Credit Ratings: Strategic Issuer Disclosure and Optimal Screening[*]

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Abstract

We study a model in which an issuer can manipulate information obtained by a credit rating agency (CRA). Better CRA screening reduces the likelihood of a high rating, but increases the value of a rated security. We find that improving the prior quality of assets can have no effect on the quality of a high-rated security, as low-type issuers manipulate more often in equilibrium. The issuer's response to anticipated CRA screening can either amplify or attenuate the effects on accuracy of increased penalties for ratings errors. Our model highlights the importance of strategic issuer disclosure in recent ratings failures.

1 Introduction

The failure of credit ratings to predict defaults of mortgage-backed securities in the lead-up to the financial crisis raises questions about the role of credit rating agencies (CRAs) in the economy. In principle, these "information intermediaries" create value by producing information in the form of ratings that allows investors to more accurately price assets. However, incentive problems may distort ratings and hence their usefulness. There has been much focus on CRAs inflating ratings of mortgage-backed securities in the pre-crisis era. However, an under-explored facet of the ratings process is the reliance of CRAs on issuers for much of the information on which they base their ratings.

We construct a theoretical model to investigate the implications of issuers' ability to distort information used to rate securities. An issuer with a low-quality asset can, at a cost, attempt to induce a favorable rating by manipulating a CRA's information. The CRA, for its part, chooses how much to invest in screening to reduce the probability that such manipulation is successful. We show that these two simple decisions interact in complex ways. The CRA, which cares about ratings accuracy, increases its screening intensity when it assigns a higher probability to manipulation by the issuer. However, the issuer may manipulate either more or less frequently when it expects the CRA to screen more intensely. The issuer's behavior can therefore either amplify or attenuate the effects of efforts to improve ratings quality by penalizing CRAs more for ratings errors.

We show that improvements in fundamental asset quality may not translate into improvements in the quality of securities actually sold. Before the late 1990s, subprime mortgages were thought of as low-quality assets. Perceptions of the quality of securities backed by these mortgages improved over the next several years. However, the crisis revealed that a large proportion of poor-quality loans had, in fact, been previously issued. Our model explains this finding by showing that a low-quality issuer increases its misreporting as the prior belief over quality improves. Information manipulation of this type is likely to be especially important for relatively new or complex assets such as mortgage-backed securities.

Many commentators have argued that CRAs were negligent - and possibly complicit - in the build-up to the financial crisis. We are by no means attempting to diminish the culpability of CRAs. However, our model puts some focus back on the role of the issuer in misleading both CRAs and investors.[1] Our results point to the need to account for issuer behavior

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[1]In a written statement before the US SEC on November 21, 2002, Raymond W. McDaniel, President of

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when using observed ratings accuracy to assess CRA diligence. We also show that penalizing issuers for distorting information can increase ratings accuracy by improving CRA screening incentives, even if these penalties are too small to have much effect on issuer behavior.

The issuer in our model has either a high-quality (positive NPV) or low-quality (negative NPV) project against which it can sell a financial claim. The issuer decides whether or not to apply for a rating. If it seeks a rating, the CRA subsequently observes a signal of project quality and assigns the security a high or low rating consistent with its signal.[2] The issuer pays a rating fee to the CRA if a high rating is assigned. It then sells its security to investors, who form rational expectations and break even on average. Finally, the project pays off. We first treat the rating fee as exogenous and then later endogenize it as the solution to a Nash bargaining game between the issuer and the CRA.

If project quality is high, the CRA observes a high signal. If project quality is low, the CRA may observe either a high or low signal. A low-type issuer (i.e., an issuer with a low-quality project) chooses whether to manipulate the information it reports to the CRA. The cost of doing so captures both the direct costs of distorting information as well as expected longer-run costs associated with sanctions for false disclosure and loss of reputation. Simultaneously, the CRA chooses how much to invest in screening in order to reduce the likelihood that such manipulation is successful.

Given the information structure of the game, either a low rating or the lack of a rating reveals the issuer to be the low type. As a low-quality project is negative NPV, an unrated or low-rated security will not be issued. The price of a high-rated security is the expected NPV of the underlying project, given the equilibrium strategies of the issuer and CRA. A high-type issuer always requests a rating since it is guaranteed to receive a high rating. The low-type issuer may or may not request a rating, but always manipulates if it seeks a rating.[3] The issuer's payoff is the expected revenue from selling its financial claim (which takes into account the probability of obtaining a high rating), less the rating fee and any manipulation costs. The CRA's payoff is the expected fee for a high rating, less the cost of screening and an expected penalty for a rating error.

Moody's Investors Service, states "Most issuers operate in good faith and provide reliable information to the securities markets, and to us. Yet there are instances where we may not believe that the numbers provided or the representations made by issuers provide a full and accurate story."

[2]Our model therefore features a binary type of issuer and binary ratings. Goel and Thakor (2015) rationalize coarse (i.e., discrete) ratings.

[3]In the absence of manipulation, the CRA assigns the low type a low rating, so not manipulating is equivalent to not requesting a rating.

We show that there are three types of equilibria in the game. When the manipulation cost is high, the issuer never manipulates. Anticipating this, the CRA does not invest in screening. When the manipulation cost is low, the low-type issuer always manipulates, and the CRA invests heavily in screening. At a moderate manipulation cost, the low-type issuer mixes between manipulating and not manipulating, and the CRA invests a moderate amount in screening. We refer to these outcomes as "zero-manipulation," "full-manipulation," and "partial-manipulation" equilibria, respectively.

The key to many of our conclusions lies in two countervailing effects of the anticipated screening intensity on the low-type issuer's incentives to manipulate. Increased screening intensity reduces the probability that the low-type issuer receives a high rating, weakening the incentive to manipulate. This direct "filtering" effect of increased screening intensity on incentives is most pronounced when the expected price of a high-rated security is high; that is, when the screening intensity itself is high. However, increased screening intensity also increases the price of a high-rated security, which strengthens the incentive to manipulate. This indirect "price improvement" effect is most pronounced when the probability of successful manipulation is high; that is, when the screening intensity is low. These countervailing effects produce an inverse U-shaped relationship between the frequency of manipulation and anticipated screening intensity in a partial-manipulation equilibrium.

The issuer's response to changes in anticipated screening intensity affects the overall impact on ratings accuracy of changes in the penalty on the CRA for inaccurate ratings. An increase in this penalty results in more intense screening; all else equal, this improves ratings accuracy. The issuer's response to an increase in expected screening intensity amplifies the positive effect on ratings accuracy when the penalty is already high, as screening intensity is high in this case. In contrast, it dampens the effect of increased screening when screening intensity is low. Thus, increasing sanctions for reporting errors from mild to only moderate levels may be ineffective at significantly improving ratings quality. On the other hand, small increases may be sufficient to generate substantial improvements in ratings accuracy when these penalties are already fairly high.

The model also produces a noteworthy conclusion regarding the quality of a high-rated security when the average quality of underlying assets improves. In a partial-manipulation equilibrium, the low-type issuer responds to an increase in the prior belief that the asset is high quality by manipulating more often, to the point that the average quality of assets for

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which issuers seek high ratings remains unchanged. Facing the same distribution of assets for which issuers seek high ratings, the CRA does not change its screening intensity, and thus the average quality of assets receiving a high rating does not change.

Finally, if the equilibrium features full manipulation, a small increase in the issuer's manipulation cost has no effect on its behavior. It can nevertheless impact equilibrium screening intensity if the fee for a high-rated asset is endogenously set through a bargaining protocol. In this case, the issuer passes part of the higher expected manipulation cost through to the CRA in the form of a lower equilibrium fee. This lower fee reduces the benefit to the CRA of assigning a high rating to a low-quality asset, and results in a higher screening intensity. Thus, making manipulation more difficult can improve ratings accuracy even if it does not affect the issuer's behavior.

Our paper contributes primarily to the literature examining the process by which CRAs produce information and assign ratings to issuers. Most of the papers in this literature focus on distortions in the ratings process due to issuers' ability to "shop" for favorable ratings (Skreta and Veldkamp, 2009; Bolton, Freixas, and Shapiro, 2012; Sangiorgi and Spatt, 2017) or CRA incentives to inflate ratings (Mathis, McAndrews, and Rochet, 2009; Fulghieri, Strobl, and Xia, 2014; Frenkel, 2015; Bouvard and Levy, 2017). Our paper adds to this literature by accounting for the ability of an issuer to distort the information on which a CRA relies in evaluating the issuer's securities and assigning ratings. Our results demonstrate that a more stringent ratings process can either mitigate or amplify these incentives. The issuer's response may therefore either enhance or undermine efforts to improve ratings quality by lowering the cost of screening or holding CRAs more accountable for ratings errors.

The audit literature has also considered the strategic interaction between a firm and a certifier (the auditor). For example, Hillegeist (1999), Laux and Newman (2010), and Deng, Melumad, and Shibano (2012) consider models in which a firm can misreport its type and the auditor chooses the quality of its own signal. The focus in this literature has been on the effect of changing auditor liability on overall audit quality.[4] An important difference in the credit ratings context is that, in the audit literature, some to all of the penalties on the auditor for underperformance represent a direct transfer to investors. This affects the ex ante value of the security to investors. On the other hand, it is very difficult for investors to win a lawsuit against credit rating agencies, as the courts have typically held that credit ratings are

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[4]Strobl (2013) considers the effect of audit quality on earnings manipulation, but treats the audit quality as fixed and observable, in contrast with our approach.

opinions protected by free speech (see, e.g., Nagy, 2009). Thus, the penalty incurred by the CRA in our model represents the cost of reputational loss and possible fines by the regulators, but is not earned by the investors. In terms of focus, the penalty on the CRA is only one feature of interest to us, and we also derive interesting comparative static implications from changing the prior probability of a high-type issuer, the manipulation cost on the issuer, the technological cost of screening, and the bargaining power of the issuer.[5]

Finally, our paper is related to the economics of crime literature. Tsebelis (1990) argues that an enforcement agency optimally responds to an increase in the penalty for crime by weakening enforcement, undoing the effect of the increased penalty. A similar phenomenon generally occurs in our setting, with the CRA optimally reducing screening intensity in response to an increase in the cost of manipulation. However, as noted, an increase in the cost of manipulation can actually result in greater screening intensity when the issuer's incentives to manipulate are overwhelming and the issuer and CRA bargain over a rating fee upfront. We also add to this literature by exploring the effects of the cost of enforcement in the form of CRA screening, showing that an increase in this cost has ambiguous effects on the extent of issuer misbehavior.

2   Model

An issuer has a project that requires an upfront investment and generates a future cash flow. The project is high quality with probability η and low quality with probability $1 - \eta$. The high-quality project has an NPV $v_h > 0$, and the low-quality project has an NPV $v_\ell < 0$. The values of η, v_h, and v_ℓ are common knowledge.

The issuer privately observes the quality of its own project. The issuer has no financial resources, so if it wishes to undertake the project, it must issue a financial claim backed by the project. If it does not undertake the project, it obtains a payoff of zero. For convenience, we refer to an issuer with a high- (low-) quality project as a high- (low-) type issuer. We normalize v_h to 1 and v_ℓ to -1.

An issuer wishing to sell a financial claim can obtain a rating from a credit rating agency

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[5]Certification of a strategic agent has been considered in other settings as well. For example, Cohn and Rajan (2013) show that the certification by a board of directors of a CEO's strategic choices can exacerbate agency conflicts due to CEO reputational concerns. Gill and Srgoi (2012) analyze a model where a seller can choose the stringency of a certification test. As in our model, greater stringency hurts the seller by reducing the probability of certification but benefits the seller by increasing the price conditional on certification. However, there is no manipulation in their model (i.e., the seller is not strategic).

(CRA). The CRA obtains a noisy signal about the type of the project and issues a rating. Investors observe the rating, update their beliefs about issuer type, and decide whether they wish to buy the financial claim being issued. Investors are risk-neutral and perfectly competitive, so if they purchase the claim, they do so at the expected NPV of the project, given their posterior beliefs.

There are two dates in the model, 0 and 1. The discount rate is normalized to zero. The sequence of events at date 0 is as follows:

1. The issuer privately observes its type, high or low.

2. Simultaneously,

    (i) The issuer chooses whether or not to approach the CRA to request a rating. If a low-type issuer requests a rating, it further chooses whether or not to take an unobserved action that we refer to as "manipulating" the information observed by the CRA (more on the consequences of this decision shortly). Manipulation incurs a cost $m > 0$, which captures both upfront costs of distorting or misrepresenting information presented to the CRA as well as expected longer-run costs associated with sanctions for false disclosure and loss of reputation.

    (ii) The CRA privately chooses a screening intensity α. If the issuer seeks a rating, the CRA incurs a screening cost $kc(\alpha)$, where $k > 0$ and $c(\cdot)$ is strictly increasing and strictly convex. We assume that $c(0) = c'(0) = 0$, so that in the absence of screening, both total and marginal costs to the CRA are zero. We also assume that $\lim_{\alpha \to 1} c'(\alpha) = \infty$, which ensures that the equilibrium screening intensity is less than 1.

