# A Dynamic Model of Characteristic-Based Return Predictability

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**ABSTRACT**

We present a dynamic model that links characteristic-based return predictability to systematic factors that determine the evolution of firm fundamentals. In the model, an economy-wide disruption process reallocates profits from existing businesses to new projects and thus generates a source of systematic risk for portfolios of firms sorted on value, profitability, and asset growth. If investors are overconfident about their ability to evaluate the disruption climate, these characteristic-sorted portfolios exhibit persistent mispricing. The model generates predictions about the conditional predictability of characteristic-sorted portfolio returns and illustrates how return persistence increases the likelihood of observing characteristic-based anomalies.

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Since the late 1970s, financial economists have identified a number of firm characteristics like valuation ratios, profitability and asset growth rates that explain the cross-section of stock returns. Historically, market-neutral portfolios that are sorted on these characteristics have exhibited very high Sharpe ratios – at least as high as the Sharpe ratio of the market. While the literature has mostly focused on these historical return patterns, there is relatively little research on the *conditional* relationships between characteristics and returns. This is somewhat surprising because the cross-section of firm characteristics in the economy, as well as the relation between characteristics and firm values, exhibits substantial time variation (e.g., the increased prevalence and valuations of growth firms in the late 1990s).

This paper explores the evolution of firm characteristics and their links to return predictability within the context of a dynamic behavioral model. The model assumes that the profitability and growth rates of firms are affected by what we refer to as the disruption climate, which is an economy-wide factor that creates losers as well as winners. The behavioral elements of the model arise because investors are overconfident about their ability to assess the disruption climate, and because of this, firms with different exposures to disruption – e.g., value versus growth firms – exhibit predictable return differences. The model's testable implications relate these return differences to observable measures of the conditional bias in investor beliefs. For instance, cross-sectional dispersions of characteristics, which reflect the hard and soft indicators of disruption that investors observe, predict subsequent returns of characteristic-sorted portfolios. The model also illustrates how the persistence of characteristic-sorted returns, which obtains due to slow-changing investor beliefs, can substantially increase the likelihood of observing characteristic-based anomalies such as the value premium in samples that are of comparable length to those in empirical studies.

Firms in the model are characterized by differences in their current access to new growth opportunities, as well as by their different histories. Growth firms are endowed with new projects each period while value firms simply harvest the profits from their existing projects. The emergence of new projects, as well as the demise of existing ones, is determined by a systematic factor that we refer to as the disruption climate. A favorable disruption climate increases the arrival rate of new projects, which benefits young growth firms, but because these new projects compete with existing businesses, a favorable disruption climate harms the profits of assets in place, and is thus detrimental to mature value firms. The model thus captures the Schumpeterian notion of creative destruction, whereby innovation creates losers as well as winners.

To abstract from differences in risk premia, investors in our model are assumed to be risk-neutral. These investors learn about the disruption climate from two sources, the realized rate of disruptive innovations, and a soft information signal that represents, for example, news reports and expert opinions. Since both sources are noisy indicators of the disruption climate, investor expectations contain estimation errors, which implies that even fully rational investors are sometimes too optimistic and sometimes too pessimistic about the rate of future disruptive innovations. However, these estimation errors do not generate predictable returns when investors are rational – some degree of irrationality is needed to generate asset pricing anomalies.

We introduce the possibility of biased inferences by assuming that investors are overconfident about the precision of their soft information, which implies that their estimates of the disruption climate puts too much weight on the soft information signal. This behavioral bias does not cause investors to systematically over or underestimate the disruption climate, that is, the *unconditional* or the long-run expected return associated with disruption rate surprises is zero. However, because overconfident investors learn slowly, *conditional* expected returns differ from zero and change slowly over time. Put differently, the mistakes that investors make in estimating the disruption climate take time to correct.

We explore the model's empirical and quantitative implications for characteristic-sorted portfolio returns through simulations. We first analyze differences in portfolio exposures to disruption and the return predictability that these differences generate. Portfolios that bet in favor of newcomers (growth firms, unprofitable firms, and high asset growth firms) are positively exposed to disruption, whereas portfolios that bet in favor of incumbents (value firms, profitable firms, and low asset growth firms) are negatively exposed. Thus, when investors overestimate the disruption climate, investment strategies that bet in favor of newcomers and against incumbents exhibit low subsequent return performance. The opposite predictability pattern obtains when investors underestimate the disruption climate. Our analysis also highlights how return predictability is affected by the interactions between different characteristics. For instance, while growth firms are positively exposed to disruption on average, profitable growth firms exhibit negligible exposures, since an increased flow of investment opportunities is offset by the demise of some of the existing profitable projects. Thus, the predictability of growth firms' returns is driven mainly by unprofitable growth firms, not the profitable ones.

Our analysis suggests three avenues for testing the model's predictions on conditional return predictability. The first is to employ measures of investor expectations that one might obtain from surveys, the media, or analyst reports. In our model, overconfident investors' expectations of the disruption climate are too dispersed relative to the rational benchmark, e.g., high expectations tend to be too high. Thus, measures of investor expectations predict subsequent returns of characteristic-sorted portfolios, similar to the "investor sentiment" effects documented in the empirical literature. Second, our analysis links return predictability to the observed distribution of firm characteristics. As we show, the cross-sectional dispersion of firms' valuation ratios, profitability, and asset growth rates capture past realizations of both hard and soft information signals. Since investors' processing of these signals is biased, measures of cross-sectional dispersion of firm characteristics predict subsequent returns. Third, characteristic-sorted portfolio returns exhibit positive autocorrelation, thus momentum strategies that buy recent winner portfolios and sell recent losers are profitable in our model.

In the final part of the paper, we consider the historical return patterns discussed at the outset, and examine the extent to which they can be replicated within the context of our model. As our simulations illustrate, the persistence of characteristic-sorted portfolio returns described above greatly increases the likelihood that these portfolios generate high Sharpe ratios. For instance, realizing a Sharpe ratio of 0.40 in a 50-year sample, an extremely unusual event under full investor rationality, occurs in up to 20% of the simulated histories. Moreover, such abnormal historical return patterns do not predict significant future returns. While these results obtain in a stylized model that assumes zero unconditional return predictability, the broader takeaway of this analysis – that investor overconfidence can substantially increase the likelihood of extreme return realizations – holds more generally, regardless of the magnitude of unconditional expected returns.

Our analysis is related to the behavioral finance literature, in particular, to Daniel, Hirshleifer, and Subrahmanyam (DHS) (1998, 2001), who describe a link between the value effect and the tendency of investors to be overconfident about the precision of their private information.[1] We also focus on overconfidence, but our channel generating mispricing is different. In the DHS papers, firms are essentially identical, and the value effect is generated

[1] Other important papers in this literature include Barberis, Shleifer, and Vishny (1998), who consider behavioral biases that influence how investors estimate the persistence of earnings shocks, and Hong and Stein (1999), who consider the effects of positive-feedback traders and traders who ignore the information embedded in market prices.

from the fact that overpriced stocks tend to have high prices relative to fundamentals. In contrast, the firms in our model differ in fundamental ways, for example, growth firms have new investment opportunities which value firms do not, and it is these fundamental differences that lead to cross-sectional differences in their exposures to sources of systematic risk.

There is also a behavioral literature that explores how fluctuations in investor sentiment can induce covariation among stocks with common characteristics.[2] In our model, overconfident investors tend to overreact to soft information about disruption shocks, and in doing so they induce excess (relative to the fully rational case) covariation among stocks with similar characteristics. In this sense, our model endogenously generates what looks like a sentiment factor.

The analysis in this paper is also related to studies that focus on asset pricing with parameter uncertainty. For instance, Lewellen and Shanken (2002) show, in a setting in which rational Bayesian investors learn about expected cash flows, that returns may appear to the econometrician to be predictable along historical sample paths. As our analysis illustrates, if we properly integrate over all possible sample paths, the null hypothesis of no predictability is rejected too often only when investors are irrational. Pastor and Stambaugh (2012) analyze the long-term variance of market returns in a model with parameter uncertainty and changing expected rates of return. Similar to our paper, they make the point that changes in expected returns can increase the volatility of long-term return realizations.[3]

[2] Barberis and Shleifer (2003) present one of the first theoretical analyses of how style or characteristic-based investor sentiment can generate excess return covariation. See Baker and Wurgler (2006) for an empirical analysis of the impact of time-varying investor sentiment on characteristic-sorted portfolio returns.

[3] Other related papers include Timmermann (1993, 1996) and Pastor and Veronesi (2003, 2006). Timmermann (1993, 1996) analyzes models in which investors use Bayes' rule to estimate unknown parameters but value assets without taking into account estimation error, resulting in predictable returns and excess volatility. Pastor and Veronesi (2003, 2006) also analyze learning effects on asset prices, though their focus is not on return predictability. The Pastor-Veronesi models illustrate how uncertainty about future growth rates can rationalize high and volatile valuations, especially for young businesses such as the technology firms of the late 1990s.

Our research also complements recent work by Gârleanu, Kogan, and Panageas (2012) and Kogan, Papanikolaou, and Stoffman (2015), who examine how the innovation process can generate sources of systematic risk that affect the prospects of different firms differently. In these models growth firms earn low expected returns as they constitute a hedge against the displacement risk brought about by technological progress. These models, which assume full rationality, require fairly strong risk preferences to rationalize the historically observed return patterns.[4] Although we solve our model with risk-neutral preferences, we can envision a hybrid model that accounts for risk preferences as well as slow learning that better explains the historical return patterns.

The remainder of the paper is organized as follows. Section I presents the model. Section II describes the model calibration and simulations. Sections III and IV present analyses of the calibrated model. Section V concludes the paper.

## I. The Model

### *A. Model Setup*

Time is continuous and denoted by $t$. Because our focus is on abnormal returns, we abstract from the possibility of risk premia and assume that investors are risk-neutral and discount cash flows at a constant rate $r$. The investors also have the same beliefs about the model parameters and observe the same information, which implies that asset prices are effectively set by a representative agent. The economy is populated by a continuum of firms; we use the superscript $i$ to denote a generic individual firm. Financing is frictionless and the Modigliani-

[4] See also Bena, Garlappi, and Grüning (2016) and Loualiche (2016), who analyze asset pricing implications of new firm creation in general equilibrium models. A number of earlier papers in the literature also provide risk-based explanations for the value premium. Examples are Berk, Green, and Naik (1999), Carlson, Fisher, and Giammarino (2004), and Zhang (2005). These papers, which specify exogenous pricing kernels that are calibrated with a high price of risk, also effectively assume extreme risk aversion.

