Portfolio Choice with Capital Gain Taxation and the Limited Use of Losses *

Paul Ehling†
BI Norwegian School of Management

Michael Gallmeyer‡
McIntire School of Commerce
University of Virginia

Sanjay Srivastava§

Stathis Tompaidis¶
McCombs School of Business
University of Texas at Austin

Chunyu Yang‖
McCombs School of Business
University of Texas at Austin

Current Draft: July 2010

*We would like to thank Victor DeMiguel, Pascal François, Lorenzo Garlappi, Urban Jermann, Spencer Martin, Jeffery Pontiff, Neng Wang, Alan White, Avi Wohl, Amir Yaron, Harold Zhang, and participants at George Washington University, Tel Aviv University, the University of Houston, the University of Stavanger, the University of Virginia, the Wharton School of the University of Pennsylvania, a BI-Oslo brownbag, an EIEF brownbag, the 2007 Bank of Canada/University of Toronto Workshop on Portfolio Management, the 2008 McGill Risk Management Conference, the 2008 Symposium on Stochastic Dynamic Models in Finance and Economics at the University of South Denmark, the 2008 Western Finance Association Meeting, the 2008 Society of Economic Dynamics Meeting, the 2008 European Finance Association Meeting, and the 2008 University of British Columbia Summer Conference for useful discussions. Paul Ehling acknowledges financial support (små driftsmidler) from the Research Council of Norway. We also thank the Texas Advanced Computing Center for providing computing resources.

†paul.ehling@bi.no
‡Corresponding author. McIntire School of Commerce, University of Virginia, Charlottesville, VA 22904, Tel.: 434-260-1498, Email: mgallmeyer@virginia.edu
§srivastavafts@gmail.com
¶Stathis.Tompaidis@mccombs.utexas.edu
‖chunyu.yang@phd.mccombs.utexas.edu

Abstract

Portfolio Choice with Capital Gain Taxation and the Limited Use of Losses

We study the consumption-portfolio problem with realized capital gain taxation and an important feature of real-world tax codes: that capital losses can only be used against capital gains. We find that this feature, which we call the limited use of losses (LUL), has striking implications for asset allocation and rebalancing of portfolios. In particular, when embedded capital gains are large, a capital lock-in effect dominates and makes it costly for the investor to trade out of a large equity position. As a result, investors in down markets hold significantly less equity than in up markets, and this creates a time-varying and path-dependent optimal equity holding that looks like increased risk aversion in down markets. These results contrast with intuition derived from existing work which assumes that use of losses is unrestricted, termed the full use of losses (FUL). Equity holdings are similar between FUL and LUL if investors have large embedded capital gains. Otherwise, even if embedded gains/losses are small, investors in an LUL world hold significantly less equity than in an FUL world, and the difference is most significant with large embedded losses. With FUL, rebates generated from capital losses artificially inflate the demand for equity, in fact even to levels above the no tax benchmark. We also show that the FUL case can lead to counterintuitive results; for example, an FUL investor can actually prefer paying capital gain taxes than being untaxed.

Keywords: time-varying portfolio choice, capital gain taxation, limited use of capital losses

JEL Classification: G11, H20

1 Introduction

Capital gain taxation is an important friction faced by taxable investors when making asset allocation decisions. However, integrating it into a portfolio choice setting is notoriously difficult. First, capital gain taxes are realization-based, or assessed only when a trading position is closed implying the tax behaves like a state-dependent transaction cost.[1] The complexity of the state dependency arises as the tax is driven by current security prices and the portfolio's tax basis, which itself is a complex function of past trading decisions. Second, in most countries, the portion of the tax code involving capital gain taxation is full of a myriad of details such as the treatment of long-term versus short-term capital gains, the computation of the tax basis, the taxation of different risky securities including derivatives, and how capital gain taxes are computed at an investor's death.