3. If the issuer does not seek a rating, the game ends. If a rating is sought, the CRA observes a binary signal of project quality. If project quality is high, then the CRA observes a high signal. If project quality is low and the issuer did not manipulate the information observed by the CRA, the CRA observes a low signal. If project quality is low and the issuer did manipulate the CRA's information, then the CRA observes a low signal with probability α and a high signal with probability $1 - \alpha$.

    The CRA then assigns a public rating, r_h ("high" rating) or r_ℓ ("low" rating), to the financial claim backed by the project. We assume that the CRA assigns a rating

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corresponding to its signal. Observe that it cannot be optimal to assign a low rating when the CRA obtains a high signal, as it only obtains a fee if the rating is high. The absence of direct rating inflation (assigning a high rating given a low signal) may be justified by relying on extremely high penalties if the CRA is found to have indulged in outright fraud (as opposed to merely being incompetent). Observe also that, in our setting, it cannot be optimal to both have $\alpha > 0$ and to assign a high rating when a low signal is obtained, as the same outcome is obtained at lower cost by reducing α instead.

If the CRA assigns a high rating, the issuer pays a fee $f \geq 0$ to the CRA.[6] We take this fee as exogenously given in Section 3 and later endogenize it as the outcome of a bargaining process in Section 4. We assume that this fee is not so large that it deters all issuance.

4. Investors compute the NPV of the project, given the CRA's report (r_h or r_ℓ), their beliefs about screening intensity α and about the reporting strategy of each type. From an investor's perspective, the financial claim is an asset, and sells at a price equal to its expected NPV. If the security is issued, the project is carried out.

If the project is undertaken, at date 1 all parties find out the type of the project and earn their respective terminal payoffs. The CRA incurs a penalty $\lambda > 0$ if its rating proves to be incorrect; that is, if a claim with a rating of r_h proves to be backed by a low-quality project.[7] This penalty captures, in reduced form, the reputational cost of reduced investor confidence as well as explicit regulatory sanctions and losses from investor lawsuits.[8]

We consider perfect Bayesian equilibria of the game. Formally, there are three players to the game: the issuer, the CRA, and investors as a group. Other than the type of the firm and the actions of the issuer and CRA, all information is common knowledge. Investors form rational expectations and compete in a perfectly competitive market, so earn zero profits in

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[6]As is standard in the literature, we assume that the issuer only pays the rating fee f if the rating is high. This assumption is consistent with the large literature that has commented on the incentive to inflate ratings in an issuer-pay model. Qualitatively, many of our results go through if the fee is paid upfront, before the rating is issued. Of course, the exact forms of the expressions we exhibit will change.

[7]In principle, a rating would also be incorrect if a claim with a rating of r_ℓ proves to be backed by a high-quality project. However, only upward rating errors are possible in our model, so an incorrect rating in our model is "inflated" in the sense that it induces a favorable belief about a low-quality security.

[8]As an example of these explicit costs, in 2015, Standard & Poor's reached a settlement with the Justice Department over inflated ratings on mortgage-backed securities over the period 2004–2007, and agreed to pay \$1.375 billion. See Department of Justice (2015).

expectation. By construction, only a low type obtains a low rating, so a low-rated security cannot be issued in equilibrium. The price of a high-rated security, p, equals the expected payoff of the project backing it, given investors' conjectures regarding the strategies of the CRA and the issuer. In equilibrium, these conjectures must be correct.

We restrict attention to equilibria that survive the $D1$ refinement. In such equilibria, a high-type issuer always requests a rating, ensuring that an unrated security cannot be issued.[9] Thus, in describing equilibrium issuer strategies, we need only specify the strategy of a low-type issuer. In principle, a low type issuer makes two choices: whether or not to request a rating and, conditional on requesting a rating, whether or not to manipulate the CRA's information. However, requesting a rating but not manipulating guarantees the issuer a low rating, which reveals the issuer to be the low type and is therefore informationally equivalent to not requesting a rating. Therefore, to simplify the strategy space, we disallow the possibility that the low-type issuer can request a rating but not manipulate. Thus, a low-type issuer may request a rating (and manipulate), not request a rating, or mix between the two actions. We denote by σ the probability with which the low-type issuer requests a rating.

The only choice the CRA makes in the game is its level of screening intensity α. Thus, when the rating fee f is exogenous, a perfect Bayesian equilibrium of the game is represented by (α, σ), where each player's strategy maximizes its own payoff, given its own beliefs and the actions of other parties in the game. When the rating fee is endogenous, it too must be determined as part of the equilibrium.

We analyze this model in the next two sections. In Section 3, we take the rating fee f as fixed and solve for the equilibrium (α, σ). Much of the intuition in the paper comes from analyzing the fixed-fee case. In Section 4, we endogenize the rating fee as the solution to a Nash bargaining game and solve for the equilibrium, (f, α, σ). We show that in equilibrium the fee is strictly less than the price of the high-rated security in this case, again ensuring that the high-type issuer always seeks a rating.

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[9]Absent the $D1$ refinement, there always exists an equilibrium in which no issuer requests a rating, supported by investors' beliefs attributing a deviation to the low-type issuer. $D1$ effectively requires that investors attribute a deviation to the type that faces a lower cost of deviating. The high type faces a lower cost of requesting a rating, since the low type must bear the manipulation cost m in order to have a chance of receiving a high rating and issuing a security.

3 Fixed Rating Fee

In this section, we take the fee f as exogenous, and solve for the equilibrium (α, σ). We do so in two steps. First, we analyze the best responses of the CRA and the low-type issuer, taking the price of a high-rated security, p, as given. We then close the model by imposing the requirement that p be set so that investors break even in expectation, and solve for the equilibrium.

3.1 Best responses of CRA and issuer

Consider the CRA's best response, taking the low-type issuer's strategy σ as given. Let $\tilde{\sigma}$ denote the CRA's conjecture of σ. The CRA's expected payoff depends in part on the quality of the pool of issuers it expects to request a rating. Define $\nu(\sigma) = \frac{\eta}{\eta + (1-\eta)\sigma}$ as the probability that the issuer is a high type, conditional on requesting a rating. Note that ν decreases with σ and can take on values between η (if $\sigma = 1$; that is, if all issuers request a rating) and 1 (if $\sigma = 0$; that is, if only high-type issuers request a rating). The total mass of issuers requesting a rating is given by $N(\sigma) = \eta + (1 - \eta)\sigma$. Thus, given $\tilde{\sigma}$, the CRA's expected payoff is

$$\Psi(\tilde{\sigma}) = N(\tilde{\sigma}) \left\{ \nu(\tilde{\sigma})f + [1 - \nu(\tilde{\sigma})](1 - \alpha)(f - \lambda) - kc(\alpha) \right\}. \tag{1}$$

The CRA's objective is to choose α to maximize Ψ. Observe that, as the CRA takes the low type's strategy as given, maximizing Ψ is equivalent to maximizing the term in the parentheses in equation (1). The following lemma characterizes the CRA's best response. Proofs of all results are in Appendix A.

Lemma 1. *The best response of the CRA satisfies* $c'(\alpha) = [1 - \nu(\tilde{\sigma})] \max \left\{ \frac{\lambda - f}{k}, 0 \right\}$.

When $f < \lambda$, the CRA's optimal screening intensity increases with $\tilde{\sigma}$, as $\nu(\cdot)$ is a decreasing function. Intuitively, since the CRA views erroneous ratings as costly, it screens more intensely when it expects the low-type issuer to manipulate with higher probability. It also screens more intensely when the cost it ascribes to an erroneous rating, λ, is higher and less intensely when the fee it receives for rating a security, f, is higher.[10]

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[10]One may note that when $\alpha = 0$, it is curious why the rating is still relevant at this point. One can

Next, consider the low-type issuer's best response, taking the CRA' strategy α and the anticipated price of a high-rated security p as given. Let $\tilde{\alpha}$ denote the low-type issuer's conjecture of α and $\tilde{p}$ the expected price p of a high-rated security. If the low-type issuer does not request a rating, then its project is revealed to be low quality. Since a low-quality project is negative NPV, the issuer does not sell an asset in this case, and its payoff is zero. If, on the other hand, the low-type issuer requests a rating, it bears the manipulation cost m, and expects to receive a high rating with probability $1 - \tilde{\alpha}$. If it obtains a high rating, it pays a fee f to the CRA and sells its asset at an anticipated price $\tilde{p}$. If it obtains a low rating, its payoff is zero. Thus, the low-type issuer's expected payoff if it requests a rating is

$$\pi_\ell(\tilde{\alpha}, \tilde{p}) = (1 - \tilde{\alpha})(\tilde{p} - f) - m. \tag{2}$$

The following lemma characterizes the low-type issuer's best response.

Lemma 2. *It is a best response for the low-type issuer to request a rating (i.e., set $\sigma = 1$) if $(1 - \tilde{\alpha})(\tilde{p} - f) \geq m$, and to not request a rating (i.e., set $\sigma = 0$) if $(1 - \tilde{\alpha})(\tilde{p} - f) \leq m$.*

Note that if $(1 - \tilde{\alpha})(\tilde{p} - f) = m$, then any $\sigma \in [0, 1]$ represents a best response for the low-type issuer. Equilibria in which the low-type issuer mixes between seeking a rating and not doing so are of particular interest to us in the rest of the paper.

3.2 Price of a high-rated security

Next, we solve for the price of a high-rated security p, given investors' beliefs about the issuer's and CRA's strategies. To keep notation simple, we assume that investors' beliefs coincide with those of the CRA and the issuer (which will be true in equilibrium), so that $\tilde{\alpha}$ is the investors' conjecture of α and $\tilde{\sigma}$ their conjecture of σ. As investors are perfectly competitive and form rational expectations, the price p must equal the expected value of an asset, conditional on a high rating. The price, then, must satisfy

$$p(\tilde{\alpha}, \tilde{\sigma}) = \frac{\nu(\tilde{\sigma})v_h + [1 - \nu(\tilde{\sigma})](1 - \tilde{\alpha})v_\ell}{\nu(\tilde{\sigma}) + [1 - \nu(\tilde{\sigma})](1 - \tilde{\alpha})} = \frac{\nu(\tilde{\sigma}) - [1 - \nu(\tilde{\sigma})](1 - \tilde{\alpha})}{\nu(\tilde{\sigma}) + [1 - \nu(\tilde{\sigma})](1 - \tilde{\alpha})}. \tag{3}$$

rationalize the existence of the CRA in this case by introducing a third type of issuer with a terrible project that has extremely negative NPV (say a Ponzi scheme) and assuming that the CRA adds value because it can perfectly and costlessly screen out the terrible type.

Observe that p is decreasing in $\tilde{\sigma}$ and increasing in $\tilde{\alpha}$. Holding $\tilde{\alpha}$ fixed, a higher anticipated probability of manipulation by the low-type issuer implies a worse pool of issuers requesting a high rating. As $\tilde{\alpha} < 1$, the pool of issuers that investors expect to survive screening and receive a high rating also worsens, resulting in a lower expected payoff from a high-rated asset, and hence a lower price. Similarly, holding $\tilde{\sigma}$ fixed, a higher anticipated screening intensity filters out more low-type issuers, improving the expected pool of high-rated assets and hence increasing the price investors are willing to pay.

3.3   Equilibrium

Let α^*, σ^*, and p^* denote the equilibrium values of α, σ, and p. In equilibrium it must be that $\tilde{\alpha} = \alpha^*$, $\tilde{\sigma} = \sigma^*$, and $\tilde{p} = p^*$.

Suppose first that the CRA and investors conjecture that the low-type issuer always requests a rating (i.e., $\tilde{\sigma} = 1$). Then, $\nu = \eta$ and the CRA's best response is $\alpha_1 \overset{d}{=} c'^{-1}\left((1-\eta)\max\left\{\frac{\lambda-f}{k}, 0\right\}\right)$. Accounting for this response, investors set the price of a high-rated security to $p(\alpha_1, 1)$. Given this price and a belief that $\alpha = \alpha_1$, it is a best response for the low-type issuer to request a rating if $m \le (1 - \alpha_1)(p(\alpha_1, 1) - f)$. The best response is unique if $m < (1 - \alpha_1)(p(\alpha_1, 1) - f)$. Thus, if the manipulation cost is sufficiently low, in equilibrium the low-type issuer always requests a rating, so that $\sigma^* = 1$. We call such an equilibrium a "full-manipulation" equilibrium.

Next, suppose that the CRA and investors conjecture that the low-type issuer never requests a rating (i.e., $\tilde{\sigma} = 0$). Then, $\nu = 1$, and the CRA's best response is to set $\alpha = 0$. Therefore, the price of a high-rated security is rationally set to 1. For the low-type issuer, in turn, it is a best response to not request a rating if $m \ge 1 - f$. So, if the manipulation cost is sufficiently high, then the low-type issuer never requests a rating in equilibrium (i.e., $\sigma^* = 0$). We call such an equilibrium a "zero-manipulation" equilibrium.

For values of m such that $(1 - \alpha_1)(p(\alpha_1, 1) - f) < m < 1 - f$, no equilibrium in which the issuer plays a pure strategy ($\sigma^* = 0$ or $\sigma^* = 1$) is sustainable. In this case, the low-type issuer mixes between requesting and not requesting a rating. For such mixing to be sustainable, the low-type issuer must be indifferent between the two actions, so that (α^*, σ^*) must satisfy the condition $(1 - \alpha^*)(p(\alpha^*, \sigma^*) - f) = m$.