Miller Theorem holds so we assume that all firms are equity financed without loss of generality.

Firms derive their values from their projects, which are infinitesimal investment opportunities that arrive exogenously. A project generates cash flows until it becomes obsolete and is terminated. Specifically, a newly arrived project requires capital investment $k_z$ and generates deterministic cash flows at rate $a_z k_z$ until it is terminated, where $a_z$ represents the project's return on investment.[5] The subscript $z$ denotes the firm's idiosyncratic growth state; as we explain below, firms in different growth states receive different projects. When an active project is terminated, a fraction $\alpha$ of the initial investment $k_z$ is recovered.[6] The net cash flows of the firm, which include the cash flow from the active projects, the cost of investing in new projects, and the value of the recovered capital of terminated projects, are immediately paid out to shareholders.[7]

Firms in this model differ for two reasons. The first is that they have different histories, that is, they received different projects in the past. The second is that they inhabit different growth states, which determine the new projects that they receive. Specifically, a firm is in one of three growth states $z$ at any given time: the *early growth* state *EG,* the *mature growth* state *MG*, and the *no-growth* state *NG*. Let $z_t^i \in \{EG, MG, NG\}$ describe the state of firm $i$ at time $t$. *New firms* are born with identical initial conditions into the early growth state and they transition

[5] Our assumption of projects with deterministic cash flows allows us to focus on projects' arrival and termination rates as the primary sources of risk. A more general version of the model could feature project cash flows that are subject to additional risk factors.

[6] We assume for simplicity that capital does not depreciate. An alternative interpretation is that capital depreciates but has to be replenished for the project to be operational; under this interpretation, the cost of depreciation is implicit in the project return $a_z$.

[7] By assuming an exogenous process governing project arrivals and terminations, we abstract from the possibility that firms' real investment choices are influenced by investor beliefs. This assumption allows us to focus on the pricing of a given set of assets. Yet the feedback from asset prices to investment choices may be relevant for some return anomalies, especially those that relate to firms' investment rates. See Altı and Tetlock (2014) for a quantitative analysis of how firms' investment decisions may amplify the effect of biased investor beliefs on return predictability.

into the mature growth state and the no growth state over time.

In our calibrated model, the firm has access to new projects in the early and mature growth states, but not in the no-growth state. Specifically, we set $k_{EG} = k_{MG} = k$ and $a_{EG} > a_{MG}$. Thus, the projects that arrive in the mature growth state require the same initial capital investment $k$ as in the early growth state but are less profitable. The assumption that firms in the no-growth state receive no projects can be stated as $k_{NG} = a_{NG} = 0$.

Let $f_t^i$ denote the firm's *profitability*, defined as the rate at which the cash flow from the firm's active projects is generated, and $K_t^i$ denote the firm's *capital stock*, which is the total capital investment incurred for the active projects. A new firm *i*, which is born into the early growth state at time *t*, has an initial capital stock that is normalized to $K_t^i = 1$ and initial profitability that is normalized to $f_t^i = 0$.[8]

After being born into the early growth state, the firm's state $z_t^i$ evolves as a continuous-time Markov process with sequential jumps. Specifically, the firm transitions from the early growth state to the mature growth state with Poisson intensity $q_{EG}$, and from the mature growth state to the no-growth state with Poisson intensity $q_{MG}$. A firm in the no-growth state dies and leaves the firm population with Poisson intensity $q_{NG}$. Each firm that dies is replaced by a new firm that is born into the early growth state, as described above. When a firm dies, its owners receive the market value of its active projects as a liquidating dividend.[9] The transition rates described above imply that firms spend $1/q_{EG}$ years on average in the early growth state, $1/q_{MG}$

[8] The assumption that firms are born with an unproductive unit of capital is motivated by the presence of firms with valuable growth opportunities but little or no profits in the data. The specific value chosen here, zero initial profitability, does not affect our results in any material way; what is important is that the model includes growth firms with low profits.
[9] Thus, a firm's death in our model resembles an asset sale to an entity outside the publicly traded corporate sector.

years on average in the mature growth state, and $1/q_{NG}$ years on average in the no-growth state. Thus, the average life expectancy of a firm is $1/q_{EG}+1/q_{MG}+1/q_{NG}$ years.

The firm's capital stock $K_t^i$ and profitability $f_t^i$ evolve according to the following laws of motion:

$$dK_t^i = k_z\left(dt + dM_t\right) - \lambda K_t^i\left(dt + dM_t\right), \tag{1}$$

$$df_t^i = a_z k_z\left(dt + dM_t\right) - \lambda f_t^i\left(dt + dM_t\right). \tag{2}$$

In equations (1) and (2), the *z* subscript refers to the firm-specific growth state. The term $dM_t$ represents a *disruption rate* that is systematic across firms. We describe the evolution of the disruption rate and investors' information about it in detail below. For now, it is sufficient to point out that the disruption rate $dM_t$ is a persistent process with a long-term mean of zero.

The first terms in equations (1) and (2) capture the arrival of new projects. Recall that firms in the early and mature growth states receive projects that each require a capital investment of $k_z = k$ and add $a_z k_z$ to the firm's profitability. The rate at which new projects arrive is stochastic and is represented by the term $dt + dM_t$. Thus, over an instantaneous time period *dt* the firm receives *dt* projects on average (i.e., one project per unit of time), with more or less new projects arriving depending on the realization of the disruption rate $dM_t$.

The second terms in equations (1) and (2) reflect the termination of active projects. Over a given period of time, a fraction of the firm's active projects become obsolete and are liquidated. When a project is terminated, the capital of the firm declines and the profitability of the firm declines by a proportional amount. Projects are terminated at an average rate $\lambda > 0$, which we assume is the same for all firms. As with the arrival of new projects, the realized termination rate depends on the disruption rate and is given by $\lambda\left(dt + dM_t\right)$.

Equations (1) and (2) illustrate how the exogenous growth state of the firm, i.e., early growth, mature growth or no growth, generates the endogenous state variables, the capital stock $K_t^i$ and profitability $f_t^i$. Transitions from one state to another, along with the termination rate of active projects, generate cross-sectional and time-series variation in firm size, profitability, and valuation ratios. Firms that have profitable active projects but are in the no-growth state expect their size and profits to decline over time. Firms that have low current profitability but are in growth states expect the opposite. Thus, the model captures in a reduced-form way the Schumpeterian notion of creative destruction: profits are redistributed from established firms to newcomers and from old to new technologies.

The economy-wide disruption rate $dM_t$, which determines the speed of this Schumpeterian reallocation process, is the main focus of the model. When $dM_t$ is high, new projects are created faster and existing projects are destroyed faster. As a result, early growth firms benefit when $dM_t$ is high and no-growth firms are hurt. Depending on parameters, mature growth firms can be either helped or hurt by more disruption, since it hurts their existing businesses while at the same time facilitates new projects.

The disruption rate, which is observable, is an exogenously specified process that has both persistent and transitory components. Specifically,

$$dM_t = \mu_t dt + \sigma_M d\omega_t^M, \tag{3}$$

where $\mu_t$, what we refer to as the *disruption climate*, is the persistent component of the disruption rate, and the Brownian process $d\omega_t^M$ is the transitory component. We model the disruption climate $\mu_t$ as a slow-moving variable with a long-term mean that is normalized to zero. Specifically, $\mu_t$ evolves according to

$$d\mu_t = -\rho_\mu \mu_t dt + \sigma_\mu d\omega_t^\mu. \tag{4}$$

Although investors observe the realized disruption rate $dM_t$, they cannot separately observe the persistent and the transitory components.[10] They do, however, observe a *soft information signal* $ds_t$ that reflects the state of technological progress, changes in the regulatory environment, and other information that may help them predict the future evolution of the disruption climate. Specifically, investors observe

$$ds_t = \eta d\omega_t^\mu + \sqrt{1-\eta^2}\, d\omega_t^s, \tag{5}$$

where the parameter $\eta \in [0,1]$ is the signal's precision and the Brownian term $d\omega_t^s$ is the signal's noise. Higher values of $\eta$ describe a more informative signal and thus less residual uncertainty about $\mu_t$. In model calibrations, we consider the possibility that investors have biased perceptions about the precision of the soft information signal. Specifically, we analyze the case

[10] Investors observe the realized disruption rate $dM_t$ because each firm's changes in capital stock and profitability are observable and can be used to back out $dM_t$ (see equations (1) and (2)). In a more general version of the model in which the change in profitability contains additional noise terms, a single firm's change in profitability would not perfectly reveal $dM_t$, but with a large cross section of firms investors would still be able to estimate it highly precisely.

in which overconfident investors believe the signal precision parameter to be $\eta_B > \eta$.

Investors in this model use the historical realizations of the disruption rate along with their soft information to learn about expected future disruption rates. We model the disruption rate $dM_t$ in a way that reflects the learning features that we would like to analyze. For learning to be relevant, the disruption rate needs to have a persistent component that is not directly observable along with a transitory component. In other words, as expressed in equation (3), the observed disruption rate equals the persistent component plus a transitory component that obscures the investor's inference problem. In principle, the persistent component $\mu_t$ could be an unknown constant $\mu$; however, when this is the case, learning effects vanish in the long run since investors eventually learn $\mu$ arbitrarily precisely. In our model, the persistent component $\mu_t$ changes over time. Investors learn about the current value of $\mu_t$, but unobservable shocks to $\mu_t$ create an additional source of uncertainty. In the steady state, these two effects cancel out, and the estimation error that investors face about $\mu_t$ remains constant over time.

The soft signal $ds_t$ plays an important role in the model. The signal summarizes all of the nonfinancial data that investors use to evaluate the current disruption climate. Investors' possibly biased perception of the signal's precision drives the return predictability patterns that the model generates. The signal is assumed to be informative about the shocks to $\mu_t$ rather than the level of $\mu_t$. This specification, which we take from Scheinkman and Xiong (2003), has two advantages. First, the signal has constant variance $\eta^2 + \left(\sqrt{1-\eta^2}\right)^2 = 1$ regardless of the value of $\eta$. Thus, the specification permits biased investor beliefs about signal precision (i.e., $\eta_B \neq \eta$) that cannot be detected directly from the time-series variance of signal realizations. Second, the specification clearly delineates the two sources of information that investors use to update their estimates. The signal is informative about shocks to $\mu_t$, whereas the realized disruption rate $dM_t$ is informative about the level of $\mu_t$. Being orthogonal, these two sources of information generate an economically meaningful two-factor structure for asset returns.