Given these complexities, theoretical work studying portfolio choice with capital gain taxation typically adopts the most significant features of the tax code and assumes the other unmodeled features are of secondary importance. One real-world feature of the tax code that has received little attention is that the use of capital losses is limited, termed the *limited use of losses* (LUL) throughout.[2] In most tax codes, capital losses can only be used to offset current or future realized capital gains. Instead, it is commonly assumed that the use of capital losses is not restricted, termed the *full use of capital losses* (FUL) throughout. If capital losses are larger than capital gains in a period, the investor receives a tax rebate that cushions the downside of holding equity. We find that even in the standard consumption-portfolio problem with multiple stocks, the limited use of losses has striking implications for asset allocation and rebalancing of portfolios as tax rebates from the full use of losses understates the "capitalization effect" of a capital gain tax.[3]

To easily compare our analysis with past work, we modify the single stock setting of Dammon, Spatt, and Zhang (2001b) and the multiple stock setting of Gallmeyer, Kaniel, and Tompaidis (2006) to accommodate the limited use of capital losses. Essential to our work is that the investor cannot

[1] Capital gain taxation systems are rarely accrual-based, with New Zealand's taxation of corporate bonds a notable exception. For a comprehensive review of capital gain taxation in several OECD countries, see Clark (2006).

[2] Our work is motivated by Gallmeyer and Srivastava (2010) who study no arbitrage restrictions on after-tax price systems in the presence of no wash sales and the limited use of capital losses. To our knowledge, this was the first work that explored the limited use of capital losses in capital gain tax problems.

[3] Much of the empirical work on capital gain taxation can be organized around studying a demand-side "capitalization effect" in that a capital gain tax lowers the demand for equity, and a supply-side "lock-in effect" in that a capital gain tax lowers the effective supply of equity due to the unwillingness of investors with embedded capital gains to trade. See for example Dai et al. (2008) and the references therein. Our work demonstrates the interplay between these two effects in a theoretical portfolio choice setting.

perfectly offset all capital gain taxes as in the seminal work of Constantinides (1983). Indeed, based on provisions in tax codes such as the 1997 Tax Reform Act in the U.S. that ruled out "shorting the box" transactions[4] as well as empirical evidence summarized in Poterba (2002), investors do realize capital gains and hence pay capital gain taxes. The role of the limited use of losses is isolated by comparing with two natural benchmarks — a no capital gain tax portfolio problem and a full use of losses capital gain tax portfolio problem. From Dammon, Spatt, and Zhang (2001b), we also know that an investor's stock position can quickly lead to a large embedded gain which could partially cancel the impact of tax rebates. So, to fully assess the impact of the limited use of losses, we solve a long-horizon portfolio choice problem with an 80 year horizon and security price dynamics chosen to be largely consistent with empirical moments of U.S. large-capitalization stock indices. For tax rates, parameters consistent with the U.S. tax code as well as the tax codes in many European countries and Canada are used.

Beginning our analysis with a one stock consumption-portfolio problem, we find that imposing the limited use of capital losses sharply impacts the after-tax risk-return trade off of holding equity. When the investor's existing portfolio contains small embedded gains or losses, an LUL investor sharply reduces equity holdings relative to an untaxed investor. Due to possible future capital gain taxes, the relative attractiveness of equity to the money market is greatly reduced. If embedded capital losses grow in the existing portfolio, the LUL investor holds equity like an untaxed investor. With the accumulated capital losses, the LUL investor can optimally trade the untaxed investor's strategy with no tax consequences. When embedded capital gains are large, tax trading costs make it difficult for the LUL investor to trade out of a large equity position.

Tax rebates artificially impact an FUL investor's demand for equity however. When an FUL investor's portfolio is not embedded with a large capital gain, the probability of receiving tax rebates increases, leading to a higher demand for equity than even the untaxed investor. Tax rebates truncate the downside risk of holding equity inflating the demand understating the "capitalization effect." On the other hand, when accumulated capital gains are large, tax trading costs, like for an LUL investor, make it difficult for an FUL investor to rebalance to a lower equity position if overexposed to equity.