Note that α_1 and $p(\alpha_1, 1)$ are functions only of exogenous parameters. We can now fully characterize the equilibrium of the game.

Proposition 1. *When $f < 1$ is fixed, the equilibrium is as follows:*

(i) *If $m \leq (1 - \alpha_1)(p(\alpha_1, 1) - f)$, the equilibrium features full manipulation, with $\alpha^* = \alpha_1$ and $\sigma^* = 1$.*

(ii) *If $m \geq (1 - f)$, the equilibrium features zero manipulation, with $\alpha^* = \sigma^* = 0$.*

(iii) *If $(1 - \alpha_1)(p(\alpha_1, 1) - f) < m < 1 - f$, the equilibrium features partial manipulation, with (α^*, σ^*) satisfying the two conditions*

$$\sigma^* = \frac{\eta}{1 - \eta} \left(\frac{1 - f - \frac{m}{1 - \alpha^*}}{(1 - \alpha^*)(1 + f + \frac{m}{1 - \alpha^*})} \right), \quad and \tag{4}$$

$$\alpha^* = c'^{-1} \left((1 - \nu(\sigma^*)) \max \left\{ \frac{\lambda - f}{k}, 0 \right\} \right). \tag{5}$$

The condition $f < 1$ ensures that, in equilibrium, the high-type issuer seeks a rating with probability one. In equilibrium, either $\sigma^* = 0$, in which case $p(\alpha^*, \sigma^*) = 1 > f$, or $\sigma^* > 0$, in which case it must be that $p > f$. If $f \geq 1$, even when the low-type issuer is fully screened out, the high-type issuer weakly prefers to not seek a rating, which implies that no ratings will be issued.

Going forward, the partial-manipulation equilibrium is of particular interest to us. We establish sufficient conditions for uniqueness of such an equilibrium. As the parameter regions in which full-, partial-, and zero-manipulation equilibria do not overlap, under these conditions the overall equilibrium is also unique. The condition $f < \lambda$ in the next lemma ensures that $\alpha^* > 0$; that is, the screening intensity of the CRA is strictly positive in equilibrium.

Lemma 3. *Suppose $f < \lambda$ and let α^* denote the equilibrium screening intensity in a partial-manipulation equilibrium. The equilibrium is unique if*

$$\frac{c''(\alpha^*)}{c'(\alpha^*)} > \frac{1 - f}{2} \left(\frac{1 + f}{m} - \frac{\eta}{1 - \eta} \right). \tag{6}$$

Although condition (6) relies on the endogenous value of α^*, for specific cost functions it is readily translatable to a function of the exogenous parameters. For example, suppose the cost function is $c(\alpha) = \alpha^z$ where $z > 1$. Further, suppose that $m < \frac{1 - \eta}{\eta}(1 + f)$, so

that the RHS of condition (6) is strictly positive. Then, condition (6) reduces to $\alpha^* <$ $(z-1)/\left(\frac{1-f}{2}\left(\frac{1+f}{m} - \frac{\eta}{1-\eta}\right)\right)$. When z is sufficiently high (i.e., the cost function is sufficiently convex), the RHS of the last inequality exceeds 1, so that the inequality is always satisfied.

For the rest of this paper, we assume that the equilibrium of the game is unique when the fee f is fixed; that is, we assume that the condition in Lemma 3 is satisfied in a partial-manipulation equilibrium. In a partial-manipulation equilibrium, the equilibrium α^* and σ^* are jointly determined as the solution to equations (4) and (5). Equation (5) can readily be interpreted as the CRA's optimal response to the equilibrium σ^*. The CRA increases its screening intensity, α^*, as σ^* increases. This response is expected, as increased manipulation by the low type worsens the pool of issuers requesting a rating.

The function on the right-hand side of equation (4) is more complex. Here, σ^* is non-monotone in α^*; it decreases with α^* for high values of α^*, but increases with α^* for low values of α^*. Loosely, this equation may be interpreted as providing the optimal equilibrium response of the low-type issuer, taking into account that investors have rational conjectures over α and σ.

Intuitively, an increase in expected screening intensity has two countervailing effects on the low-type issuer's incentives to request a rating. On the one hand, it lowers the probability of successful manipulation, which (given that the manipulation cost has stayed fixed) dissuades the low type from requesting a rating. This direct screening effect is large when screening intensity is high, because all else equal, a high α implies a high price for the security. On the other hand, in equilibrium more intense screening increases the price conditional on a high rating, encouraging the low type to request a rating. This indirect "price improvement" effect is large when screening intensity is low, since the probability that manipulation is successful is then high. These countervailing effects affect the response of α^* and σ^* to changes in the exogenous parameters.

3.4 Comparative statics

The exogenous parameters of the model are η, k, λ, m and (in this section) f. We now consider how a small change in each of these parameters affects the equilibrium (α^*, σ^*) as well as the incidence of ratings errors in a partial-manipulation equilibrium. An error occurs whenever the issuer is the low type, the issuer manipulates, and the CRA fails to detect manipulation in its screening. The incidence of errors is $q(\alpha^*, \sigma^*) = (1 - \alpha^*)\sigma^*$. Note that

the price of a high-rated security can be written as $p = \frac{\eta - q}{\eta + q}$, so that the price is strictly decreasing in q.

In our model, welfare depends both on the mix of projects that are funded, and on the costs incurred by the issuer and CRA. Manipulation costs incurred by the low-type issuer and reputational penalties suffered by the CRA reduce welfare, to the extent that m and λ do not represent transfers to the regulator or other agents in the economy. Similarly, the screening cost $kc(\alpha)$ reduces welfare. The first-best outcome is obtained if α^* and σ^* are both zero in equilibrium. In this case, only high-quality projects are funded, and no manpiulation, screening, or reputational costs are incurred.

In equilibrium in our model, all high-quality projects (which have positive NPV) are funded. Thus, keeping the screening intensity constant, welfare is inversely related to the number of low-quality projects (which have negative NPV) that obtain funding. As a result, the error frequency q is inversely related to welfare, and the price p is positively related to welfare.

In our discussion of the comparative statics, we restrict attention to the case where $f < \lambda$, so that the screening intensity of the CRA is strictly positive. We begin by considering the effect of a change in λ, the penalty for a rating error, on (α^*, σ^*) as well as on $q(\alpha^*, \sigma^*)$. To the extent that regulation has an impact on λ, this effect has policy implications. Since k and λ have directly opposing effects on CRA incentives, we simultaneously consider the effect of changes in k, which affects the cost of screening, on the quantities of interest. Note that the technical condition $(f + m)^2 + 2m < 1$ in the following proposition is sufficient but not necessary for the conclusions to hold.

Proposition 2. *Consider a partial-manipulation equilibrium with fixed $f < \lambda$. Suppose that $(f + m)^2 + 2m < 1$. Then,*

(i) $\frac{d\alpha^*}{d\lambda} > 0$. *Further, there exists a $\hat{\lambda} > 0$ such that, if $\lambda < \hat{\lambda}$, then $\frac{d\sigma^*}{d\lambda} > 0$, and if $\lambda > \hat{\lambda}$, then $\frac{d\sigma^*}{d\lambda} < 0$. That is, as λ increases, the low-type issuer manipulates more often when λ is low and less often when λ is high. In both cases, $\frac{dq(\alpha^*, \sigma^*)}{d\lambda} < 0$ and $\frac{dp(\alpha^*, \sigma^*)}{d\lambda} > 0$. That is, ratings become more accurate as λ increases, and the price of the high-rated security increases.*

(ii) $\frac{d\alpha^*}{dk} < 0$. *Further, there exists a $\hat{k} > 0$ such that, if $k < \hat{k}$, then $\frac{d\sigma^*}{dk} > 0$, and if $k > \hat{k}$, then $\frac{d\sigma^*}{dk} < 0$. That is, as k increases, the low-type issuer manipulates more often when*

k is low and less often when k is high. In both cases, $\frac{dq(\alpha^*, \sigma^*)}{dk} > 0$ and $\frac{dp(\alpha^*, \sigma^*)}{dk} < 0$. That is, ratings become less accurate as k decreases, and the price of the high-rated security decreases.

Not surprisingly, the CRA responds to an increase in λ (i.e., a larger penalty for a rating error), or a decrease in k (i.e., a fall in the cost of screening), by increasing screening intensity. Changes in λ or k do not directly impact the issuer's strategy. However, they do so indirectly through their effects on α^*. That is, in equilibrium, the issuer anticipates the impact of changes in λ or k on the CRA's strategy and alters its own strategy accordingly.

As noted earlier, the issuer responds to an increase in expected screening intensity by manipulating less often when α^* is high but more often when α^* is low. Since screening intensity is high when λ is high and low when λ is low, σ^* is first an increasing and then a decreasing function of λ. Similarly, σ^* increases with k for high k and decreases with k for low k.

An increase in σ^* partially, but not fully, offsets the effect of an increase in α^* on the ratings error rate q. That is, overall, the error rate q decreases in response to an increase in λ. Suppose, instead, that q were to increase with λ. Then, the price of a high-rated security, p, would fall. A low-type issuer would respond to a combination of lower probability of successful manipulation (due to a higher α) and smaller benefit of successful manipulation (due to a lower p) by manipulating less often. However, more intense CRA screening and less manipulation would then together unambiguously lower the error rate. Thus, the equilibrium error rate can only fall in response to an increase in λ. However, the issuer's response to the change in expected screening intensity can either amplify or attenuate the direct effect of a change in λ on the error rate.

Figure 1 plots α^*, σ^*, and $q(\alpha^*, \sigma^*)$ for different values of λ in a specific example, where we set $\eta = 0.4$, $m = 0.1$, $f = 0.1$, and $kc(\alpha) = 0.05\alpha^2$. Consistent with Proposition 2, the figure shows that for $\lambda > f$, α^* increases with λ and that σ^* first increases and then decreases with λ. It also shows the unambiguous decline in q. However, the rate of decline in q (i.e., the slope of the curve depicting q) varies with the level of λ.

The error frequency is moderately flat for $\lambda \in (0.1, 0.2)$. In this region, σ^* increases with α^*, attenuating the effect of the increase in α^* on the error frequency. Thus, increasing penalties on CRAs for inaccurate ratings may have little effect on ratings accuracy in settings

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where CRAs currently face limited penalties for ratings errors. In contrast, q declines steeply in λ when the penalty is approximately in the range (0.2, 0.25). In this region, σ^* declines with α^*, amplifying the effect of the increase in α on the error frequency. Thus, when penalties are already moderately high, only a small further increase may be necessary to effect a significant improvement in ratings accuracy. When λ is very high (greater than approximately 0.25), further increases in λ have limited effect on α and therefore q because the marginal cost of increasing α from an already-high level is high.[11]
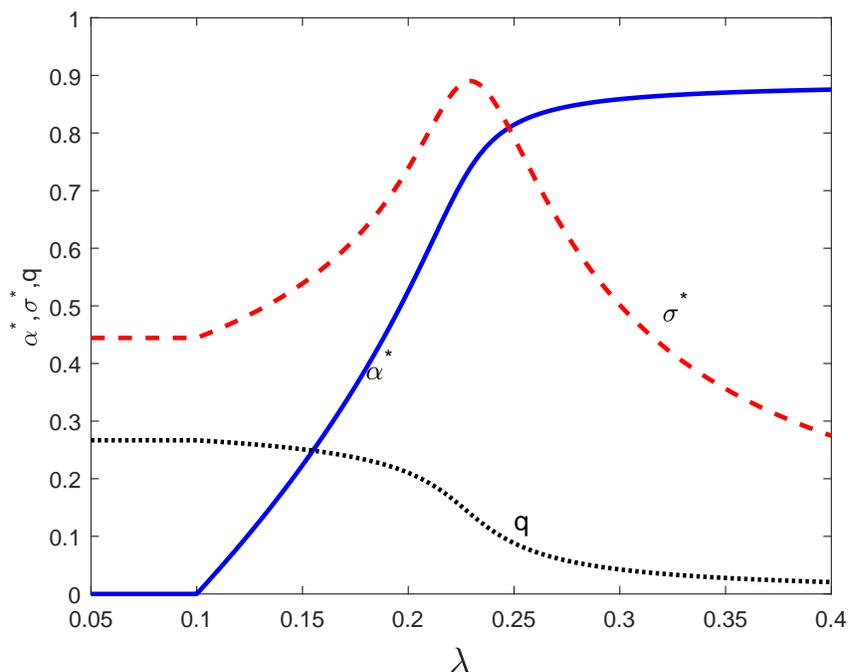
This figure illustrates the equilibrium values the low-type issuer's strategy σ^*, the CRA's screening intensity α^*, and the intensity of rating errors $q(\alpha^*, \sigma^*)$ as λ, the reputational penalty if the CRA makes an error, changes. The other parameters are $\eta = 0.4, m = 0.1, f = 0.1$, and $kc(\alpha) = 0.05\alpha^2$.