*B. Information Processing*

As discussed above, investors update their beliefs about the disruption climate $\mu_t$ based on two pieces of information, namely, the realized disruption rate $dM_t$ and the signal $ds_t$. In this section we characterize the steady state of the model in which the precision of the conditional estimate of $\mu_t$ is constant over time.

Let $\hat{\mu}_t$ denote investors' *expected disruption rate*, defined as the conditional estimate of $\mu_t$ given all available information at time *t*. Let $\gamma$ denote the steady-state variance of the estimation error $\hat{\mu}_t - \mu_t$. The law of motion of $\hat{\mu}_t$ is given by

$$d\hat{\mu}_t = -\rho_\mu \hat{\mu}_t dt + \sigma_\mu \eta ds_t + \frac{\gamma}{\sigma_M} d\bar{\varpi}_t, \tag{6}$$

where

$$d\bar{\varpi}_t \equiv \frac{dM_t - \hat{\mu}_t dt}{\sigma_M}. \tag{7}$$

The *disruption surprise* $d\bar{\varpi}_t$ is a standard Brownian motion that reflects the unexpected component of the realized disruption rate. Recall that the signal $ds_t$ is also a standard Brownian motion by construction. Therefore, the disruption surprise $d\bar{\varpi}_t$ and the signal $ds_t$ constitute the two sources of systematic risk that are orthogonal to each other.

The steady-state variance of the estimation error $\gamma$ solves

$$\frac{\sigma_\mu^2}{2\rho_\mu} = \frac{1}{2\rho_\mu}\left(\sigma_\mu^2\eta^2 + \frac{\gamma^2}{\sigma_M^2}\right) + \gamma. \tag{8}$$

The solution is given by

$$\gamma = \sigma_M\left(-\rho_\mu\sigma_M + \sqrt{\rho_\mu^2\sigma_M^2 + \left(1-\eta^2\right)\sigma_\mu^2}\right). \tag{9}$$

When investors have biased signal precision, the parameter $\eta$ in equations (6), (8), and (9) is replaced by its biased counterpart $\eta_B > \eta$, which results in $\gamma_B < \gamma$. When this is the case, investors overestimate the precision of their soft information, which implies that they believe that their disruption rate estimate $\hat{\mu}_t$ is more precise than it actually is. Inspecting equation (6), we see that this bias leads investors to place too much weight on their soft signal $ds_t$ in updating $\hat{\mu}_t$, and too little weight on *hard information* as reflected by the disruption surprise $d\bar{\varpi}_t$.

*C. Valuation and Returns*

We now turn to firms' valuations and stock returns. To keep our discussion focused on the economic intuition, we present the relevant equations and provide their derivations in the

Internet Appendix.[11]

The value of a firm can be expressed as the discounted value of its expected cash flows conditional on all available information:

$$V_t^i = E_t\left(\int_{u=t}^{\infty} e^{-r(u-t)}\left[ f_u^i du + \left(\alpha\lambda K_u^i - k_z\right)\left(du + dM_u\right)\right]\right). \tag{10}$$

The cash flows in equation (10) consist of three components: $f_u^i$, the profits accruing to firm *i* from its active projects, $\alpha\lambda K_u^i$, the capital recovered from terminated projects, and $k_z$, the outflows arising from capital investments for new projects given the firm-specific state $z = z_u^i$. Note that the profits from active projects accrue at an instantaneously deterministic rate, whereas capital flows for new and terminated projects are stochastic and determined by the economy-wide disruption rate $dM_u$.

Firm value in equation (10) can be decomposed as

$$V\left(z_t^i, K_t^i, f_t^i, \hat{\mu}_t\right) = f_t^i V_f\left(\hat{\mu}_t\right) + K_t^i V_K\left(\hat{\mu}_t\right) + V_{g,z}\left(\hat{\mu}_t\right), \tag{11}$$

where the functions $V_f\left(\hat{\mu}_t\right)$, $V_K\left(\hat{\mu}_t\right)$, and $V_{g,z}\left(\hat{\mu}_t\right)$ are solutions to a set of differential equations that we derive in the Internet Appendix. The first two terms in equation (11) represent the present value of the cash flows from the firm's active projects and the value derived from the expected partial recovery of capital tied to those projects, respectively. Note that these two terms

[11] The Internet Appendix is available in the online version of the article on the Journal of Finance website.

are linear in the firm's profitability $f_t^i$ and capital stock $K_t^i$. The third term, which is a function of the firm's idiosyncratic growth state *z*, reflects the NPV of future projects.[12] When investors expect a relatively high degree of disruption (i.e., $\hat{\mu}_t$ is high), firms in the early and mature growth states enjoy higher valuations of their growth opportunities (i.e., $V_{g,EG}\left(\hat{\mu}_t\right)$ and $V_{g,MG}\left(\hat{\mu}_t\right)$ are relatively high). In these same high $\hat{\mu}_t$ states, active projects are expected to be terminated sooner, which implies that the present values of firm cash flows $V_f\left(\hat{\mu}_t\right)$ are low and the values derived from the partial recovery of capital $V_K\left(\hat{\mu}_t\right)$ are high.

The firm's excess return (i.e., its realized rate of return in excess of the discount rate *r*) consists of three components:

$$\begin{aligned} dr_t^i - rdt &= \left[\frac{f_t^i V_f'\left(\hat{\mu}_t\right) + K_t^i V_K'\left(\hat{\mu}_t\right) + V_{g,z}'\left(\hat{\mu}_t\right)}{V_t^i}\right]\left(\sigma_\mu \eta ds_t + \frac{\gamma}{\sigma_M} d\bar{\varpi}_t\right) \\ &+ \left[\frac{\left(\alpha\lambda K_t^i - k_z\right) + \left(k_z - \lambda K_t^i\right)V_K\left(\hat{\mu}_t\right) + \left(a_z k_z - \lambda f_t^i\right)V_f\left(\hat{\mu}_t\right)}{V_t^i}\right]\sigma_M d\bar{\varpi}_t + d\varepsilon_t^i. \end{aligned} \tag{12}$$

The first two terms in equation (12) characterize the exposure of the firm's return to the two systematic risk factors in the model, the disruption surprise $d\bar{\varpi}_t$ and the signal $ds_t$. The last term, $d\varepsilon_t^i$, is the firm's idiosyncratic return, which is driven by growth state transitions.

[12] The functions $V_{g,z}$ account for future transitions of the growth state and future projects' initial capital investments, profits, and eventual capital recoveries. Also, note that growth opportunities are worth zero in the no-growth state, that is, $V_{g,NG} \equiv 0$.

A comparison of the first two terms in equation (12) provide some intuition for the factor structure of asset returns in the model. The first term in equation (12) reflects the Bayesian updating of the expected disruption climate $\hat{\mu}_t$, which is relevant for predicting subsequent disruption rates and thus valuing future cash flows. The second term in equation (12) captures the *immediate impact* of the disruption process on the arrival and termination of projects. Note that both the disruption surprise $d\bar{\varpi}_t$ and the signal $ds_t$ contribute to the updating of $\hat{\mu}_t$, but only the disruption surprise $d\bar{\varpi}_t$ has an immediate effect on the firm's projects. This asymmetry is what generates a two-factor structure in asset returns. Having experienced different histories and being in different growth states, firms differ in their relative sensitivities to the expected disruption climate versus the realized disruption rate. Thus, firms' relative exposures to the two systematic risk factors differ in the cross section.

*D. Return Dynamics with Biased Investor Beliefs*

Thus far we have characterized firm values and returns conditional on investors' beliefs, which may or may not be biased. In the rest of this section we consider the case in which investors have biased beliefs, but characterize expected returns from the perspective of a fully rational observer. In particular, we examine the link between the two systematic risk factors in the model – the signal $ds_t$ and the disruption surprise $d\bar{\varpi}_t$ – and return predictability.

First, consider the signal $ds_t$. From equation (12) we see that the effect of the signal $ds_t$ on returns is proportional to $\eta$. This implies that if investors have biased beliefs about the signal's precision, that is, $\eta_B > \eta$, the sensitivity of returns to the signal is amplified. This amplification effect, however, does not influence the conditional expected rates of return that prevail at the time the signal is realized. This is because the signal $ds_t$ has an expected value of zero regardless of the perceived or actual precision. Even with biased perceptions of the precision of the signal, investors are observing something that is a random walk so their expectation of what the signal will be in the next instance is unbiased.

In contrast to the signal, the disruption surprise $d\bar{\varpi}_t$ can generate conditional return predictability when investors have biased beliefs. To illustrate this predictability formally, let $\hat{\mu}_t^B$ denote investors' estimate of $\mu_t$ in the case in which they have a biased perception of the signal precision $\eta_B > \eta$. Let $\hat{\mu}_t^R$ denote the unbiased estimate that a fully rational observer (i.e., someone who knows the true signal precision) would have given the same history. Similarly, let $d\bar{\varpi}_t^B$ and $d\bar{\varpi}_t^R$ denote the disruption surprise from the perspectives of biased investors and the rational observer, respectively. Substituting the new notation into equation (7) yields

$$d\bar{\varpi}_t^B \equiv \frac{dM_t - \hat{\mu}_t^B dt}{\sigma_M} = \frac{\hat{\mu}_t^R - \hat{\mu}_t^B}{\sigma_M} dt + \frac{dM_t - \hat{\mu}_t^R dt}{\sigma_M} = \frac{\hat{\mu}_t^R - \hat{\mu}_t^B}{\sigma_M} dt + d\bar{\varpi}_t^R. \tag{13}$$

Investors with biased signal precision perceive $d\bar{\varpi}_t^B$ to be a standard Brownian motion, since under their beliefs $\hat{\mu}_t^B$ is an unbiased estimate of $\mu_t$. However, because the rational observer's estimate $\hat{\mu}_t^R$ will typically differ from biased investors' estimate $\hat{\mu}_t^B$, *the conditional Sharpe ratio* of $d\bar{\varpi}_t^B$ in equation (13), that is, the term $\left(\hat{\mu}_t^R - \hat{\mu}_t^B\right) / \sigma_M$, is (almost always) nonzero. For example, after a string of positive realizations of the signal, the biased estimate $\hat{\mu}_t^B$ is likely to exceed the rational estimate $\hat{\mu}_t^R$, resulting in a negative conditional Sharpe ratio. In such cases, biased investors will be disappointed on average by subsequent realizations of the disruption rate. The opposite predictability pattern will obtain after a string of negative signal realizations.