By correctly modeling the role of capital losses on the demand for equity, the LUL investor displays a time-varying demand for equity. Specifically, an LUL investor trading in a down market will trade to

[4] A "shorting the box" transaction involves realizing a capital gain with no tax consequences. This is achieved by taking an offsetting short position in the security that the investor would like to sell. Before the 1997 Tax Reform Act in the U.S., such a trade was not viewed as a sale of the security and not subject to capital gain taxation.

significantly lower equity holdings than in an up market. In other words, an LUL investor experiences large swings in equity demands between when the “capitalization effect” or the “lock-in effect” dominates. This effect is masked by tax rebates for an FUL investor. This simple tax-induced time-varying demand for equity reproduces behavior that looks like time-varying risk aversion as in habit formation models like Campbell and Cochrane (1999) and Chan and Kogan (2002). Given heterogeneity in capital gain tax codes through time and across countries, empirical work could potentially use this time-varying equity demand feature to better understand the economic forces behind the dynamics of stock market returns.

Considering the case when two stocks are traded, we find that this time-varying demand for equity is not mitigated for an LUL investor. Trading multiple stocks also does not hinder the artificial demand for equity driven by tax rebates for the FUL investor. Although a two stock portfolio generates scenarios with simultaneous capital gains and losses, we find that asymmetric trade occurs for stocks with embedded gains and losses. For stocks with capital losses, it is always optimal to liquidate the entire position to generate realized capital losses. For overinvested stocks with capital gains, any selling will be small to minimize realized capital gains. Combining these two types of trades leads to scenarios where realized losses are larger than realized gains. For FUL investors, this continues to generate tax rebates that artificially elevate optimal wealths and equity holdings relative to LUL investors.

From these differences in optimal trading strategies across the LUL and FUL investors in the one and two stock cases, the total equity exposure over the investor’s lifetime tends to be higher for the FUL investor. Additionally, the FUL investor’s wealth distribution over his lifetime is artificially higher given the ability to collect tax rebates. From an investor welfare perspective, we also document the cost of imposing each form of capital gain taxation on an untaxed investor. The tax rebate generated for an FUL investor generates a counterfactual result — an untaxed investor would actually prefer to pay capital gain taxes if the full use of losses were allowed. Such behavior is not exhibited if a capital gain tax is imposed with a limited use of losses. Under this form of capital gain taxation, no tax rebates are generated leading to the untaxed investor never preferring such a taxation scheme. Overall, these results are robust to a variety of different comparative static exercises.

Given the complexity of our portfolio optimization problem, we numerically solve it by extending the methodologies of Brandt et al. (2005) and Garlappi and Skoulakis (2008) to incorporate endogenous state variables and constraints on portfolio weights. Our two stock portfolio choice problem is a dynamic programming problem with five endogenous state variables plus time. Each stock contributes two endogenous state variables — that stock's equity-to-wealth ratio and its tax basis-to-price ratio. Since the state variable evolution is given by functions that are piecewise linear, the Bellman equation corresponds to a singular stochastic control problem solved by employing a domain decomposition of the state space. A full description of the technique used can be found in Yang (2010).

The novelty of our work is in analyzing capital gain taxation with the limited use of losses. Several other papers have examined portfolio choice with capital gain taxation when the use of capital losses is not restricted. When "shorting the box" trades are allowed, Constantinides (1983) shows that an investor can optimally defer all gains and immediately realize all losses without influencing his portfolio decision. Central to Constantinides' analysis is the valuation of the cash stream created from tax-loss selling, commonly called the tax-loss option. With no short-selling, Dybvig and Koo (1996) provide a numerical study of after-tax portfolio choice. Due to computational issues, they study the problem for a limited number of time periods. Later work, based on Dammon, Spatt, and Zhang (2001b), assumes the tax basis follows the weighted-average of past purchase prices as in this paper. By doing so, after-tax portfolio choice can be studied by numerical dynamic programming for longer horizons. This work includes studies with multiple stocks (Dammon, Spatt, and Zhang (2001a); Garlappi, Naik, and Slive (2001); Gallmeyer, Kaniel, and Tompaidis (2006)) and studies that explore investing simultaneously in taxable and tax-deferred accounts (Dammon, Spatt, and Zhang (2004)).