Figure 1: Equilibrium values of α^*, σ^*, and q as λ, the reputational penalty on the CRA for a rating error, changes

Next, consider the effect of an increase in η, the ex ante proportion of high-type issuers. Somewhat surprisingly, we show that both the level of screening and the price of the high-rated security are invariant to small changes in η in a partial-manipulation equilibrium.

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[11]Note that with the quadratic cost function, although $c'(\alpha) = 2\alpha$ stays bounded even when $\alpha \to 1$, we ensure numerically in this example and in the other examples in this paper that α^* stays strictly below 1.

Proposition 3. *Consider a partial-manipulation equilibrium with fixed $f < \lambda$. Then, $\frac{d\sigma^*}{d\eta} > 0$, $\frac{d\alpha^*}{d\eta} = 0$, and $\frac{dq(\alpha^*,\sigma^*)}{d\eta} = \frac{dp(\alpha^*,\sigma^*)}{d\eta} = 0$. That is, as η increases, the CRA's screening intensity remains the same, and the low-type issuer increases its manipulation intensity just enough to keep the rating accuracy and the price of the high-rated security unchanged.*

Recall that in a partial-manipulation equilibrium, the low-type issuer must be indifferent between requesting a rating and manipulating, and not requesting a rating. This indifference requires that the price of a high-rated security p satisfy $(1 - \alpha^*)(p - f) = m$. Suppose that η increases by a small amount. By continuity, in the new equilibrium, the low-type issuer must continue to mix between seeking a rating and not doing so. Observe that ν, the pool of issuers requesting a rating, is strictly increasing in η and strictly decreasing in σ. Thus, a small increase in η can be exactly offset by an increase in σ just large enough to ensure that ν remains constant. In turn, if ν is unchanged, the CRA's best response is unchanged. Finally, with ν and α^* unchanged, the error rate q and price p remain constant as well. Thus, in the new equilibrium, σ^* increases just enough that the price of the high-rated security is unchanged, whereas α^* is unchanged.

Next, consider changes in the manipulation cost, m. In a partial-manipulation equilibrium, α^* and σ^* both fall as the manipulation cost m increases. Here, the effects work in the expected direction. Further, as m increases, the price of the high-rated security increases; that is, the ex post pool of high-rated issuers improves in quality. Details on this case are provided in Appendix B. In contrast, in a full-manipulation equilibrium (i.e., when $\sigma^* = 1$), if the low-type issuer has a strict preference to seek a rating, small increases in m have no effect on either σ^* or α^*. If the manipulation cost increases by a small amount and the price of the high-rated security remains the same, the low-type issuer still strictly prefers to manipulate, so sets $\sigma^* = 1$. In turn, m can affect the CRA's first-order condition only through $\nu(\sigma^*)$, the conditional probability that an issuer applying for a rating is the high type. This probability does not change when σ^* is constant, so the issuer chooses the same value of α^* as before. In the next section, we note that this comparative static is quite different when the fee for a high rating, f, is endogenous rather than fixed.

Finally, consider a small increase in f, the fee for a high rating. We show in Appendix B that, in a partial-manipulation equilibrium, the screening intensity of the CRA, α^*, decreases when f increases. This is equivalent to rating inflation. In our model, the CRA has no incentive to set a high screening intensity and then misreport its signal, as it can obtain

the exact same combination of fee revenue and ex post penalties at a lower cost simply by reducing α appropriately. Rating inflation in our model therefore takes the form of the CRA setting a low screening intensity, rather than it directly misreporting its signal. A higher fee for a high rating results in more rating inflation and a greater proportion of incorrect ratings. As we note in Appendix B, the equilibrium effect on σ^* cannot be unambiguously signed. Numerically, we find that the price of a high-rated security decreases, and in many examples σ^* increases in f when f is low and decreases in f when f is high.

4   Rating Fee Set by Bargaining

It seems reasonable to treat a CRA's fees as fixed in the short run, as the fee schedule is generally standardized and is not negotiated transaction-by-transaction. However, in the long run, we expect the fee to adjust based on the surplus generated in the transaction between the issuer and the CRA.

In this section, we model the fee as the outcome of a Nash bargaining game between the CRA and issuer. We assume that the bargaining takes place (and the fee is set) before the issuer learns its type. The timing captures the intuition that the screening intensity of the CRA is based on a standard process it applies to all issuers. It also sidesteps the problematic effects of asymmetric information on the analysis of bargaining.

Let $\phi \in [0, 1]$ denote the bargaining power of the CRA, with $1 - \phi$ being the bargaining power of the issuer. The disagreement payoff for each party, issuer and CRA, is zero. Therefore, the surplus of each party relative to its disagreement payoff is its payoff if the two parties reach an agreement. Let $H(f) = \Pi(f)^{1-\phi} \, \Psi(f)^{\phi}$ denote the Nash product, where $\Pi(f)$ and $\Psi(f)$ are the issuer's and CRA's expected payoffs as functions of f, respectively. Formally, the rating fee f is the solution to the following problem:

$$\max_{f} \ \ H(f) = \Pi(f)^{1-\phi} \, \Psi(f)^{\phi} \tag{7}$$

The CRA's expected payoff if an agreement is reached, Ψ, is given by the expression in equation (1),

$$\Psi(f) = N(\sigma^*)\Big\{\nu(\sigma^*)f + [1 - \nu(\sigma^*)](1 - \alpha^*)(f - \lambda) - kc(\alpha^*)\Big\}, \tag{8}$$

where $N(\sigma^*) = \eta + (1 - \eta)\sigma^*$ is the mass of issuers seeking a rating in equilibrium. The issuer's expected payoff if agreement is reached depends on the probability that it will have a high-quality asset, and may be written as

$$\Pi(f) = \eta(p(\alpha^*, \sigma^*) - f) + (1 - \eta)\sigma^* \left[(1 - \alpha^*)(p(\alpha^*, \sigma^*) - f) - m\right]. \tag{9}$$

If the Nash product $H(f)$ is differentiable at the optimal value of f, the first-order condition is given by:

$$\phi \frac{\Psi'(f)}{\Psi(f)} = -(1 - \phi) \frac{\Pi'(f)}{\Pi(f)}. \tag{10}$$

Multiplying both sides by f, the optimal value of f equates the magnitudes of the bargaining-power-weighted elasticities of the payoffs of the CRA and issuer with respect to f. When the issuer has most of the bargaining power (low ϕ), the bargaining outcome will be favorable to the issuer in the sense that the CRA is not able to command a higher fee even though a small upward deviation in the fee would benefit the CRA more than it costs the issuer.

In many settings, Nash bargaining simply represents a division of surplus. However, in our setting, the fee over which the issuer and CRA bargain affects the behavior of the two parties in the screening subgame and therefore the size of the surplus as well. Therefore, the overall impact of the fee on the respective payoffs includes any indirect effects arising from the induced change in the equilibrium of the screening game, (α^*, σ^*). Specifically, we have

$$\Pi'(f) = \frac{\partial \Pi}{\partial f} + \frac{\partial \Pi}{\partial \sigma} \frac{d\sigma^*}{df} + \frac{\partial \Pi}{\partial \alpha} \frac{d\alpha^*}{df},$$

$$\text{and} \quad \Psi'(f) = \frac{\partial \Psi}{\partial f} + \frac{\partial \Psi}{\partial \sigma} \frac{d\sigma^*}{df} + \frac{\partial \Psi}{\partial \alpha} \frac{d\alpha^*}{df}.$$

We assume going forward that $\lambda + m < 1$. Under this assumption, there are three cases to consider, corresponding to different ranges of f. First, suppose that $f < \lambda$, which implies that $m < 1 - f$. In this case, as seen from Proposition 1 parts (i) and (iii), and equation (5), in equilibrium the CRA screens with positive intensity (i.e., $\alpha^* > 0$) and the low-type issuer manipulates with positive probability (i.e., $\sigma^* > 0$). When $f \in [\lambda, 1 - m)$, the CRA no longer finds it worthwhile to screen, since the fee it receives for assigning a high rating exceeds the penalty for an incorrect rating. Hence, $\alpha^* = 0$. The low-type issuer continues to manipulate with positive probability ($\sigma^* > 0$). Finally, when $f \geq 1 - m$, the low-type issuer no longer finds it worthwhile to manipulate, so $\sigma^* = 0$ and $\alpha^* = 0$. In this region, the price of the

high-rated security is one. It is worth noting that $f \in [1 - m, 1]$ implements the first-best outcome, because it eliminates costly manipulation and screening, and only positive-NPV projects are financed in equilibrium.

The indirect effect that f has on the payoffs of the two parties (through its impact on α^* and σ^*) complicates the maximization problem. As the following lemma shows, $H(f)$ may be non-monotone in f, exhibiting multiple local maxima.

**Lemma 4.** *(a) The Nash product $H(f)$ has a local maximum at $f = \max\{\phi, 1 - m\}$.*

(b) There exists a $\hat{\phi}$ such that, if $\phi < \hat{\phi}$, then the Nash product has an interior local maximum in the interval $(0, \lambda)$. In this case, the locally optimal fee f satisfies the first-order condition

$$\phi \frac{\Psi'(f)}{\Psi(f)} = -(1 - \phi) \frac{\Pi'(f)}{\Pi(f)}. \tag{11}$$

The intuition for part (a) is as follows. When $f > \lambda$, the CRA prefers to assign a high rating to the low-type issuer, so $\alpha^* = 0$. If, in addition, f is sufficiently close to $1 - m$, a partial manipulation equilibrium obtains, so that $\sigma^* \in (0, 1)$. In such an equilibrium, the expected payoff of the low-type issuer is zero. Further, $p - f = m$, so the ex ante payoff Π is invariant to f. However, the CRA's payoff strictly increases as σ^* decreases, due to the fall in the expected reputational cost. Hence, the Nash product increases in f when f is sufficiently close to $1 - m$. If $\phi < 1 - m$, then $f = 1 - m$ is a local maximizer of the Nash product. However, if $\phi > 1 - m$, then $H(f)$ continues to increase with f for values of f beyond $1 - m$. In this case, the Nash product is maximized at $f = \phi$.

Now, consider values of f such that $f < \lambda$ (part (b)). In this case, $\alpha^* > 0$. Further, α^* decreases as f increases. All else equal, the decrease in screening intensity decreases the issuer's expected payoff, because it decreases the price of a high-rated security. Of course, in equilibrium, σ^* responds as well. Taking this into account, we show in the proof of Lemma 4 that, as $f \to \lambda$ from below, $\Pi'(f) < 0$. If the issuer has a sufficiently high bargaining power (i.e., if ϕ is small enough), then the fall in the issuer's expected payoff with f outweighs the increase in the CRA's payoff, and the Nash product $H(f)$ also falls with f for values of f close to λ. In this case, $H(f)$ will have a local maximum at a value of f less than λ as well as one at $f = 1 - m$.

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We illustrate Lemma 4 with the following example. Set $\eta = 0.6, m = 0.3, \lambda = 0.2, k = 0.05$, and $c(\alpha) = \alpha^2$. Figure 2 shows the equilibrium values of α^* and σ^* (panel (a)) and of the Nash product (panel (b)) as the fee f varies. As can be seen, the Nash product has a maximum at approximately $f = 0.1$ and another one at $f = 1 - m = 0.7$.

(a)   (b)
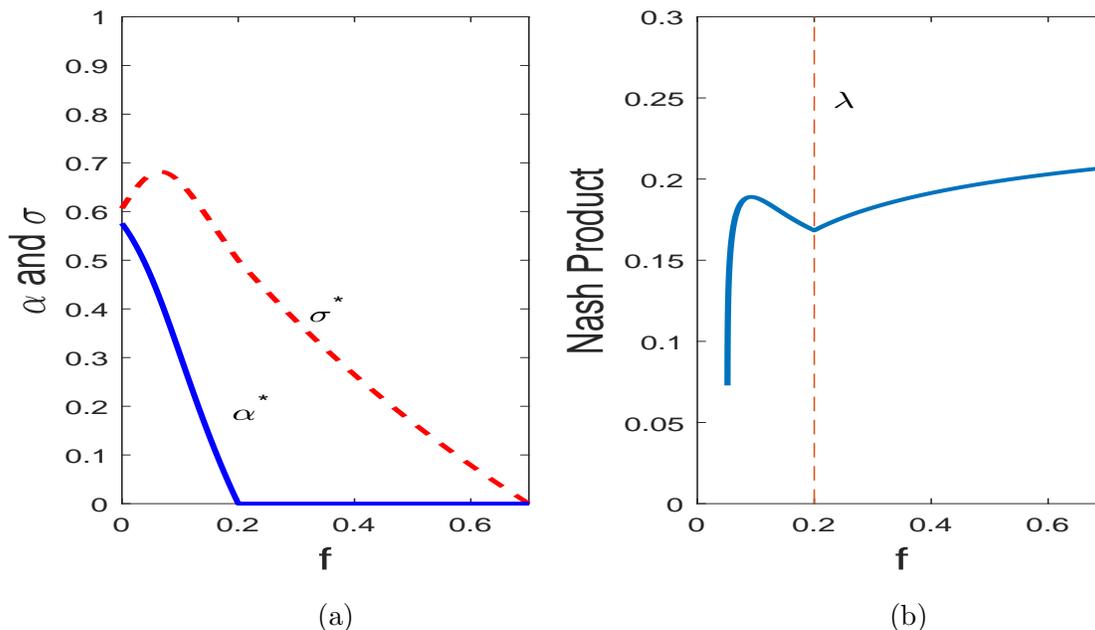
Panel (a) shows the equilibrium values of α^* and σ^* as f changes, and panel (b) the resulting Nash product. The parameter values are $\eta = 0.6, m = 0.3, \lambda = 0.2, k = 0.05$, and $c(\alpha) = \alpha^2$.