Since the disruption climate is a persistent state variable, the biases in investors' estimates do not get corrected immediately. As a result, the conditional Sharpe ratio exhibits persistence as well. Formally, the law of motion of the conditional Sharpe ratio is given by

$$d\left(\frac{\hat{\mu}_t^R-\hat{\mu}_t^B}{\sigma_M}\right)\equiv-\left(\rho_\mu+\frac{\gamma_B}{\sigma_M^2}\right)\left(\frac{\hat{\mu}_t^R-\hat{\mu}_t^B}{\sigma_M}\right)dt+\frac{\sigma_\mu}{\sigma_M}\left(\eta-\eta_B\right)ds_t+\left(\frac{\gamma-\gamma_B}{\sigma_M^2}\right)d\bar{\omega}_t^R. \tag{14}$$

As equation (14) shows, the conditional Sharpe ratio evolves stochastically in response to the soft information signal $ds_t$ and the disruption surprise $d\bar{\omega}_t^R$, but tends to revert to its long-term mean of zero over time. Specifically, since overconfident investors overreact to soft information relative to the fully rational case (i.e., $\eta-\eta_B$ is negative), positive realizations of the soft information signal reduce the conditional Sharpe ratio. In contrast, since overconfident investors underreact to hard information relative to the fully rational case (i.e., $\gamma-\gamma_B$ is positive), positive realizations of the disruption surprise increase the conditional Sharpe ratio. The rate of mean reversion of the conditional Sharpe ratio reflects both changes in the disruption climate over time ($\rho_\mu$) and biased investors' learning from disruption surprises ( $\gamma_B/\sigma_M^2$ ).

The firm's expected excess return from the perspective of the rational observer can be computed by substituting $d\bar{\omega}_t^B$ from equation (13) into equation (12) and taking expectations. We skip the formula for brevity. Intuitively, cross-sectional differences in factor exposures, as characterized by equation (12), generate cross-sectional return predictability. For instance, early growth firms, which accumulate new projects at a high rate relative to their capital base, are highly sensitive to disruption surprises and exhibit stronger conditional return predictability relative to other firms. We analyze these predictability patterns numerically in Section III.

## II. Model Calibration and Simulation

*A. Calibration*

The model has 13 parameters. Most of these parameters are difficult to calibrate based directly on data observations, and in any case the model structure is too simplistic to provide a fully realistic description of the true data-generating process. We thus pick parameters that broadly replicate the salient features of the data and highlight the mechanisms that our model is intended to capture. The empirical sample we use as reference for our calibrations includes firms traded in U.S. stock exchanges over the 50-year period from July 1964 to June 2014.[13] The calibrated parameters for our simulations are described in Table I.

The discount rate *r* is set to 0.050. Because we think of the disruption climate as a slow-moving variable, we pick a relatively small mean-reversion rate for $\mu$, $\rho_\mu = 0.070$, which implies a half-life of shocks to $\mu$ that is approximately 10 years. We set the volatility of $\mu$ to $\sigma_\mu = 0.100$. The parameter choices for $\rho_\mu$ and $\sigma_\mu$ imply a standard deviation of 0.267 for $\mu$ in a long time series. We pick the volatility of the transitory component of the disruption rate to be $\sigma_M = 0.250$. Thus, the long-term variation in the persistent component of the disruption rate and the short-term variation in its transitory component are similar in magnitude.

We calibrate a moderately informative signal by choosing its precision to be $\eta = 0.500$. Overconfident investors perceive the signal precision to be $\eta_B = 0.934$, which we calibrate based

[13] See the Internet Appendix for details on the empirical sample construction and a discussion of the historical stylized facts.

on survey evidence that we discuss in the next paragraph. Given these parameter choices and using equation (14), the conditional Sharpe ratio of the disruption surprise has a rate of mean reversion of $-\left(\rho_{\mu}+\gamma_{B} / \sigma_{M}^{2}\right)=-0.1589$, which implies a half-life of about 4.36 years. The time-invariant distribution of the conditional Sharpe ratio is normal with a mean of zero and a standard deviation of 0.462. Using the corresponding percentile values, the conditional Sharpe ratio is within the interval [−0.6, 0.6] about 80% of the time, and within [−1, 1] about 97% of the time.[14]

One can compare the degree of overconfidence assumed in our calibrated model to results from surveys that ask participants to make predictions and report their perceived confidence intervals. In particular, Ben-David, Graham, and Harvey (2013) ask financial executives to predict one-year S&P 500 returns and provide an 80% confidence interval. The authors find that the executives' reported confidence intervals include the realized outcome only 36.3% of the time, suggesting that they tend to be quite overconfident about the precision of their estimates.[15] We calibrate the biased signal precision in our base-case simulations to match this level of overconfidence. Specifically, given the true signal precision $\eta=0.500$, investors with biased precision $\eta_{B}=0.934$ compute 80% confidence intervals that include the realized outcome (i.e., shocks to $\mu_{t}$) 36.3% of the time on average.[16]

The remaining parameters of the model describe firms and their investment opportunities.

[14] Note that, due to its persistence, the conditional Sharpe ratio varies less in finite samples compared to its time-invariant distribution.
[15] The standard error associated with this point estimate is 7.8%.
[16] In earlier studies, Alpert and Raiffa (1969) ask Harvard Business School students to answer general knowledge questions, and Russo and Schoemaker (1992) ask money managers to answer questions about their industry. These studies respectively find the participants' 98% and 90% confidence intervals to include the correct answer 54% and 50% of the time. Our base-case overconfidence calibration implies 98% and 90% confidence intervals to include the realized outcome 60.8% and 45.5% of the time, respectively.

We set the life expectancy of firms to 10 years, with an average of $1/q_{EG}=3$ years spent in the early growth state, $1/q_{MG}=4$ years spent in the mature growth state, and $1/q_{NG}=3$ years spent in the no-growth state.[17] The average project termination rate is $\lambda=0.150$, which implies that the average half-life of firms' active projects is 4.621 years ($=-\ln(0.5)/\lambda$).

Firms receive their projects in the early and mature growth states. The initial investment required for a project is the same in both states, $k_{EG} = k_{MG} = k$. A higher value of $k$ generates more cross-sectional dispersion in firms' growth rates by allowing young firms to grow faster. Accordingly, we calibrate $k$ to match the dispersion in asset growth rates in our empirical sample. Specifically, setting $k = 0.950$ approximately replicates the difference between the average asset growth rates of the median firms in the top and bottom asset growth portfolios, which is 0.519 in our empirical sample.

We assume that the projects that firms receive in the mature growth state are half as profitable as those that they receive in the early growth state: $a_{MG} = a_{EG} / 2$. We calibrate $a_{EG} = 0.250$ (and thus $a_{MG} = 0.125$) to approximately match the median operating profitability of firms in our empirical sample, which is 0.145. Finally, we calibrate the capital recovery rate of terminated projects, $\alpha=0.650$, to approximately match the median Tobin's $q$ of firms in our empirical sample, which is 1.376.

*B. Simulation Procedure and Formation of Characteristic-Sorted Portfolios*

Using the parameters described in the previous subsection, we simulate sample paths for a set of hypothetical firms. Specifically, we begin by solving for the firm value in equation (11) numerically, as described in the Internet Appendix. We then start each simulation with 10,000

[17] The average number of years an individual firm appears in our empirical sample is 10.56.

firms in the early growth state. As the economy evolves, these firms are endowed with new projects, some of their existing projects are terminated, they can transition to new states, and they may die. As described in Section I.A, firms that die are replaced with new firms born into the early growth state.

The initial values of the disruption climate, $\mu_0$, and investors' estimate of it, $\hat{\mu}_0$, are drawn randomly from their time-invariant distributions. We simulate 200 years of data by approximating the continuous passage of time with 48 discrete time periods per year (i.e., four periods per month). We drop the first 150 years so as to allow firm characteristics to reach their steady-state distributions, and use the remaining 50 years of data (the length of our empirical sample) for analysis. We repeat this procedure to generate 10,000 simulated samples.

In forming portfolios, we focus on three firm characteristics that have received substantial attention in the empirical literature. First, to capture the value anomaly, we examine *market-to-book* portfolios, which are formed by sorting firms with respect to their ratio of market value to capital stock. Second, we examine *asset growth* portfolios, which are formed by sorting firms with respect to the growth rate of their capital stock over the prior year. Finally, we examine *profitability* portfolios, which are formed by sorting firms with respect to their ratio of profitability to capital stock.[18] The cutoffs for portfolio assignments are chosen based on the quintile breakpoints of the underlying characteristics. For instance, the low market-to-book (i.e., value) portfolio consists of firms in the bottom quintile of the cross-sectional distribution of the

[18] See Fama and French (1992) for the value anomaly, Cooper, Gulen, and Schill (2008) for the asset growth anomaly, and Fama and French (2015) and Novy-Marx (2013) for the profitability anomaly. Our measurement of firm characteristics differs from these and other studies in two respects. First, our value and profitability measures are scaled by book assets instead of book equity, since capital structure is irrelevant and firms are all-equity financed in our model. Second, unlike in the empirical literature where profitability can be measured in multiple ways (e.g., net versus gross), there is only one profitability measure in our model, which can be interpreted as net of all expenses. In the Internet Appendix, we show that applying our characteristic measures to empirical data yields results that are qualitatively similar to those in the above-cited studies.

market-to-book ratio, while the high market-to-book (i.e., growth) portfolio consists of firms in the top quintile.

*C. Summary Statistics of Simulated Samples*

Table II presents summary statistics for the simulated data samples. As shown in Panel A, the value-weighted portfolio of all simulated firms earns an average annual return of 5.00%, matching the discount rate *r*. Since our model does not include an aggregate market factor, our simulated returns are not as volatile as actual stock returns. Nevertheless, the average volatility of 2.61% is not negligible relative to the model's discount rate. The other statistics reported in Panel A show that the median firm's capital stock, profitability, and Tobin's *q* do not exhibit substantial variation across or within samples.

