly affects q. In effect, then, the value of long-term growth options adds noise to the part of q that measures near-term investment.

In the current continuous-time setup, it is difficult to isolate the exact value of growth options, since these options are implicitly defined. Nevertheless, one can derive an approximate value for these implicit options and analyze whether they are indeed responsible for the poor performance of q. Consider the firm value at time t, $V(K_t, m^\theta_t, m^z_t, \gamma_t)$. The uncertainty in growth prospects is captured by γ_t, the variance of the estimation error of project quality. Since it is this uncertainty that creates the implicit options, the part of firm value that is due to γ_t, that is, $V(K_t, m^\theta_t, m^z_t, \gamma_t) - V(K_t, m^\theta_t, m^z_t, 0)$, represents the option value of growth. The remaining part, $V(K_t, m_t^\theta, m_t^z, 0)$, should better reflect the near-term investment plans, since it is free of the noise introduced by long-term growth options.

Let

$$q_t^* = \frac{V(K_t, m_t^\theta, m_t^z, 0)}{K_t} \tag{13}$$

be the corresponding "noise-free" component of q_t. Panel C of Table IV presents the results of the investment regression where the independent variables are q^* and the expected cash flow. The results confirm the hypothesis that the source of the noise in q is the value of real options relating to long-term growth. For all firm classes, the coefficient of q^* is higher than the coefficient of q (from Panel B). Also, the sensitivity of investment to expected cash flow declines once q is replaced by q^*. Expected cash flow is closely linked to current profitability, but not to the value of long-term growth options. This weak relationship to the "noise" in q makes expected cash flow a useful instrument in the investment regressions. Hence, investment is highly sensitive to expected cash flow in regressions where q is the control variable. When q is replaced by the less noisy q^*, the effect of cash flow diminishes.[13] Notice that the decline in the sensitivity to expected cash flow from Panel B to Panel C is the largest for Class 1, and it is very small for Class 3. This makes sense, since the option value is substantial for young firms (Class 1) but negligible for mature ones (Class 3).[14,15]

Erickson and Whited (2000) show that positive investment–cash flow sensitivities may obtain when Tobin's q is a noisy proxy for marginal q. The option value of long-term growth in the current discussion illustrates an important economic source for the noise in Tobin's q, which is modeled generically in Erickson and Whited. Unfortunately, the value of growth options is not observable in actual data, making an empirical evaluation of the idea difficult. One indirect test may involve regressing the sum of next few years' investment, rather than only next year's, on q and expected cash flow. If its failure is indeed related to the option value of long-term growth, then q should fare better once the investment figure accounts for the optimal exercise of this option.[16]

[13] The sensitivity to expected cash flow is not completely eliminated when q^* controls for investment. This is because the calculation of the option value is only approximate, and, hence, even q^* contains noise (albeit less than q).

[14] The option value, as a fraction of firm value, is large and highly variable in Class 1 (mean 8 percent, standard deviation 5.5 percent), whereas it is small and does not vary as much in Class 3 (mean 1 percent, standard deviation 0.8 percent). For very young firms, real options represent as high as 33 percent of firm value.

[15] It should be emphasized that q as a stand-alone variable is highly informative about investment opportunities of young firms. In univariate regressions, investment-q sensitivity is the highest for Class 1 (results unreported). The poor proxy quality of q for these firms becomes apparent only after cash flow is included in the regressions.

[16] The results of this experiment on model-generated data, which are not reported to save space, confirm the above intuition. When the dependent variable is the sum of the next four years' investment, the q sensitivity increases substantially relative to Panel B of Table IV, whereas the cash flow sensitivity turns negative.

IV. Conclusion

This paper analyzes the sensitivity of investment to cash flow in the benchmark case where financing is frictionless. Overall, the results indicate that the frictionless benchmark is able to account for the observed magnitudes of the investment–cash flow sensitivity, and the patterns it exhibits. Investment is sensitive to cash flow, even after controlling for its link to profitability by conditioning on Tobin's q. Furthermore, the sensitivity is substantially higher for young, small firms with high growth rates and low dividend payout ratios, as it is in the data. The uncertainty these firms face about their growth prospects amplifies the investment–cash flow sensitivity in two ways. First, the uncertainty is resolved in time as cash flow realizations provide new information about investment opportunities. This makes investment highly sensitive to cash flow surprises. Second, the uncertainty creates implicit growth options, whose values show up in q. Since these options relate to long-term growth potential but not to investment in the near-term, q performs as a noisy measure of short-term investment expectations. Having a weaker relationship with the value of long-term growth options, cash flow acts as a useful instrument in investment regressions.

Both factors discussed above contribute to the failure of Tobin's q to control for the investment opportunity set, rendering the economic interpretation of empirical cash flow sensitivity findings difficult. The first issue, that is, the informativeness of cash flow shocks, is an econometric one, and is relatively easy to handle; one can remove the effects of the surprise component of cash flow by using lagged instruments. The second problem is more fundamental; it illustrates the limitations of q as a composite measure of both short- and long-term investment expectations. Future work could address the issue of providing observable variables that account for different dimensions of growth separately, in effect breaking down q into its components.

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