Figure 2: Effect of Fee on Nash Product

The global maximum of the Nash bargaining problem occurs at the local maximum that produces the highest value of $H(f)$. Since the exact values of the local maxima change continuously in the underlying parameters, small changes in the parameters may result in the global maximum shifting from one local maximum to another. This is then manifested as a discrete change in the fee, the screening intensity, and the manipulation probability as the parameters change.

4.1   Comparative statics with endogenous fee

We now consider how the equilibrium of the full game $(f^*, \alpha^*, \sigma^*)$ varies with the underlying parameters m, η, k, λ, and ϕ.

We begin with the manipulation cost m, and an example. Set $\eta = 0.6, \lambda = 0.2, \phi = 0.1, k = 0.05$, and $c(\alpha) = \alpha^2$. Figure 3 exhibits the equilibrium values of f, σ, and α at

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different values of m.
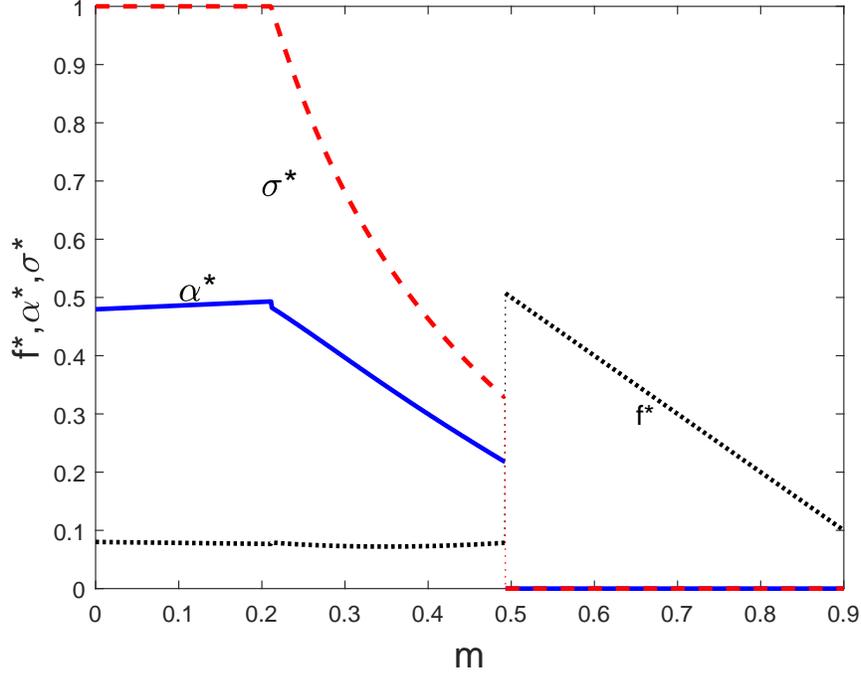
This figure illustrates the equilibrium values of the fee f, the low-type issuer's strategy σ, and the CRA's screening intensity α as the manipulation cost m changes. The other parameters are $\eta = 0.6, \lambda = 0.2, \phi = 0.1, k = 0.05$, and $c(\alpha) = \alpha^2$.

Figure 3: Equilibrium values of f, α, and σ as manipulation cost m changes

When $m \leq 0.21$ (approximately), the equilibrium features full-manipulation ($\sigma^* = 1$), with $f^* \approx 0.08$. For m between 0.21 and 0.48, the equilibrium features partial manipulation, with $\sigma^* \in (0, 1)$. Here, the equilibrium fee f^* remains approximately between 0.07 and 0.1, and less than the value of λ (0.2). In both cases, the equilibrium screening intensity α^* satisfies the CRA's first order condition as shown in Lemma 1. For $m \geq 0.48$, the fee jumps to $1 - m$, and the equilibrium features no-manipulation ($\sigma^* = 0$) and no screening ($\alpha^* = 0$).

Another feature of Figure 3 worth noting is that α^* increases with m when m is low enough to induce a full-manipulation equilibrium ($\sigma^* = 1$). As we comment on page 17, when f is fixed, a small change in m has no effect in a full-manipulation equilibrium, provided the low-type issuer strictly prefers to manipulate. In contrast, when f is endogenous, a small increase in m unambiguously increases the equilibrium screening intensity of the CRA, α^*.

Proposition 4. *Suppose that the fee f is endogenously determined and m is low enough that*

a full-manipulation equilibrium obtains. Then, $\frac{d\sigma^*}{dm} = 0$ and $\frac{d\alpha^*}{dm} > 0$. That is, the CRA's screening intensity increases with m. Further, the price of a high-rated security, p, increases with m, and the equilibrium rating fee, f^*, decreases with m.

While an increase in m does not affect σ^* in a full-manipulation equilibrium, it decreases the issuer's ex ante payoff. Hence, the bargaining at the initial stage results in a lower equilibrium rating fee f^*. As a result, the CRA screens more intensely. Finally, the fact that σ^* is unchanged whereas α^* has increased immediately implies that the price of the high-rated security, p, increases.

This result demonstrates the potential importance of accounting for the effects of changes in exogenous parameters on the bargaining outcome: With an exogenous rating fee, the CRA's strategy does not change with m. It also demonstrates that imposing penalties on the issuer can, in some circumstances, improve the overall quality of the screening process and reduce the incidence of ratings errors by lowering the benefit to the CRA of making such errors.

Our second comparative static result considers changes in η and augments Proposition 3.

Proposition 5. *When the fee f is endogenously determined,*

(i) *In a partial-manipulation equilibrium, $\frac{d\sigma^*}{d\eta} > 0$, $\frac{d\alpha^*}{d\eta} = 0$, and $\frac{dq(\alpha^*, \sigma^*)}{d\eta} = \frac{dp(\alpha^*, \sigma^*)}{d\eta} = 0$, as in Proposition 3 when f is taken as fixed. In addition, $\frac{df^*}{d\eta} = 0$. That is, small changes in η, the ex ante probability of the high type, have no effect on the equilibrium fee.*

(ii) *In a full-manipulation equilibrium, small changes in η lead to a decrease in α^*, an increase in p, and an increase in f^*.*

We illustrate this comparative static in the context of an example. Set $\lambda = 0.2, m = 0.3, \phi = 0.1, k = 0.05$ and $c(\alpha) = \alpha^2$. As seen in Figure 4, when $\eta < \frac{1}{\sqrt{2}} \approx 0.707$, the equilibrium features partial manipulation by the low-type issuer, with $\sigma^* \in (0, 1)$. In this region, σ^* increases with η, whereas α^* is flat in η. At first glance, the latter property is puzzling. As the expected quality of the issuer improves, one may expect that the need for screening declines. However, the endogenous response of the low-type issuer implies that the

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average quality of the pool that applies for a rating, in fact, remains constant as η increases in this range. In contrast, when $\eta > 0.707$, the equilibrium features full manipulation, with $\sigma^* = 1$. In this region, as expected, α^* decreases with η.
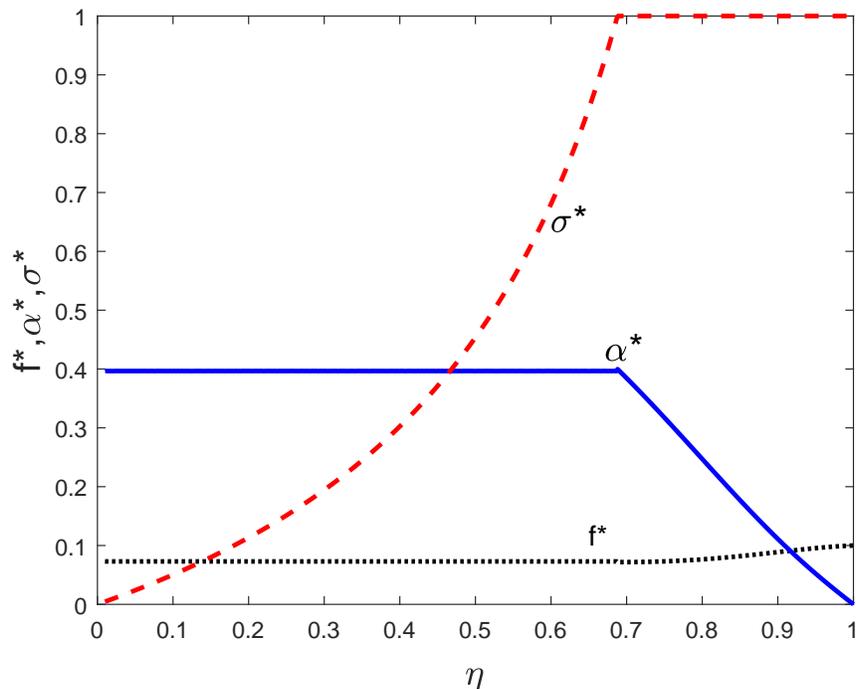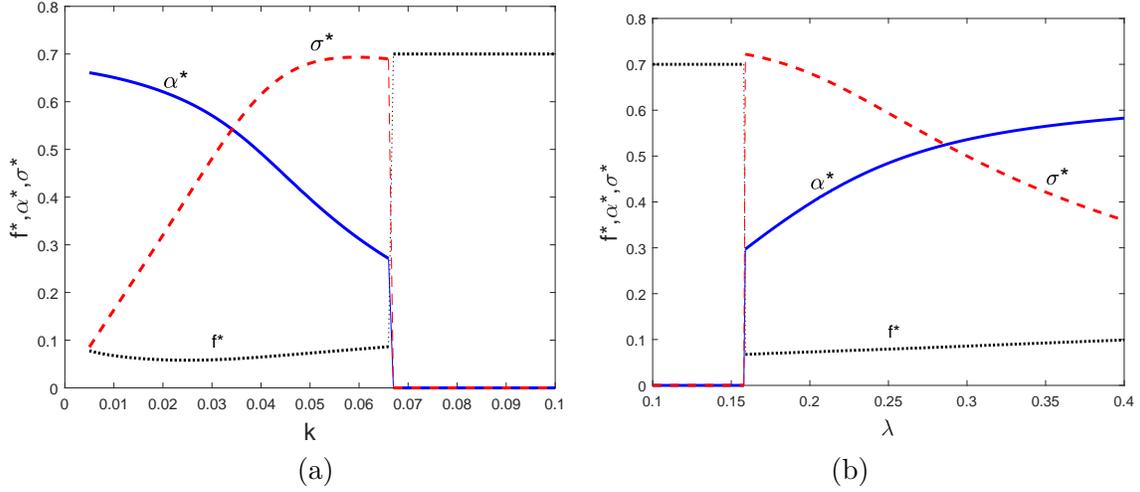

This figure illustrates the equilibrium values of the fee f, the low-type issuer's strategy σ, and the CRA's screening intensity α as η, the prior probability of a high-type issuer, changes. The other parameters are $\lambda = 0.2, m = 0.3, \phi = 0.1, k = 0.05$, and $c(\alpha) = \alpha^2$.

Figure 4: Equilibrium values of f, α, and σ as η, the prior probability of a high-type issuer, changes

We illustrate through examples how the rating fee f^* and equilibrium strategies of the low-type issuer (σ^*) and the CRA (α^*) vary with the exogenous parameters k, λ, and ϕ when the rating fee is determined endogenously. In these examples, we work with the base set of parameters $\eta = 0.6$, $\lambda = 0.2$, $m = 0.3$, $\phi = 0.1$, $k = 0.05$, and $c(\alpha) = \alpha^2$, unless otherwise noted. In each example, we vary one of the parameters and plot α^*, σ^*, and f^* as functions of that specific parameter. Figure 5 presents plots where we vary first (a) k and then (b) λ.

These figures provide a contrast to the results in Proposition 2. In Figure 5 (a), σ^* increases in k until k is approximately equal to 0.05, and then flattens out over the range $k \in (0.05, 0.065)$. For k above 0.065, the optimal value of f jumps discretely to $1 - m = 0.7$. At

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These figures illustrate the equilibrium values of the fee f, the low-type issuer's strategy σ, and the CRA's screening intensity α as k and λ vary. The other parameters are $\eta = 0.6, m = 0.3, \phi = 0.1$, and $c(\alpha) = \alpha^2$. For figure (a), $\lambda = 0.2$, and for figure (b), $k = 0.05$

Figure 5: Effect on α^* and σ^* as k and λ vary

this point, therefore, σ^* falls to zero and stays there for higher levels of k. From Proposition 2, if f were held constant, we would expect to see a continuous decrease in σ^* as k increased above 0.065.

A similar contrast occurs in Figure 5 (b), when λ varies. For low values of λ (below about 0.16), the optimal value of f is at $1 - m = 0.7$, so that we have zero manipulation. As λ increases above 0.16, f^* falls discretely to about 0.08. From this point on, σ^* decreases in λ.