Panel B reports the time-series averages of the median values of firm characteristics in each of the three growth states. The mature growth firms are the largest and the early growth firms are the smallest on average. Profitability is highest for firms in the early growth state. Since firms do not receive any projects following their transition from mature growth to no growth, and since such transition occurs with the same likelihood for any firm in the mature growth state, by construction the mature growth and the no-growth states exhibit the same average profitability. Asset growth rates decline as firms transition from early growth to mature growth, and become negative in the no-growth state. Similarly, Tobin's *q* values are highest in the early growth state, followed by the mature growth state and then the no-growth state.

Panel C reports statistics on characteristic-sorted portfolios. The first three columns report the percentages of firms in a given characteristic-sorted portfolio that belong to each of the three growth states. The next four columns are the median values of the firm characteristics for each portfolio. The high market-to-book (i.e., growth) portfolio consists solely of firms in the early growth state, whereas the low market-to-book (i.e., value) portfolio consists mainly of firms in the no-growth state. Growth firms also tend to be smaller, more profitable, and exhibit higher growth rates relative to value firms. High asset growth firms, which are primarily in the early growth state, are larger and have higher Tobin's *q* values. However, there is no difference between the profitability rates of high versus low asset growth firms. Profitability portfolios are somewhat more evenly distributed across the three growth states. Relative to low profitability firms, high profitability firms are larger and exhibit higher Tobin's *q* values and slightly lower growth rates. In comparison to our empirical sample, the simulated characteristic-sorted portfolios exhibit less cross-sectional dispersion in market-to-book and profitability ratios, and somewhat lower asset growth rates.[19]

Panel D reports the return volatilities and correlations of characteristic-sorted long-short portfolios.[20] The simulated returns of the characteristic-sorted portfolios are less volatile than their empirical counterparts (not reported in the table), and exhibit strong positive correlations with each other. These discrepancies between the simulated and empirical data are not surprising. For parsimony, our model focuses on a single disruption factor. In reality, there are likely to be several such factors that are imperfectly correlated with each other and that affect different portfolios differently. In addition, our model abstracts from other risk factors that are unrelated to disruptive innovations (e.g., short-term shocks to profitability). Such risk factors are likely to increase the return volatilities of the characteristic-sorted portfolios and dampen the correlations between their returns.[21]

[19] For a more complete comparison of portfolio characteristics in the simulated and empirical data samples, see Table IA.I in the Internet Appendix.
[20] The reported volatilities and correlations are average values of in-sample estimates based on monthly data.
[21] A second reason the model generates relatively low volatilities is that it abstracts from financial leverage effects. In unreported analyses we find that adjusting for financial leverage in historical data reduces volatilities of characteristic-sorted portfolio returns by up to 30%.

## III. Conditional Returns of Characteristic-Sorted Portfolios

*A. The Exposures of Characteristic-Sorted Portfolios to Disruption*

As discussed in Section I.C, in our model characteristic-based return predictability arises from firms' exposure to the disruption surprise factor $d\bar{\omega}_t^B$. We start our analysis by examining these factor exposures. In Table III, we report the betas of various characteristic-sorted portfolios with respect to the disruption surprise factor. Since the portfolios' factor exposures can vary across different states of the world, we report conditional betas as well as the unconditional betas that capture average exposures.[22]

As the signs of the reported unconditional betas indicate, high disruption rates are good news for high market-to-book, high asset growth, and low profitability firms, and bad news for low market-to-book, low asset growth, and high profitability firms. The table also reports the unconditional betas for portfolios that are formed based on the interaction of two characteristics. As these betas show, a single characteristic is often not sufficient to characterize firms' factor exposures.

For instance, while high market-to-book firms are positively exposed to disruption on average, firms can have high market-to-book ratios for different reasons, and depending on the reason, their exposure to the disruption factor will vary. Some are unprofitable firms whose stock prices largely reflect future expected growth. These firms' exposure to the disruption factor is strongly positive. Other firms with high market-to-book ratios are relatively more mature and derive high market values from highly profitable existing projects as well as access to future

[22] Factor betas are estimated via monthly return regressions. Note that the disruption surprise factor has a volatility of one by construction (see equation (13)), which is much larger than portfolio return volatilities. Thus, the factor betas are small in absolute value. For ease of interpretation, we report the estimated betas multiplied by 100 (or equivalently, betas with respect to the factor renormalized to have a volatility of 1%).

opportunities. These firms' exposure to disruption can be close to zero, since the increased flow of new projects is offset by the demise of some of the existing projects. Thus, the extent to which high market-to-book firms are exposed to disruption depends on their profitability. Similarly, among highly profitable firms, those with low asset growth rates are strongly negatively exposed to disruption, whereas those with high asset growth rates exhibit near-zero exposure.

The next four columns in Table III report conditional betas that capture firms' exposure to disruption in different states of the model economy. Specifically, we consider states in which the conditional Sharpe ratio of the disruption surprise factor $\left(\hat{\mu}_t^R - \hat{\mu}_t^B\right)/\sigma_M$ is low or high, and similarly, states in which investors' expected disruption rate $\hat{\mu}_t^B$ is low or high.[23] Firms that benefit from disruption, that is, those with high asset growth rates, low profitability, and high market-to-book ratios, exhibit higher disruption betas when investors' expected disruption rates are low rather than high. The intuition is that these firms are proportionally more affected by disruption shocks when their valuations are relatively low, which is the case when investors expect low disruption rates. A similar rationale explains why the disruption betas of these firms are higher when the conditional Sharpe ratio is low. The realized disruption rates disappoint investors in such states of the world on average, lowering the valuations of growth-oriented firms and thus increasing their exposure to subsequent disruption shocks.[24]

In summary, the disruption betas are particularly high for young unprofitable firms with high growth rates, and particularly low for mature profitable firms with low growth rates. Firms'

[23] In Tables III and IV, low and high values of the conditioning variable are defined as those that are less than one standard deviation below and more than one standard deviation above its mean, respectively.

[24] The conditional beta patterns discussed in the text imply that growth firms are better hedges against future disruption shocks in relatively more tranquil times when incumbents are expected to be in a strong position. While there are no hedging demands in our risk-neutral model, similar insights can be explored within the context of asset pricing models with priced disruption risk. See Gârleanu, Kogan, and Panageas (2012) and Kogan, Papanikolaou, and Stoffman (2015).

exposures to disruption change over time as the model economy transitions from states of low to high expected disruption rates or low to high conditional mispricing.

*B. Conditional Predictability of Characteristic-Sorted Portfolio Returns*

In this section we analyze the predictability of the characteristic-sorted portfolio returns that are generated by our model and compare these implications to existing empirical evidence. For brevity, we focus hereafter on the long-short portfolios that are formed based on firms' market-to-book ratios, asset growth rates, and profitability. Since the unconditional return premium associated with disruption is zero, our analysis focuses on conditional predictability. Specifically, we examine the predictability of portfolio returns by computing their Sharpe ratios in different subsamples of simulated months that reflect different states of the model economy. The results are reported in Table IV.[25]

As the first two rows of Table IV show, the predictability of characteristic-sorted portfolio returns strongly depends on the conditional Sharpe ratio of the disruption surprise factor. All three long-short portfolios exhibit highly positive (negative) return performance as measured by their Sharpe ratios when the conditional Sharpe ratio associated with disruption is low (high). Recall that low conditional Sharpe ratio states are those in which biased investors' estimate of the disruption climate is too high relative to the rational estimate. Accordingly, market-to-book, asset growth, and profitability strategies that bet in favor of incumbents and against newcomers in such states of the world exhibit positive abnormal performance.

The conditional Sharpe ratios discussed above are not directly observable to the econometrician. Thus, to test the model's implications, we need a set of observable or at least

[25] Sharpe ratios are computed as the ratio of the subsample mean to the subsample standard deviation of the monthly returns of long-short portfolios and are reported in annualized terms.

indirectly observable conditioning variables. Consider first investors' beliefs about the expected disruption rate, which might be indirectly inferred from surveys, textual analysis of media or analyst reports, and observed investor or management choices. The third and fourth rows in Table IV show that the characteristic-sorted portfolios exhibit positive performance when investors' disruption expectations are high and negative performance when those expectations are low. These patterns reflect investor overconfidence. While overconfidence does not generate an unconditional bias in investor expectations, it does cause the distribution of those expectations to be more dispersed. As a result, expectations in the right (left) tail are too high (low) relative to the fully rational case. In other words, when investors are very optimistic about the disruption rate they tend to be too optimistic, and when they are very pessimistic they tend to be too pessimistic. Hence, our model predicts that measures of investor beliefs about growth opportunities, such as the "sentiment" proxies used in the empirical literature, predict subsequent returns.[26]

Next, we examine the predictability of returns based on the cross-sectional distributions of firm fundamentals. The disruption process in our model generates persistent differences in the cross-sectional dispersions of firms' valuation ratios, asset growth rates, and profitability. To evaluate the information content of cross-sectional dispersion, we analyze returns in subsamples in which market-to-book spread, asset growth spread, and profitability spread are low or high.[27] The results are reported in the last six rows of Table IV.

Consider first the dispersion in asset growth and profitability rates. Being non-price measures of firm fundamentals, asset growth and profitability rates in the model reflect the *hard*

[26] See, for instance, Baker and Wurgler (2006), who find that investor sentiment proxies such as the number of IPOs or the equity share in new issues predict the subsequent performance of characteristic-sorted portfolios.
[27] The market-to-book spread is defined as the difference between the market-to-book ratios of the median firms in the high market-to-book (i.e., growth) and low market-to-book (i.e., value) portfolios. The asset growth and the profitability spreads are defined similarly based on the high/low asset growth and profitability portfolios.

*information* revealed by realized disruption rates. Specifically, asset growth rates are more dispersed when realized disruption rates are high, and profitability is more dispersed when realized disruption rates are low. Since overconfident investors underreact to these hard information signals, the dispersions of asset growth and profitability rates proxy for investor underreaction in our model. Confirming this intuition, Table IV shows that the three characteristic-sorted portfolios in our model perform well following periods of low asset growth spreads and high profitability spreads, whereas they perform poorly in the opposite circumstances.

Unlike asset growth and profitability, market-to-book ratios reflect market prices and thus are affected by both hard and soft information. This makes the relation between the cross-sectional dispersion of market-to-book ratios and the future returns of characteristic-sorted portfolios more subtle. First, high realized disruption rates increase the assets of growth firms, increasing their book values, and thereby narrowing the spread between the market-to-book ratios of growth and value firms. Since investors underreact to this hard information, narrowing the dispersion of market-to-book ratios through this channel is associated with low future returns to characteristic-sorted portfolios that bet on incumbent firms. Second, negative realizations of the soft information signal decrease growth firms' valuations and thus decrease the cross-sectional dispersion of market-to-book ratios. Since investors overreact to such signals, narrowing the dispersion of market-to-book ratios through this channel is also associated with low future returns of characteristic-sorted portfolios that bet on the incumbents. Thus, both channels contribute to a negative relationship between the market-to-book spread and the future returns of the three characteristic-sorted portfolios analyzed in Table IV. The results in the table confirm this negative relationship and show that the effect is asymmetric: the abnormal performance of the characteristic-sorted portfolios is especially large following periods of high market-to-book spreads.

To summarize, the cross-sectional dispersion of firm characteristics reflects past realizations of both hard and soft information and thus can predict future returns. In particular, dispersion in non-price characteristics such as asset growth and profitability rates relate to investors' underreaction to hard information, whereas dispersion in price-related characteristics such as market-to-book reflect both underreaction to hard information and overreaction to soft information. As far as we know, there has been little empirical inquiry into these predictions. One exception is Cohen, Polk, and Voulteenaho (2003), who show that the value spread, which measures the cross-sectional dispersion of book-to-market ratios, positively predicts the subsequent returns of value-minus-growth strategies. Our model's predictions above are consistent with their findings.[28]

*C. Autocorrelations of Characteristic-Sorted Portfolio Returns*

As we discuss in Section I.C, expected returns are persistent in our model. Specifically, the conditional Sharpe ratio of the disruption surprise factor is a persistent process, as equation (14) shows. In this subsection, we examine the implications of persistence for the autocorrelations of characteristic-sorted portfolio returns.

Table V reports return autocorrelations for the three characteristic-sorted long-short portfolios. Autocorrelations are calculated for returns measured at one- and five-year intervals.[29] In addition to population autocorrelations that are estimated by combining all simulated 50-year

[28] It should be noted that expected returns in our model vary because investors have biased estimates of future cash flows. Given this, one might also be able to test the implications of our model by directly exploring the link between characteristics and observed biases in earnings forecasts, as in La Porta (1996). Alternatively, one can indirectly infer biases by looking at the link between characteristics and abnormal stock returns on earnings announcement dates, as in Chopra, Lakonishok, and Ritter (1992) and La Porta et al. (1997).

[29] In unreported results, we find that monthly return autocorrelations are close to zero. This is not surprising, as short-term return variation in our model is determined primarily by Brownian diffusion terms.

data samples, the table also reports statistics on in-sample autocorrelations that are estimated within each 50-year simulated sample.

Population autocorrelations are positive at one-year intervals and become even larger at five-year intervals. Thus, characteristic-sorted portfolio returns are highly persistent. However, strong persistence is revealed only by the population estimates. The in-sample autocorrelation estimates are small on average and become weaker at longer return intervals. Furthermore, the in-sample estimates are negative in 20% or more of the simulated samples. Thus, the strong autocorrelation patterns in the population may not be easy to detect even with 50 years of data.

The persistence of characteristic-sorted portfolio returns implies that momentum strategies that buy recent winner portfolios and sell recent losers tend to be profitable. In unreported analysis, we find that this is indeed the case in our simulated data samples. The predictions of the model in this regard are consistent with the findings in Lewellen (2002), who documents that book-to-market (as well as size) portfolios exhibit momentum, and the analysis of Chen and Hong (2002), who highlight the role of positive autocorrelation for Lewellen's momentum findings.

## IV. Average Returns of Characteristic-Sorted Portfolios

Our focus up to this point has been on the conditional predictability of characteristic-sorted portfolio returns. In this section we turn our attention to the extent to which our model can explain the historical evidence linking characteristics and average returns, for example, the fact that value has significantly beaten growth. Recall that our model is designed so that characteristics do not predict returns over sufficiently long sample periods. The question we ask in this section is whether we are likely to observe a significant relationship between characteristics and average returns in samples that approximately match the length of historical data samples.

We start our analysis by computing annualized Sharpe ratios for the three characteristic-sorted long-short portfolios using 600 months (50 years) of return data in each simulated sample. We then characterize the probability distributions of these Sharpe ratios using the 10,000 simulated samples. The results are reported in Table VI. Panel A of the table provides the benchmark for these Sharpe ratio distributions under the null of no return predictability. Specifically, the panel reports the *t*-statistic values for conventional statistical significance levels, and the annualized Sharpe ratios that these *t*-statistic values correspond to in a 50-year sample period.[30]

Panel B of Table VI reports the Sharpe ratio distributions for the three characteristic-sorted long-short portfolios that are generated in simulations with rational investors. Relative to the benchmark distribution in Panel A, the simulated distributions of the characteristic portfolios have slightly negative means and are also left-skewed. These deviations from the benchmark, which are quite small in magnitude, are likely to result in part from the fact that the returns of characteristic-sorted portfolios are not distributed i.i.d. normal.[31] The basic takeaway from these simulations is that extreme Sharpe ratio realizations (e.g., magnitudes that are comparable to historically observed realizations) are extremely unlikely in our model when investors are rational.

[30] A long-short portfolio's Sharpe ratio is the *t*-statistic on its mean return divided by the square root of $T$, which is the number of return observations. Our sample period is 50 years, or 600 months. The relevant $t$ distribution therefore has 599 degrees of freedom. The reported Sharpe ratios are annualized by multiplying by the square root of 12.

[31] Indeed, in unreported analyses we find that the realized mean returns of the characteristic-sorted portfolios are positively correlated with the realized time-series return standard deviations of those portfolios. Thus, in simulated samples where the portfolios earn positive returns on average, those returns are more volatile, dampening the realized Sharpe ratio. Such deviations from constant return volatility are not surprising in a model like ours, where the systematic factors affect firm valuations in nonlinear ways.

The main results of our analysis are shown in Panel C of Table VI and Figure 1. Panel C reports the distributions of characteristic-sorted portfolios' Sharpe ratios in simulations with overconfident investors. Figure 1 plots the Sharpe ratio distributions for the low-minus-high market-to-book portfolio with rational and overconfident investors.[32] As both the table and the figure show, introducing the overconfidence bias results in substantially more dispersed Sharpe ratio distributions relative to the rational case. The economic magnitudes of the tail Sharpe ratios are quite large. For instance, the 90$^{th}$ percentile of the Sharpe ratio of the low-minus-high market-to-book portfolio is more than doubled, from 0.155 with rational investors to 0.367 with overconfident investors.

To put things into a more concrete perspective, consider a Sharpe ratio of 0.40, which is within the range of the historical Sharpe ratios documented in empirical studies (a Sharpe ratio of 0.40 over a 50-year period corresponds to a *t*-statistic of $0.40 \times \sqrt{50} = 2.83$). Based on the distributions reported in Table VI, what is the likelihood of observing a 50-year sample in which characteristic-sorted portfolios achieve a Sharpe ratio of 0.40 or above in absolute value (i.e., in either tail of the distribution)? When investors are rational, this likelihood is extremely low. For instance, market-to-book sorts generate a Sharpe ratio of 0.40 or above in only 0.63% of the sample paths. With overconfident investors, however, market-to-book sorts generate a Sharpe ratio of 0.40 or above in 18.23% of the sample paths, which is a 29-fold increase relative to the case with rational investors. Similarly, at least one portfolio sorted based on the three characteristics we consider generates a Sharpe ratio of 0.40 or above in 20.64% of the sample paths. Thus, although the likelihood of generating the magnitude of historically observed

[32] For visual ease, Figure 1 plots smoothed probability distribution function estimates that are obtained by applying a normal kernel function to simulated Sharpe ratios. Figures for the asset growth and the profitability portfolios are omitted for brevity.

characteristic-based anomalies is negligible when investors have unbiased beliefs, these magnitudes arise quite frequently when investors are overconfident.

The increased dispersion of the Sharpe ratio distributions of characteristic-sorted portfolios is a consequence of return persistence. As discussed earlier, the conditional expected returns of characteristic-sorted portfolios are persistent in our model because overconfident investors' beliefs about the disruption climate adjust too slowly. An implication of this persistence is the increased variance of long-term return realizations relative to short-term return volatility. As a result, Sharpe ratios, which scale averages of long-term return realizations by short-term volatility, exhibit greater dispersion.

Historical returns are of interest to academics and practitioners in part because they may reveal patterns that will repeat in the future. For instance, one might try to predict the future performance of a value strategy based on its past performance. We now briefly turn to the power of such predictions within the context of our model simulations. Specifically, we ask: conditional on a characteristic-sorted portfolio achieving a Sharpe ratio of 0.40 or above in a 50-year sample, what is its likely Sharpe ratio over the next 10 years?[33]

The results, which we report without tabulating, indicate that in our model historical returns do not significantly predict future returns. For instance, conditional on achieving a historical Sharpe ratio that exceeds 0.40, the low-minus-high market-to-book portfolio generates an expected future Sharpe ratio of only 0.14 (which translates into a *t*-statistic of 0.43 over the 10-year sample period). Furthermore, the realized Sharpe ratio over the 10-year period is negative with a probability of 0.41. Similarly, the low-minus-high asset growth portfolio's future Sharpe ratio is 0.11 in expectation (*t*-statistic of 0.34) and negative with probability 0.41, and the

[33] Our choice of a 10-year evaluation period reflects the typical length of track records that investors take into account in practice. We obtain similar results when we consider shorter evaluation periods.

high-minus-low profitability portfolio's future Sharpe ratio is 0.07 in expectation (*t*-statistic of 0.21) and negative with probability 0.46. In short, strong historical return patterns do not repeat in the future in our model simulations.

The results that we describe in this section obtain in a stylized model with zero unconditional return premia. In reality, it is likely that characteristic-sorted portfolios have unconditional return premia that partly explain their historical returns. Our analysis is not intended to argue that such premia do not exist. Rather, we make a general point – extreme return realizations are more likely when overconfident investors adjust their beliefs slowly – that holds regardless of the magnitudes of unconditional premia. We believe that this insight may help us understand why many of the anomalies examined in the empirical literature exhibit such extreme average returns.