Finally, Figure 6 presents a plot where we vary ϕ, again setting the other parameter values to those noted at the beginning of this section. An increase in ϕ increases the bargaining power of the CRA, which results in a higher rating fee f^*. As the fee for a high rating increases, the CRA's incentives to screen decline, resulting in a lower α^*. Recall that throughout, there remains a local optimum for f at $1 - m$ (in the example, $1 - m = 0.7$ remains well greater than the values of ϕ we consider). At ϕ approximately equal to 0.135, the equilibrium value of f jumps discretely to $1 - m$, with σ^* and α^* both equal to zero at this point.

5   Empirical Implications

This section discusses some empirical implications of our model. These implications relate to the model's predictions about the effects of four exogenous parameters on four outcomes.
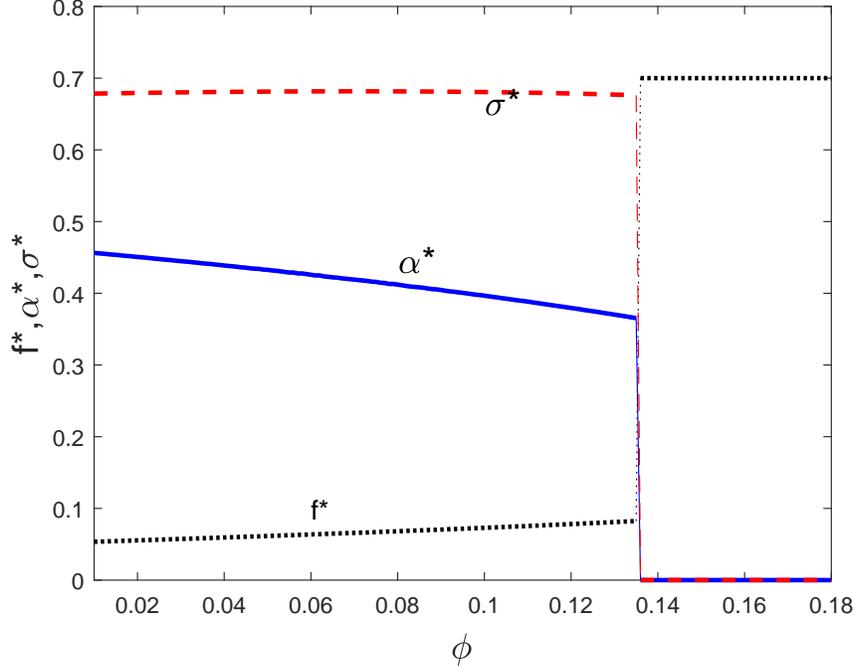
This figure illustrates the equilibrium values of the fee f, the low-type issuer's strategy σ, and the CRA's screening intensity α as ϕ, the bargaining power of the CRA, changes. The other parameters are $\eta = 0.6, \lambda = 0.2, m = 0.3, k = 0.05$, and $c(\alpha) = \alpha^2$.

Figure 6: Equilibrium values of f, α, and σ as ϕ, the bargaining power of the CRA, varies

The exogenous parameters are λ (the cost to the CRA of ratings errors), k (the CRA's cost of screening), m (the issuer's cost of manipulation), and η (the fraction of projects in the economy that are high quality). The four outcomes are α (the CRA's screening intensity), σ (the frequency of issuer manipulation), q (the frequency of ratings errors), and p (the price for a high-rated security).

We first discuss how the inputs and outputs may be measured. A comparative statics test would rely on changes in the inputs. A potentially measurable source of variation in λ is the severity of any explicit sanctions CRAs face for inaccurate ratings. Another is the importance to the CRA of maintaining a reputation for ratings quality. For example, the advent of the SEC's NSRSO designation in the United States is plausibly a positive shock to reputational costs, since it carries with it the threat of decertification. An increase in competition among CRAs may also drive down the value of maintaining a reputation. Along these lines, Becker and Milbourn (2011) find that ratings become more favorable and less accurate in response to

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an increase in competition due to the material entry of Fitch as a third major rating agency.

One might imagine capturing information about the direct cost of screening ($kc(\alpha)$) through measures of the opacity of the assets being rated. All else equal, more opaque assets are likely to be more difficult to evaluate, necessitating greater CRA effort to screen out lower-quality assets.

The manipulation cost m varies with at least two factors. First, the cost of manipulation is likely to be lower for newer securities (such as securities backed by subprime mortgages in the late 1990s and early 2000s), as the CRA (and the rest of the market) will learn about the features of a security over time. Second, manipulation is easier (and hence less costly) when a security is more complex. Expected manipulation costs should also increase with the ex post legal sanctions for committing fraud, the prosecutorial resources available to the regulator, and the willingness of courts to convict in fraud cases. For example, the 2002 Sarbanes-Oxley Act can be interpreted as a large positive shock to the cost of manipulating corporate debt ratings, as manipulation of accounting information is an important means of influencing CRA beliefs.[12]

In testing the model's predictions regarding changes in asset quality, one could imagine capturing asset quality at a broad level through measures of the state of the economy such as GDP or the aggregate market-wide Tobin's Q. One could also imagine measuring asset class-specific quality through current default rates in the asset class. Note that the model predicts the absence of a relationship between overall asset quality and both the incidence of ratings errors and the price of highly-rated securities. This predicted non-relationship represents a sharp prediction regarding the issuer's and CRA's strategic response to an increase in asset quality, as an improvement in asset quality should mechanically reduce ratings errors and increase the price of high-rated securities absent these strategic responses.

Turning to the outputs, screening intensity (α in the model) can be measured in several different ways: by the number of analysts working for a CRA per security rated, the CRA's expenses per security rated, and the length and detail of reports accompanying ratings. Higher σ in the model is naturally interpreted as a greater incidence of issuer manipulation. Manipulation is unobservable in real time in our model by assumption. However, ex-post prosecutions or regulatory actions for fraudulent reporting might be a useful indicator of the

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[12]As Begley (2015) shows, some firms even incur real costs (such as reducing R&D expenditures) to reduce their debt-to-EBITDA ratio in the year before issuing a new security, in an attempt to obtain a more favorable credit rating.

frequency with which such manipulation occurs.

The frequency of ratings errors (q) can be measured directly, at least ex post. For example, one can compare the default rate of securities in a given class to the stated default rate that a given rating is supposed to imply. In doing so, it would be useful to filter out the effects of factors affecting default rates such as the state of the economy that would have been difficult for a CRA to forecast at the time of the rating.

Finally, the price conditional on a high rating (p) could be measured as the inverse of the spread on investment-grade securities over the risk-free rate, which could then be compared across different settings or over time.

We divide our implications into two parts. First, we consider the rating fee f as fixed, as in Section 3. These implications are relevant for formulating short-run tests of the effects of factors that affect issuer and CRA behavior, as fees are unlikely to adjust immediately to small changes in such factors. Next, we consider long-run effects when the fee can adjust as well, as in Section 4.

5.1 Short-run implications

First, with a fixed fee, our model predicts that CRA screening intensity should rise with the cost to the CRA of ratings errors, while manipulation frequency should increase in response to a cost increase when the cost is low and decrease with a cost increase when the cost is high (Proposition 2). The first prediction is straightforward and would be common to most models of CRA screening. However, the second prediction follows from a more nuanced argument—in considering the issuer's manipulation decision, the price increase due to greater screening intensity can more than offset the direct effect of a reduction in the probability that manipulation succeeds, and it does so in particular when the cost to the CRA of a ratings error is low. Together, these predictions suggest that when the cost of ratings errors is low, we should observe both more manipulation and more intense screening in response to an increase in this cost.

As a practical consideration, testing the model's prediction of a non-monotone relationship between the frequency of manipulation and the cost of ratings errors would require observing this cost over a sufficiently large range to encompass the level at which the sign of the relationship changes. The model makes similar, though opposite, predictions about the relationships between the four outcomes described above and the cost of screening (k in the

model).

Second, the model also makes specific predictions about the variation in strategies and outcomes with m, the cost of manipulation. However, holding fixed the ratings fee, these predictions are straightforward and not distinctive features of our model. An increase in the cost manipulation generally results in less manipulation, to which the CRA responds by screening less intensely. The net effect is fewer ratings errors and a higher price for high-rated securities. As we describe next, the model yields more distinct predictions, at least in some cases, when the ratings fee is endogenously determined through bargaining.

5.2   Long-run implications

First, both in the long- and short-run, our model predicts that the overall incidence of manipulation does not change over a wide range of asset quality, as captured by the parameter η. More precisely, in a partial-manipulation equilibrium, the probability that a low-type issuer manipulates increases enough to perfectly offset the effect of a reduction in the fraction of low-quality assets on the overall rate of manipulation. The model also predicts that screening intensity, frequency of ratings errors, and price and quality of high-rated securities do not change with the underlying quality of assets.

It is surprising that the mix of high- and low-quality projects backing a high-rated security remains unchanged when the prior belief over asset quality improves. The key is that low-type issuers manipulate more often as the prior belief improves, in response to the indirect price improvement effect. This result therefore turns the focus back on issuers in explaining the mortgage-backed security market in the period before the 2008-09 financial crisis. As the volume of high-rated MBS increased in the 2000s, one could argue that prior beliefs over quality were improving. Increased manipulation by the low-type issuer would ensure that the pool of loans backing a high-rated security in fact was not improving in quality.[13] Thus, there was an increasing gap between the potential and actual quality of MBS securities in the build-up to the crisis.

Second, with an endogenously determined fee, CRA screening intensity may increase with the cost of manipulation (m) when that cost starts from a fairly low level (Proposition 4). The novel policy implication here is that imposing penalties on a firm for misreporting

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[13]Note that, with an improvement in the prior quality of loans, lax screening would also imply that additional low-quality loans would enter this pool.

information can improve the effort put in by the CRA as well.

Third, examination of Figure 6 suggests that an increase in CRA bargaining power (ϕ) in the issuer-pays model generally leads to less intense CRA screening but has ambiguous implications for the incidence of manipulation. Specifically, when the CRA has little bargaining power, an increase in that bargaining power causes an increase in the incidence of manipulation. However, when the CRA already has a strong bargaining position, a further increase in its bargaining power causes a decrease in the incidence of manipulation. One could measure ϕ directly by observing the level of CRA fees and profitability. In addition, variation in competition among CRAs over time or across markets would also provide a source of variation in CRA bargaining power vis-à-vis issuers and hence their share of proceeds.

Finally, the analysis in Section 4 yields predictions about the long-run responses of various outcomes to changes in different parameters that are sharp but may be difficult to test. Several of the figures in that section show discontinuities in α, σ, and f at threshold parameter values when f is determined endogenously. While identifying these specific thresholds in the data would be challenging, a sharp drop in manipulation incidence and screening intensity accompanied by an increase in rating fee following a small increase in the cost of manipulation (Figure 3) or cost of screening (Figure 5) would support the model. Similarly, a sharp rise in manipulation incidence and screening intensity and a fall in the rating fee following a small increase in the CRA's cost of ratings errors would also support the model.

6 Conclusion

We argue that strategic disclosure by issuers is an important friction to consider in the ratings process. Our broad message is that the quality of credit ratings depends on both the quality of screening and the type and disclosure strategy of the issuer. In our model, an endogenous increase in CRA screening intensity may be accompanied by either a decrease or increase in issuer manipulation in response. A decrease in manipulation magnifies the effect of increased screening intensity on ratings accuracy, while an increase in manipulation dampens the effect. In addition, the need to account for issuer behavior can undo the effects of shocks that might otherwise impact the quality of CRA screening.

When assessing periods during which the quality of ratings has been perceived to be low, it is important to remember that issuers are likely to know more about their own asset

qualities than a CRA. Any policy design intended to improve the overall ratings process must include providing incentives to issuers to truthfully report the quality of their assets as an important component.

Appendix A: Proofs

Proof of Lemma 1. As mentioned in the text, the payoff of the CRA is $\Psi = N(\tilde{\sigma})\{\nu f + (1-\nu)(1-\alpha)(f-\lambda) - kc(\alpha)\}$. Holding f fixed, the derivative with respect to α is $\frac{\partial \Psi}{\partial \alpha} = (1-\nu)(\lambda - f) - kc'(\alpha)$. If $\lambda \geq f$, the first-order condition implies that the CRA's optimal choice of α satisfies $c'(\alpha) = \frac{1-\nu}{k}(\lambda - f)$. As $c''(\alpha) > 0$, it follows immediately that the second-order condition for a maximum is satisfied.

If $\lambda < f$, we have $\frac{\partial \Psi}{\partial \alpha} < 0$. The optimal choice of α is therefore zero. $\square$

Proof of Lemma 2. Suppose the low-type issuer seeks a rating. Seeking a rating and not manipulating is equivalent to not seeking a rating (since the issuer always gets a low rating, and is therefore revealed to be a low type). If the issuer seeks a rating and manipulates its information, it obtains an expected payoff of $(1-\tilde{\alpha})(\tilde{p}-f)-m$. If it does not seek a rating, it does not issue a financial claim and obtains zero. The statement of the lemma follows immediately. $\square$

Proof of Proposition 1. (i) Suppose that $m \leq (1-\alpha_1)(p(\alpha_1, 1) - f)$, where α_1 is defined as $\alpha_1 = c'^{-1}\left(\frac{1-\eta}{k} \max\{\lambda - f, 0\}\right)$. If the CRA chooses $\alpha^* = \alpha_1$ and the low-type issuer chooses $\sigma^* = 1$ (i.e., seeks a rating with probability one), the price of the high-rated asset is $p(\alpha_1, 1)$. From Lemma 2, it follows that if the CRA chooses $\alpha^* = \alpha_1$, it is a best response for the low-type issuer to set $\sigma^* = 1$. When $\sigma^* = 1$, $\nu(\sigma) = \eta$, so from Lemma 1, it then follows that it is a best response for the CRA to set $\alpha^* = \alpha_1$. Thus, there is an equilibrium in which $(\alpha^*, \sigma^*) = (\alpha_1, 1)$. Further, α_1 is a best response only if $\sigma^* = 1$, and $\sigma = 1$ is a best response only if $m \leq (1 - \alpha^*)(p(\alpha^*, 1) - f)$. Therefore, in equilibrium, we can have $\alpha^* = \alpha_1$ only if $m \leq (1-\alpha_1)p(\alpha_1, 1) - f$.

(ii) Suppose that $m \geq 1 - f$. If the CRA chooses $\alpha^* = 0$ and the low-type issuer chooses $\sigma^* = 0$, the price of the high-rated security is 1. Now, from Lemma 2, it is a best response for the low-type issuer to not seek a rating if $m \geq 1 - f$. Further, when $\sigma^* = 0$, $\nu(\sigma^*) = 1$, so that from Lemma 1, the best response of the CRA is to set $\alpha^* = 0$. Therefore, there is an equilibrium in which $(\alpha^*, \sigma^*) = (0, 0)$. Further, an equilibrium with $\alpha^* = 0$ can exist only when $m \geq 1 - f$. At any lower value of m, $\sigma^* > 0$ when $\alpha^* = 0$, so that $\alpha^* = 0$ cannot be a best response.

(iii) Suppose that $m \in ((1-\alpha_1)(p(\alpha_1, 1) - f), 1 - f)$ and consider the conjectured equilibrium given by $\alpha = c'^{-1}\left(\frac{1-\nu(\sigma)}{k} \max\{\lambda - f, 0\}\right)$ and $\sigma = \frac{\eta}{1-\eta}\left(\frac{1-f-\frac{m}{1-\alpha}}{(1-\alpha)(1+f+\frac{m}{1-\alpha})}\right)$. The price of the high-rated security is then $p(\alpha, \sigma) = \frac{\eta-(1-\eta)\sigma(1-\alpha)}{\eta+(1-\eta)\sigma(1-\alpha)}$. From Lemma 1, it is immediate that α is a best response to σ. Suppose that the low-type issuer is indifferent between seeking a rating and not seeking a rating. Then, it must be that $(1 - \alpha)(p(\alpha, \sigma) - f) = m$, or that $(1 - \alpha)\left(\frac{\eta-(1-\eta)\sigma(1-\alpha)}{\eta+(1-\eta)\sigma(1-\alpha)} - f\right) = m$. Solving the last equation for σ yields the value of σ^* in the statement of the proposition. It is immediate that $\sigma^* > 0$. Further, $\sigma^* \geq 1$ implies that $m \leq (1 - \alpha_1)(p(\alpha_1, 1) - f$. Thus, when $m > (1 - \alpha_1)(p(\alpha_1, 1) - f)$, we have $\sigma^* < 1$. To complete the proof, note that given σ^*, the value of α^* must satisfy the CRA's first-order condition. $\qquad\square$

Proof of Lemma 3. Suppose that $f < \lambda$, and let (α^*, σ^*) denote the equilibrium values in a partial-manipulation equilibrium. Observe that $\nu(\sigma^*) = \left(1 + \left(\frac{1-\eta}{\eta}\right)\sigma^*\right)^{-1} = \left(1 + \frac{1-f-\frac{m}{1-\alpha^*}}{(1-\alpha^*)(1+f)+m}\right)^{-1}$, where the last equation uses the expression for σ^* in equation (4). Substituting this expression for $\nu(\sigma^*)$ into the RHS of equation (5) and re-writing the resulting equation, we have that α^* satisfies the condition $h(\alpha^*) = 0$, where

$$h(\alpha) = \left(2 - \alpha\left(1 + f + \frac{m}{1-\alpha}\right)\right)c'(\alpha) - \frac{\lambda - f}{k}\left(1 - f - \frac{m}{1-\alpha}\right). \qquad (12)$$

Observe that $h(0) < 0$. Thus, a sufficient condition for uniqueness is that whenever $h(\alpha) = 0$, we also have $h'(\alpha) > 0$. This condition is equivalent to

$$\left(2 - \alpha\left(1 + f + \frac{m}{1-\alpha}\right)\right)c''(\alpha) > \left(1 + f + \frac{m}{(1-\alpha)^2}\right)c'(\alpha) - \frac{\lambda - f}{k}\frac{m}{(1-\alpha)^2}. \qquad (13)$$

In a partial-manipulation equilibrium, it must be that $\alpha^* < 1 - \frac{m}{1-f}$ (otherwise $\sigma^* = 0$). Thus, it follows that $2 - \alpha\left(1 + f + \frac{m}{1-\alpha}\right) > \frac{2m}{1-f}$, so a sufficient condition for inequality (13) to hold is that

$$\left(\frac{2m}{1-f}\right)c''(\alpha) > \left(1 + f + \frac{m}{(1-\alpha)^2}\right)c'(\alpha) - \frac{\lambda - f}{k}\frac{m}{(1-\alpha)^2}. \qquad (14)$$

To obtain condition (6), observe that the equilibrium α^* satisfies $\frac{\lambda-f}{k} = \frac{c'(\alpha^*)}{1-\nu}$. Hence, inequality (14) can be written as $\left(\frac{2m}{1-f}\right)c''(\alpha) > \left(1 + f - \frac{\nu}{1-\nu}\frac{m}{(1-\alpha)^2}\right)c'(\alpha)$. Since $\nu > \eta$ in

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any partial-manipulation equilibrium and since $\frac{m}{(1-\alpha)^2} > m$, the RHS of this inequality is therefore strictly less than $\left(1 + f - \frac{\eta\, m}{1-\eta}\right) c'(\alpha)$. Therefore, inequality (13) is satisfied if

$$\left(\frac{2m}{1-f}\right) c''(\alpha) > \left(1 + f - \frac{\eta\, m}{1-\eta}\right) c'(\alpha). \tag{15}$$

This condition is equivalent to the one stated in the lemma. $\qquad\square$

Proof of Proposition 2. Let $y = \frac{\lambda - f}{k}$ and recall the definition of $h(\alpha)$ from equation (12) in the proof of Lemma 3. The Implicit Function Theorem implies that $\frac{d\alpha^*}{dy} = -\frac{\partial h/\partial y}{\partial h/\partial \alpha}$. Further, the second-order condition for α^* requires that $\frac{\partial h}{\partial \alpha} > 0$. By inspection, $\frac{\partial h}{\partial y} < 0$. Thus, it follows immediately that $\frac{d\alpha^*}{dy} > 0$, which implies that $\frac{d\alpha^*}{d\lambda} > 0$ and $\frac{d\alpha^*}{dk} < 0$.

Next, let
$$g(\alpha) = \frac{\eta}{1-\eta}\left(\frac{1 - f - \frac{m}{1-\alpha}}{(1-\alpha)(1+f) + m}\right). \tag{16}$$

From equation (4) in Proposition 1, it follows that in a partial-manipulation equilibrium we have $\sigma^* = g(\alpha^*)$. Further, $\frac{d\sigma^*}{dy} = \frac{\partial g}{\partial \alpha}\frac{d\alpha^*}{dy}$ since $\frac{\partial g}{\partial y} = 0$ when $y = \frac{\lambda - f}{k}$. From equation (16), we have

$$\frac{\partial g}{\partial \alpha} = \frac{\eta}{1-\eta}\left(\frac{1 - f^2 - \frac{2m}{1-\alpha}(1+f) - \frac{m^2}{(1-\alpha)^2}}{[(1-\alpha)(1+f) + m]^2}\right). \tag{17}$$

It is immediate to see that $\frac{\partial g}{\partial \alpha}$ is strictly decreasing in α. Further, $\frac{\partial g}{\partial \alpha}$ is strictly negative at $\alpha = 1 - \frac{m}{1-f}$ (the value of α at which σ^* becomes zero) and strictly positive at $\alpha = 0$ if $(f+m)^2 + 2m < 1$. Thus, if $(f+m)^2 + 2m < 1$, there exists a threshold $\hat{\alpha} \in (0, 1 - \frac{m}{1-f})$ such that $\frac{d\sigma^*}{dy} > 0$ for $\alpha < \hat{\alpha}$ and $\frac{d\sigma^*}{dy} < 0$ for $\alpha > \hat{\alpha}$. Since α^* increases in y and since y decreases in k, this implies that there exists a $\hat{k}$ such that $\frac{d\sigma^*}{dk} > 0$ for $k < \hat{k}$ and $\frac{d\sigma^*}{dk} < 0$ for $k > \hat{k}$. Similarly, since y increases in λ, there exists a $\hat{\lambda}$ such that $\frac{d\sigma^*}{d\lambda} > 0$ for $\lambda < \hat{\lambda}$ and $\frac{d\sigma^*}{d\lambda} < 0$ for $\lambda > \hat{\lambda}$. $\qquad\square$

Proof of Proposition 3. Consider the function $h(\alpha)$, as defined in equation (12) in the proof of Lemma 3. Recall that a partial-manipulation equilibrium is defined by $h(\alpha^*) = 0$. Thus, for any $x \in \{\eta, k, \lambda, m, f\}$, from the Implicit Function Theorem we have $\frac{d\alpha^*}{dx} = -\frac{\partial h/\partial x}{\partial h/\partial \alpha}$.

By inspection, $\frac{\partial h}{\partial \eta} = 0$, so it follows that $\frac{d\alpha^*}{d\eta} = 0$.

To calculate $\frac{d\sigma^*}{d\eta}$, recall from equation (4) in Proposition 1 that

$$\sigma^* = \frac{\eta}{1-\eta}\left(\frac{1-f-\frac{m}{1-\alpha^*}}{(1-\alpha^*)(1+f)+m}\right). \tag{18}$$

It follows that, for any $x \in \{\eta, k, \lambda, m, f\}$, we have $\frac{d\sigma^*}{dx} = \frac{\partial\sigma^*}{\partial x} + \frac{\partial\sigma^*}{\partial\alpha^*}\frac{d\alpha^*}{dx}$. As $\frac{d\alpha^*}{d\eta} = 0$ and (by inspection) $\frac{\partial\sigma^*}{\partial\eta} > 0$, it is immediate that $\frac{d\sigma^*}{d\eta} > 0$.

Finally, note that in a partial-manipulation equilibrium, it must be that $(1-\alpha^*)(p-f) = m$. For small changes in the exogenous parameters, the equilibrium continues to feature partial manipulation, so this equation must continue to hold. As α^* is invariant to η, it follows that p must also remain constant for small changes in η. $\qquad\square$

Proof of Lemma 4. (a) Consider values of f close to but strictly less than $1-m$. From Proposition 1, for some $f_1 < 1-m$, whenever $f > f_1$ we have $\sigma^* \in (0,1)$. Then, the indifference condition of the low-type issuer implies that $p-f = m$, or $f = p-m$. Observe that when $\alpha^* = 0$, we can write the ex ante payoff of the issuer as $\Pi = \eta(p-f)+(1-\eta)\sigma(p-f-m) = \eta(p-f) = \eta m$ when $\sigma \in (0,1)$. Thus, over the region $(f_1, 1-m]$, Π is invariant to f.

Now, the payoff of the CRA may be written as $\Psi = \left(\eta + (1-\eta)\sigma\right)(p-m) - (1-\eta)\sigma\lambda$. When $\alpha^* = 0$, we have $p = \frac{\eta-(1-\eta)\sigma}{\eta+(1-\eta)\sigma}$. Thus, $\Psi = \eta(1-m) - (1-\eta)\sigma(1+\lambda-m)$. When $\lambda < 1-m$, it follows that $m < 1+\lambda$. Therefore, Ψ is strictly decreasing in σ, and hence is maximized at $\sigma = 0$. When $\alpha^* = 0$, to induce $\sigma = 0$, it must be that $f = 1-m$. Hence, over the region $[f_1, 1-m]$, the Nash product $H(f)$ is maximized at $f = 1-m$.

Now, consider values of $f > 1-m$. At these values, from Proposition 1, we have $\sigma = 0$ and $\alpha = 0$. Therefore, for f in this region, we have $\Psi = \eta f$, and $\Pi = \eta(1-f)$. From the Nash maximization problem, it now follows that the optimal value of f is $\max\{\phi, 1-m\}$.

Hence, the Nash product has a local optimum at $f = \max\{\phi, 1-m\}$.