## V. Conclusion

Over the past 50 years, portfolios formed on firm characteristics such as value, profitability, and asset growth have generated very high Sharpe ratios. Motivated by these observations, financial economists have developed a number of rational and behavioral asset pricing models. We contribute to this literature by offering a dynamic behavioral model that explicitly links characteristic-sorted portfolio returns to systematic risk factors that determine the evolution of firm fundamentals. Our model provides testable implications about the conditional predictability of characteristic-sorted portfolio returns, and illustrates how persistence of these returns increases the likelihood of observing abnormally high Sharpe ratios.

For the sake of parsimony we make a number of assumptions. In particular, we assume risk neutrality and we design a model in which one economic concept generates multiple anomalies. Given our simplifications, it is not surprising that our model does not capture all salient features of the data; for example, the correlations of characteristic-sorted portfolio returns are too high in our model. Moreover, the most extreme Sharpe ratios documented in the empirical literature cannot be generated as a likely sample path in our model. These discrepancies can potentially be addressed by incorporating insights from our model into existing models that generate a richer structure of expected returns.

Our framework can also be extended to consider additional behavioral biases. In unreported work, we explore two potential biases. Under the first, suggested by Shiller (2000), investors may be overly optimistic about the commercial potential of new technologies. This bias can be incorporated in our model by assuming that investors' initial expectations are too optimistic, that is, investors initially believe that the disruption climate is stronger than it actually is. We find that this form of optimism has a substantial effect on characteristic-sorted portfolio returns only when investors are also overconfident and thus shed their optimism slowly. Under the second bias, suggested by Barberis, Shleifer and Vishny (1998), the Tversky and Kahneman (1974) representativeness heuristic can lead investors to overreact to hard information, such as earnings. In a similar spirit, our model can incorporate the possibility that investors overreact to realized disruption rates. Our preliminary investigation of such a model reveals that overreaction and overconfidence lead to very distinct predictions. For instance, the overconfidence model implies that the cross-sectional dispersion of market-to-book ratios positively predicts the subsequent returns of value firms – in contrast, the overreaction model predicts the opposite. Future work that explores the different implications of alternative biases is clearly warranted.

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## REFERENCES


Alpert, Marc, and Howard Raiffa, 1969, A progress report on the training of probability assessors, Unpublished manuscript, Harvard University.

Altı, Aydoğan, and Paul C. Tetlock, 2014, Biased beliefs, asset prices, and investment: A structural approach, *Journal of Finance* 69, 325-361.

Baker, Malcolm, and Jeffrey Wurgler, 2006, Investor sentiment and the cross-section of stock returns, *Journal of Finance* 61, 1645-1680.

Barberis, Nicholas, and Andrei Shleifer, 2003, Style investing, *Journal of Financial Economics* 68, 161-199.

Barberis, Nicholas, Andrei Shleifer, and Robert Vishny, 1998, A model of investor sentiment, *Journal of Financial Economics* 49, 307‒343.

Ben-David, Itzhak, John R. Graham, and Campbell R. Harvey, 2013, Managerial miscalibration, *Quarterly Journal of Economics* 128, 1547-1584.

Bena, Jan, Lorenzo Garlappi, and Patrick Grüning, 2016, Heterogeneous innovation, firm creation and destruction, and asset prices, *Review of Asset Pricing Studies* 6, 46-87.

Berk, Jonathan B., Richard C. Green, and Vasant Naik, 1999, Optimal investment, growth options, and security returns, *Journal of Finance* 54, 1553-1607.

Carlson, Murray, Adlai Fisher, and Ron Giammarino, 2004, Corporate investment and asset price dynamics: Implications for the cross-section of returns, *Journal of Finance* 59, 2577-2603.

Chen, Joseph, and Harrison Hong, 2002, Discussion of "Momentum and autocorrelation in stock returns," *Review of Financial Studies* 15, 565-573.

Chopra, Navin, Josef Lakonishok, and Jay R. Ritter, 1992, Measuring abnormal performance: Do stocks overreact? *Journal of Financial Economics* 31, 235-268.

Cohen, Randoph B., Christopher Polk, and Tuomo Voulteenaho, 2003, The value spread, *Journal of Finance* 58, 609-641.

Cooper, Michael J., Huseyin Gulen, and Michael J. Schill, 2008, Asset growth and the cross section of stock returns, *Journal of Finance* 63, 1609-1651.

Daniel, Kent, David Hirshleifer, and Avanidhar Subrahmanyam, 1998, Investor psychology and security market under- and overreactions, *Journal of Finance* 53, 1839‒1886.

Daniel, Kent, David Hirshleifer, and Avanidhar Subrahmanyam, 2001, Overconfidence, arbitrage, and equilibrium asset pricing, *Journal of Finance* 56, 921-965.

Fama, Eugene F., and Kenneth R. French, 1992, The cross-section of expected stock returns, *Journal of Finance* 47, 427-465.

Fama, Eugene F., and Kenneth R. French, 2015, A five-factor asset pricing model, *Journal of Financial Economics* 116, 1-22.

Gârleanu, Nicolae, Leonid Kogan, and Stavros Panageas, 2012, Displacement risk and asset returns, *Journal of Financial Economics* 105, 491-510.

Hong, Harrison, and Jeremy C. Stein, 1999, A unified theory of underreaction, momentum trading, and overreaction in asset markets, *Journal of Finance* 54, 2143-2184.

Kogan, Leonid, Dimitris Papanikolaou, and Noah Stoffman, 2015, Winners and losers: Creative destruction and the stock market, Working paper, Massachusetts Institute of Technology.

La Porta, Rafael, 1996, Expectations and the cross-section of stock returns, *Journal of Finance* 51, 1715-1742.

La Porta, Rafael, Josef Lakonishok, Andrei Shleifer, and Robert Vishny, 1997, Good news for value stocks: Further evidence on market efficiency, *Journal of Finance* 52, 859-874.

Lewellen, Jonathan, 2002, Momentum and autocorrelation in stock returns, *Review of Financial Studies* 15, 533-563.

Lewellen, Jonathan, and Jay Shanken, 2002, Learning, asset-pricing tests, and market efficiency, *Journal of Finance* 57, 1113-1145.

Loualiche, Erik, 2016, Asset pricing with entry and imperfect competition, Working paper, Massachusetts Institute of Technology.

Novy-Marx, Robert, 2013, The other side of value: The gross profitability premium, *Journal of Financial Economics* 108, 1-28.

Pastor, Lubos, and Robert F. Stambaugh, 2012, Are stocks really less volatile in the long run? *Journal of Finance* 67, 431-477.

Pastor, Lubos, and Pietro Veronesi, 2003, Stock valuation and learning about profitability, *Journal of Finance* 58, 1749-1789.

Pastor, Lubos, and Pietro Veronesi, 2006, Was there a Nasdaq bubble in the late 1990s? *Journal of Financial Economics* 81, 61-100.

Russo, J. Edward, and Paul J. H. Schoemaker, 1992, Managing overconfidence, *Sloan Management Review* 33, 7–17.

Scheinkman, Jose A., and Wei Xiong, 2003, Overconfidence and speculative bubbles, *Journal of Political Economy* 111, 1183−1219.

Shiller, Robert J., 2000, *Irrational Exuberance* (Princeton University Press, Princeton).

Timmermann, Allan, 1993, How learning in financial markets generates excess volatility and predictability in stock prices, *Quarterly Journal of Economics* 108, 1135-1145.

Timmermann, Allan, 1996, Excess volatility and predictability of stock prices in autoregressive dividend models with learning, *Review of Economic Studies* 63, 523−557.

Tversky, Amos, and Daniel Kahneman, 1974, Judgement under uncertainty: Heuristics and biases, *Science* 185, 1124-1131.

Zhang, Lu, 2005, The value premium, *Journal of Finance* 60, 67−103.

**Table I**
**Model Parameters**

The table reports the parameter values in the calibrated model. More detailed descriptions of the parameters and their calibration are provided in Section II.A.

| Parameter | Value |
|---|---|
| Discount rate $r$ | 0.050 |
| Mean reversion rate of disruption climate $\rho_\mu$ | 0.070 |
| Volatility of disruption climate $\sigma_\mu$ | 0.100 |
| Volatility of transitory disruption shocks $\sigma_M$ | 0.250 |
| True signal precision $\eta$ | 0.500 |
| Biased signal precision $\eta_B$ | 0.934 |
| Expected time in the early growth state $1 / q_{EG}$ | 3 years |
| Expected time in the mature growth state $1 / q_{MG}$ | 4 years |
| Expected time in the no-growth state $1 / q_{NG}$ | 3 years |
| Project capital investment $k$ | 0.950 |
| Project profitability in the early growth state $a_{EG}$ | 0.250 |
| Project profitability in the mature growth state $a_{MG}$ | 0.125 |
| Average project termination rate $\lambda$ | 0.150 |
| Capital recovery rate $\alpha$ | 0.650 |

**Table II**
**Summary Statistics of Simulated Data**

The table reports summary statistics for the simulated data samples. The statistics reported in the table are described in Section II.C.

Panel A: All firms

| | Value-weighted portfolio (annual %) | Median capital stock $K$ | Median profitability $f/K$ | Median Tobin's $q$ $V/K$ |
|---|---|---|---|---|
| Mean across all simulated samples | 5.00 | 3.150 | 0.144 | 1.371 |
| Standard deviation of sample means | 0.75 | 0.232 | 0.005 | 0.055 |
| Mean of sample standard deviations | 2.61 | 0.267 | 0.004 | 0.077 |

Panel B: Median firm characteristics

| | Capital stock $K$ | Profitability $f/K$ | Asset growth (%) | Tobin's $q$ $V/K$ |
|---|---|---|---|---|
| Early Growth | 2.418 | 0.171 | 22.35 | 2.242 |
| Mature Growth | 4.057 | 0.138 | 9.21 | 1.285 |
| No Growth | 2.670 | 0.138 | -13.91 | 1.192 |

Panel C: Characteristic-sorted portfolios

| | % Early Growth | % Mature Growth | % No Growth | Capital stock $K$ | Profitability $f/K$ | Asset growth (%) | Tobin's $q$ $V/K$ |
|---|---|---|---|---|---|---|---|
| Low Market-to-Book | 0.0 | 33.5 | 66.5 | 2.515 | 0.113 | -12.22 | 1.106 |
| High Market-to-Book | 100.0 | 0.0 | 0.0 | 1.911 | 0.138 | 37.54 | 2.312 |
| Low Asset Growth | 0.1 | 0.1 | 99.8 | 1.673 | 0.122 | -13.94 | 1.299 |
| High Asset Growth | 57.5 | 39.8 | 2.7 | 2.007 | 0.122 | 37.60 | 2.249 |
| Low Profitability | 38.5 | 34.8 | 26.7 | 1.579 | 0.092 | 12.15 | 1.296 |
| High Profitability | 58.1 | 24.0 | 17.9 | 3.884 | 0.212 | 8.53 | 2.049 |

**Table II – continued**

| Panel D: Characteristic-sorted portfolio returns | | | | |
|---|---|---|---|---|
| | Volatility (annualized %) | Correlations | | |
| | | Low minus High Market-to-Book | Low minus High Asset Growth | High minus Low Profitability |
| Low minus High Market-to-Book | 3.85 | 1 | 0.952 | 0.909 |
| Low minus High Asset Growth | 3.49 | - | 1 | 0.892 |
| High minus Low Profitability | 3.36 | - | - | 1 |

**Table III**
**The Exposure of Characteristic-Sorted Portfolio Returns to Disruption**

The table reports betas of characteristic-sorted portfolios with respect to the disruption surprise factor. The betas are estimated via monthly return regressions. The unconditional betas are estimated using all simulated sample months. The conditional betas are estimated using subsamples of simulated months in which the conditioning variables are less than one standard deviation below or more than one standard deviation above their respective means. For ease of interpretation, the table reports the estimated betas multiplied by 100.

| | Unconditional | Low Conditional Sharpe Ratio | High Conditional Sharpe Ratio | Low Expected Disruption | High Expected Disruption |
|---|---|---|---|---|---|
| Low Market-to-Book | −1.57 | −1.45 | −1.67 | −1.19 | −1.72 |
| High Market-to-Book | 1.90 | 2.23 | 1.57 | 3.35 | 0.65 |
| Low Asset Growth | −2.34 | −2.17 | −2.47 | −2.48 | −2.10 |
| High Asset Growth | 0.60 | 0.81 | 0.40 | 2.18 | −0.43 |
| High Profitability | −1.17 | −1.26 | −1.12 | −1.06 | −1.29 |
| Low Profitability | 1.34 | 1.77 | 0.97 | 3.79 | −0.18 |
| Unprofitable / High Market-to-Book | 3.62 | 3.61 | 3.64 | 5.88 | 2.07 |
| Profitable / High Market-to-Book | 0.19 | 0.15 | 0.15 | 0.84 | −0.35 |
| Profitable / High Asset Growth | −0.04 | −0.40 | 0.09 | −0.03 | −0.38 |
| Profitable / Low Asset Growth | −2.89 | −2.79 | −3.01 | −3.17 | −2.63 |

**Table IV**
**Conditional Predictability of Characteristic-Sorted Portfolio Returns**

The table reports Sharpe ratios of long-short characteristic-sorted portfolio returns. The Sharpe ratios are computed using monthly returns in subsamples of simulated months in which the conditioning variables are less than one standard deviation below or more than one standard deviation above their respective means. The reported Sharpe ratios are annualized.

| | Low minus High Market-to-Book | Low minus High Asset Growth | High minus Low Profitability |
|---|---|---|---|
| Low Conditional Sharpe Ratio | 0.601 | 0.542 | 0.553 |
| High Conditional Sharpe Ratio | −0.622 | −0.621 | −0.488 |
| Low Expected Disruption | −0.128 | −0.143 | −0.074 |
| High Expected Disruption | 0.183 | 0.152 | 0.261 |
| Low Asset Growth Spread | 0.213 | 0.147 | 0.181 |
| High Asset Growth Spread | −0.164 | −0.179 | −0.091 |
| Low Profitability Spread | −0.129 | −0.140 | −0.095 |
| High Profitability Spread | 0.255 | 0.173 | 0.210 |
| Low Market-to-Book Spread | −0.084 | −0.117 | 0.008 |
| High Market-to-Book Spread | 0.449 | 0.400 | 0.429 |

**Table V**
**Autocorrelations of Characteristic-Sorted Portfolio Returns**

The table reports statistics on autocorrelations of characteristic-sorted long-short portfolio returns measured at one-year and five-year intervals. The population autocorrelations are computed by combining all simulated data samples, whereas the in-sample autocorrelations are computed within each 50-year simulated sample. The table also reports the percentage of simulated samples in which the estimated autocorrelation is negative.

| | Low minus High Market-to-Book | Low minus High Asset Growth | High minus Low Profitability |
|---|---|---|---|
| One year: | | | |
| Population | 0.209 | 0.202 | 0.165 |
| In-sample mean | 0.129 | 0.127 | 0.076 |
| % of samples negative | 20.1% | 20.9% | 33.7% |
| Five years: | | | |
| Population | 0.337 | 0.324 | 0.278 |
| In-sample mean | 0.073 | 0.069 | 0.022 |
| % of samples negative | 39.4% | 39.9% | 47.0% |

**Table VI**
**Predictability of Average Returns of Characteristic-Sorted Portfolios**

The table reports percentiles of Sharpe ratio distributions in simulated 50-year samples. Panel A reports percentiles of the *t* distribution and the corresponding annualized Sharpe ratios. Panels B and C report percentiles of the distributions of annualized Sharpe ratios for characteristic-sorted long-short portfolios in simulations with rational and overconfident investors, respectively.

| | Percentile | | | | | | |
|---|---|---|---|---|---|---|---|
| | 1st | 5th | 10th | 50th | 90th | 95th | 99th |
| Panel A: Benchmark | | | | | | | |
| *t*-statistic | −2.333 | −1.647 | −1.283 | 0 | 1.283 | 1.647 | 2.333 |
| Sharpe Ratio | −0.330 | −0.233 | −0.181 | 0 | 0.181 | 0.233 | 0.330 |
| Panel B: Rational Investors | | | | | | | |
| Low minus High Market-to-Book | −0.361 | −0.265 | −0.212 | −0.034 | 0.155 | 0.211 | 0.311 |
| Low minus High Asset Growth | −0.383 | −0.285 | −0.231 | −0.056 | 0.120 | 0.171 | 0.268 |
| High minus Low Profitability | −0.329 | −0.243 | −0.196 | −0.032 | 0.126 | 0.169 | 0.237 |
| Panel C: Overconfident Investors | | | | | | | |
| Low minus High Market-to-Book | −0.712 | −0.505 | −0.402 | −0.012 | 0.367 | 0.470 | 0.656 |
| Low minus High Asset Growth | −0.743 | −0.533 | −0.428 | −0.039 | 0.326 | 0.428 | 0.610 |
| High minus Low Profitability | −0.568 | −0.405 | −0.322 | 0.008 | 0.332 | 0.422 | 0.571 |

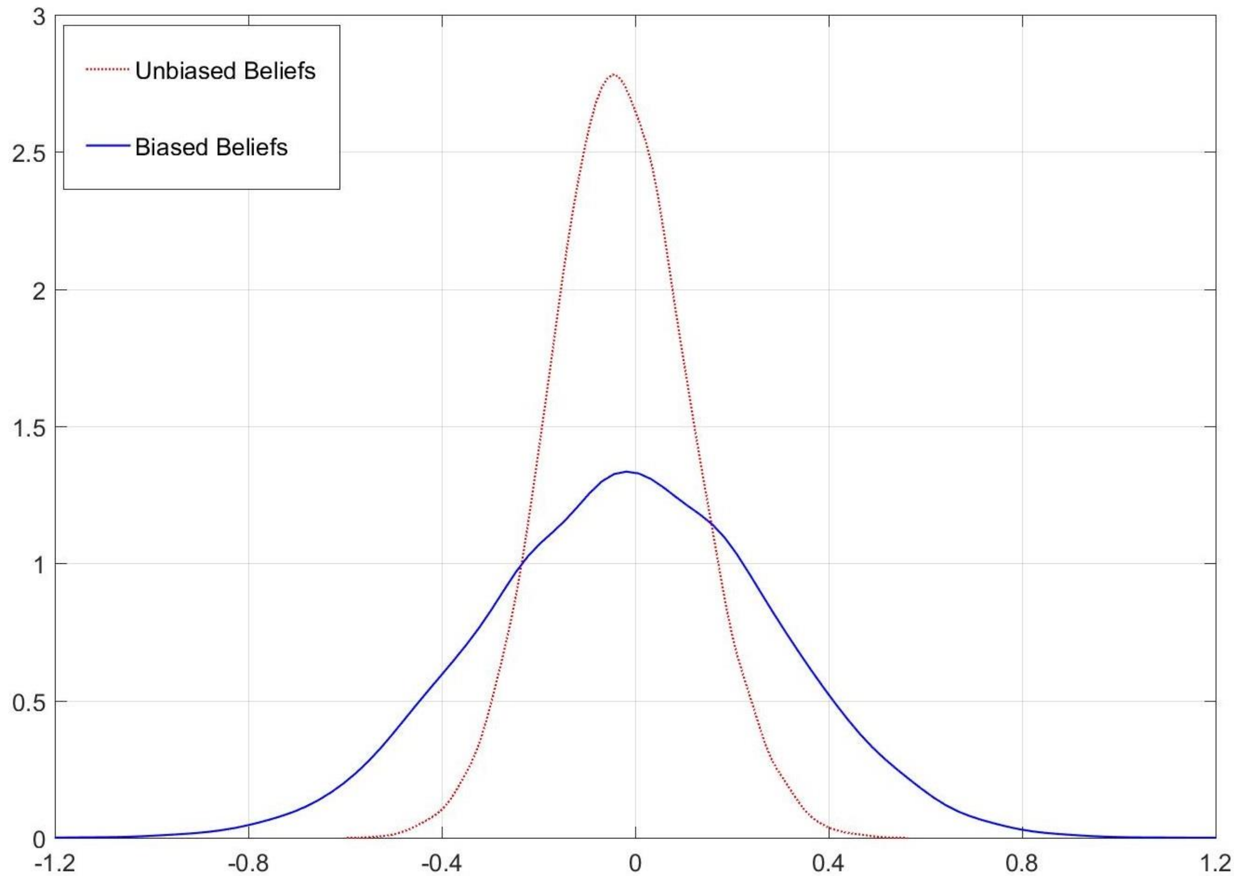


**Figure 1. Sharpe ratio distributions of the low-minus-high market-to-book portfolio**