(b) For f close to zero, it follows that $\Psi < 0$, so the Nash product is undefined. At $f = \lambda$, we have $\alpha^* = 0$, so that $\Psi(f) > 0$. It is immediate that there must be a minimal level of f, which we denote $\underline{f}$, such that $\Psi \geq 0$ if and only if $f \geq \underline{f}$. Therefore, a sufficient condition to ensure that $H(\cdot)$ has an interior maximum on $(0,\lambda)$ is that $\lim_{f\to\lambda^-} H'(f) < 0$.

Now, observe that

$$H'(f) = (1 - \phi) \left[\frac{\Psi(f)}{\Pi(f)} \right]^{\phi} \Pi'(f) + \phi \left[\frac{\Pi(f)}{\Psi(f)} \right]^{1-\phi} \Psi'(f) \qquad (19)$$

Thus, when $\Psi > 0$ and $\Phi > 0$, the condition $H'(f) < 0$ is equivalent to

$$(1 - \phi) \frac{\Pi'(f)}{\Pi(f)} + \phi \frac{\Psi'(f)}{\Psi(f)} > 0. \qquad (20)$$

Now, consider the CRA's expected payoff Ψ, shown in equation (8), as $f \to \lambda^-$. When $\alpha^* > 0$ (which will obtain as long as $f < \lambda$ and $\sigma^* > 0$), using the CRA's first-order condition $kc'(\alpha) = (1 - \nu)(\lambda - f)$ and the definition of $\nu(\sigma^*)$, we can write

$$\Psi(f) = \eta f - kN(\sigma^*)[(1 - \alpha^*)c'(\alpha^*) + c(\alpha^*)] \qquad (21)$$

Thus, $\Psi'(f) = \eta - k \left\{ N(\sigma^*)(1 - \alpha^*)c''(\alpha^*) \frac{d\alpha}{df} - N'(\sigma^*)[(1 - \alpha)c'(\alpha^*) + c(\alpha^*)] \right\} \frac{d\alpha^*}{df}$. Now, differentiating the CRA's first-order condition, we have $kc''(\alpha) \frac{d\alpha}{df} = -(1 - \nu(\sigma^*)) - (\lambda - f)\nu'(\sigma^*) \frac{d\sigma}{df}$.

Now, when $f \to \lambda^-$, in the limit $\alpha^* = 0$. Further, let σ_λ denote denote the equilibrium value of σ^* in the screening game when $f = \lambda$. Making the relevant substitution and taking the limit as $f \to \lambda^-$, we obtain

$$\lim_{f \to \lambda^-} \Psi'(f) = \eta + (1 - \eta)\sigma_\lambda = N(\sigma_\lambda), \qquad (22)$$

Finally, note that $\lim_{f \to \lambda^-} \Psi(f) = \eta\lambda$. Therefore, $\lim_{f \to \lambda^-} \frac{\Psi'(f)}{\Psi(f)} = \frac{\lambda}{\nu(\sigma_\lambda)}$.

For the term $\frac{\Pi'(f)}{\Pi(f)}$ in condition (20), there are two cases to consider. First, suppose that $\lambda + m > 2\eta - 1$. Then, from equation (4) in Proposition 1, it follows that $\sigma_\lambda = \frac{\eta}{1-\eta} \frac{1-\lambda-m}{1+\lambda+m} < 1$.

Now, observe that $\sigma^* < 1$ implies that $(1 - \alpha^*)(p(\alpha^*, \sigma^*) - f) = m$ (else the low type is not indifferent between manipulating and not). Therefore, whenever $\sigma^* < 1$, the expected surplus of the issuer is $\Pi(f) = N(\sigma)^* \nu(\sigma^*)(p - f) = \frac{\eta m}{1-\alpha^*}$. Hence, $\Pi'(f) = \frac{\eta m}{(1-\alpha^*)^2} \frac{d\alpha^*}{df}$. Further, totally differentiating the CRA's first-order condition,

$$\frac{d\alpha^*}{df} = -\frac{[1 - \nu(\sigma^*)] + \nu'(\sigma^*)(\lambda - f)\frac{d\sigma^*}{df}}{kc''(\alpha^*)} \qquad (23)$$

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Putting all this together, in the limit as $f \to \lambda^-$, we obtain:

$$\lim_{f \to \lambda^-} \Pi'(f) = -\eta m \frac{1 - \nu(\sigma_\lambda)}{kc''(0)}. \tag{24}$$

Further, $\lim_{f \to \lambda^-} \Pi(f) = \eta m$, so that $\lim_{f \to \lambda^-} \frac{\Pi'(f)}{\Pi(f)} = -\frac{1-\nu(\sigma_\lambda)}{kc''(0)}$.

Making all the appropriate substitutions, when $\lambda + m > 2\eta - 1$ we have $\lim_{f \to \lambda^-} H'(f) < 0$ if and only if $-(1 - \phi) \frac{1-\nu(\sigma_\lambda)}{kc''(0)} + \phi \frac{1}{\nu(\sigma_\lambda)\lambda} < 0$, or

$$\phi < \frac{\lambda\nu(\sigma_\lambda)[1 - \nu(\sigma_\lambda)]}{\lambda\nu(\sigma_\lambda)[1 - \nu(\sigma_\lambda)] + kc''(0)} \tag{25}$$

Now, from $\sigma_\lambda = \frac{\eta}{1-\eta} \frac{1-(\lambda+m)}{1+(\lambda+m)}$, it follows that $\nu(\sigma_\lambda) = \frac{1+m+\lambda}{2}$ and so $\nu(\sigma_\lambda)(1 - \nu(\sigma_\lambda)) = \frac{1-(m+\lambda)^2}{4}$. Substituting this expression into (25) and simplifying gives the following condition for $\lim_{f \to \lambda^-} H'(f) < 0$:

$$\phi < \frac{\lambda[1 - (m + \lambda)^2]}{\lambda[1 - (m + \lambda)]^2 + 4kc''(0)}. \tag{26}$$

Denote the right-hand side as ϕ_1.

Next, suppose that $m + \lambda \le 2\eta - 1$, so that $\sigma_\lambda = 1$. In this case, we can write $\Pi(f) = R(\alpha^*)(p - f) - (1 - \eta)m$, where $R(\alpha^*) = n + (1 - \eta)(1 - \alpha^*)$ is the proportion of issuers who succeed on obtaining a high rating. Thus, $\Pi'(f) = (\frac{\partial p}{\partial \alpha} \frac{d\alpha^*}{df} - 1)R - (1 - \eta)(p - f)\frac{d\alpha^*}{df}$. Now, observe that $p = \frac{2\eta - R}{R} = \frac{2\eta}{R} - 1$. Thus, $\frac{\partial p}{\partial \alpha} = \frac{2\eta(1-\eta)}{R^2}$. Therefore, we have $\Pi'(f) = -R + (1-\eta)(1+f)\frac{d\alpha^*}{df}$. When $\sigma^* = 1$, differentiation of the CRA's first-order condition yields $\frac{d\alpha^*}{df} = -\frac{1-\eta}{kc''(\alpha^*)}$. Making the relevant substitutions, and recognizing that $\alpha^* \to 0$ and $p \to 2\eta - 1$ as $f \to \lambda^-$, we obtain $\lim_{f \to \lambda^-} \frac{\Pi'(f)}{\Pi(f)} = -\frac{1 + \frac{(1-\eta)^2(1+\lambda)}{kc''(0)}}{2\eta - 1 - \lambda - (1-\eta)m}$. Denote $Z = \frac{1 + \frac{(1-\eta)^2(1+\lambda)}{kc''(0)}}{2\eta - 1 - \lambda - (1-\eta)m}$. Recalling that $\lambda + m \le 2\eta - 1$, it follows that $Z > 0$.

Now, the condition for $H'(f) < 0$ is equivalent to $-(1 - \phi)Z + \phi\frac{\lambda}{\eta} > 0$, or $\phi < \frac{Z}{Z + \frac{\lambda}{\eta}} \overset{d}{=} \phi_2$.

Define $\hat\phi = \phi_1$ if $\lambda + m > 2\eta - 1$ and $\hat\phi = \phi_2$ otherwise. Then, it follows that when $\phi < \hat\phi$, the Nash product $H(f)$ has an interior optimum on $(0, \lambda)$. It follows immediately that the first-order condition $H'(f) = 0$, or $\phi\frac{\Pi'(f)}{\Pi(f)} = -(1 - \phi)\frac{\Psi'(f)}{\Psi(f)}$, is satisfied at this optimum. $\qquad \square$

Proof of Proposition 4. Suppose we are in a full-manipulation equilibrium, so that $\sigma^* = 1$. Recall that $\nu(1) = \eta$. Consider a small increase in m. For a sufficiently small change, it continues to be the case that $\sigma^* = 1$ in the new equilibrium. However, keeping α fixed, the

expected surplus of the issuer falls, as the payoff of the low type decreases in m. Therefore, f^* must fall. Further, the first-order condition of the CRA, given by $kc'(\alpha^*) = (1 - \eta)(\lambda - f^*)$, implies that as f^* falls, α^* increases. As α^* increases, and σ^* remains at 1, it is immediate that the price p must increase. $\qquad\square$

Proof of Proposition 5. (i) As in the case with a fixed rating fee f, the screening intensity α^* in a partial-manipulation equilibrium has to satisfy the condition $h(\alpha^*) = 0$, where $h(\alpha)$ is defined in equation (12) (with $f = f^*$). In addition, α^* and f^* have to satisfy the equation $\phi\Pi\frac{d\Psi}{df} = -(1 - \phi)\Psi\frac{d\Pi}{df}$. Note that $h(\alpha)$ does not depend on η; further, f^* depends on η only through $\nu(\sigma^*)$. Recall from the proof of Lemma 3 that in a partial-manipulation equilibrium $\nu(\sigma^*)$ can be expressed as $\nu(\sigma^*) = \left(1 + \frac{1 - f - \frac{m}{1-\alpha^*}}{(1-\alpha^*)(1+f)+m}\right)^{-1}$. This means that the effect of a (small) increase in η on ν is offset by an increase in σ^*, leaving ν unchanged. Of course, if ν is unchanged, so are α^*, f^*, and $p(\alpha^*, \sigma^*)$.

(ii) Consider a full-manipulation equilibrium. Since $\bar{m}$ is increasing in η, an increase in η means that we continue to have $\sigma^* = 1$. The equilibrium screening intensity satisfies $kc'(\alpha^*) = (1 - \eta)(\lambda - f^*)$. As η increases, keeping σ^* fixed at 1, ν increases. Therefore, α^* decreases. Nevertheless, the increase in η implies that p increases, and in turn f^* increases. $\qquad\square$

Appendix B: Additional Comparative Statics with Fixed f

In this section, we consider the comparative statics of α^* and σ^* for the case in which the rating fee f is fixed, as the manipulation cost m and rating fee f vary. Suppose throughout that we have a partial-manipulation equilibrium, and suppose further that $f < \lambda$.

First, consider changes in f. Following the technique used in the proof of Proposition 2, observe that when $y = \frac{\lambda - f}{k}$, it is immediate that $\frac{dy}{dk} < 0$. As $\frac{d\alpha^*}{dy} > 0$ in a partial-manipulation equilibrium, it follows that $\frac{d\alpha^*}{df} < 0$.

Now, as in the proof of Proposition 2, we have $\frac{d\sigma^*}{df} = \frac{\partial g}{\partial f} + \frac{\partial g}{\partial \alpha}\frac{d\alpha^*}{df}$, where $g(\alpha) = \frac{\eta}{1-\eta}\left(\frac{1-f-\frac{m}{1-\alpha}}{(1-\alpha)\left(1+f+\frac{m}{1-\alpha}\right)}\right)$. It is immediate that $\frac{\partial g}{\partial f} < 0$. Thus, if $\frac{\partial g}{\partial \alpha} > 0$ (i.e., when $(f+m)^2 + 2m < 1$ and α is small), it follows that $\frac{d\sigma^*}{df} < 0$. However, if α is large or $(f+m)^2 + 2m > 1$, then $\frac{\partial g}{\partial \alpha} < 0$, so it is not immediate to determine the sign of $\frac{d\sigma^*}{df}$.

Next, consider changes in m, the manipulation cost for the low-type issuer. We have $\frac{\partial h}{\partial m} = \frac{-\alpha c'(\alpha) + \frac{\lambda - f}{k}}{1-\alpha}$. Recalling that α^* is defined by the equation $c'(\alpha^*) = \frac{\lambda - f}{k}(1 - \nu(\sigma^*(\alpha^*)))$, we have $\frac{\partial h}{\partial m} = \frac{\lambda - f}{k(1-\alpha)}(-\alpha(1-\nu)+1) = \frac{\lambda - f}{k(1-\alpha)}(1 - \alpha + \nu\alpha) > 0$. As $\frac{\partial h}{\partial \alpha} > 0$, it follows that $\frac{d\alpha^*}{dm} = -\frac{\partial h/\partial m}{\partial h/\partial \alpha} < 0$.

Now, observe that α^* and σ^* must satisfy the equation $c'(\alpha^*) = \frac{\lambda - f}{k}(1 - \nu(\sigma^*))$. Consider a small increase in m. As α^* decreases, so does $c'(\alpha^*)$. Therefore, it must be that $\nu(\sigma^*)$ increases, which in turn implies that σ^* decreases. Therefore, $\frac{d\sigma^*}{dm} < 0$. ∎

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