Other papers study a variety of issues pertaining to portfolio choice with capital gain taxation. Using numerical nonlinear programming techniques, DeMiguel and Uppal (2005) study the utility cost of using the weighted-average of past purchase prices as a tax basis compared to the exact share identification rule.[5] Bergstresser and Pontiff (2010) take a different approach by studying the after-tax returns of benchmark portfolios such as the Fama-French portfolios. In their setting, capital gain taxation is paid using the exact share identification rule. For exact solutions to capital gain tax portfolio problems under restrictive conditions, see Cadenillas and Pliska (1999), Jouini, Koehl, and Touzi (2000), and Hur (2001). For a theoretical analysis of the optimal location of assets between taxable and tax-deferred accounts, see Huang (2008) for the case of no portfolio constraints and

[5] As a consistency check of our two stock results, we use the same numerical algorithm as DeMiguel and Uppal (2005) to solve our limited use of losses portfolio problem for four periods with two stocks and for two periods with five stocks. Due to computational reasons, it is not possible to extend this algorithm to the 80 trading periods we consider. These results are consistent with the results we present.

Garlappi and Huang (2006) for the case with portfolio constraints. Again, all of this previous work assumes the use of capital losses is unrestricted.

One paper that does study the limited use of capital losses is concurrent work by Marekwica (2009), which we became aware of after preparing the first draft of our manuscript. His work only studies the single risky stock case and has a different focus than our own. His primary objective is to study how optimal trading strategies under the average purchase price basis rule are influenced when capital losses can be used to offset against other forms of taxable income. For example, in the U.S. tax code, $3,000 of taxable income per year can be offset using realized capital losses.

The paper is organized as follows. Section 2 describes the portfolio problem. Section 3 provides an example that highlights the intuition behind the role of the limited use of capital losses. A conditional analysis of optimal portfolios is presented in Section 4. Section 5 reports lifetime properties of the optimal portfolios, while Section 6 analyzes the economic costs of capital gain taxation under both the full use of losses and the limited use of losses. Section 7 concludes. Appendix A gives a thorough description of the problem studied. Appendix B discusses the numerical procedure used.

2 The Consumption-Portfolio Problem

The investor chooses an optimal consumption and investment policy in the presence of realized capital gain taxation at trading dates $t = 0, ..., T$. The framework is a multiple stock extension of the single risky asset model of Dammon, Spatt, and Zhang (2001b) based on Gallmeyer, Kaniel, and Tompaidis (2006) where we modify capital gain taxation to accommodate for the limited use of capital losses. Our assumptions concerning the exogenous price system, taxation, and the investor's portfolio problem are outlined below. The notation and model structure are based on the setting in Gallmeyer, Kaniel, and Tompaidis (2006). A full description of our partial equilibrium setting is given in Appendix A.

2.1 Security Market

The set of financial assets available to the investor consists of a riskless money market and multiple dividend-paying stocks. In particular, we consider scenarios where the investor's risky opportunity set consists of one to two stocks. The money market pays a continuously-compounded pre-tax rate of return r. The stocks pay dividends with constant dividend yields. The ex-dividend stock prices evolve as lognormal distributions.

2.2 Taxation

Interest income is taxed as ordinary income on the date that it is paid at the rate τ_I. Dividends are also taxed on the date that they are paid, but at the rate τ_D to accommodate for differences in taxation between interest and dividend income.

Our analysis centers around a feature of the tax code that has received little attention in the academic literature, namely that most capital gain tax codes restrict how realized capital losses are used. However, the most common assumption used in the portfolio choice literature is that there are no restrictions on the use of capital losses, which we term the *full use of capital losses* (FUL) case.

Definition 1 (Full Use of Capital Losses (FUL) Case)**.** *Under the full use of capital losses (FUL) case, an investor faces no restrictions on the use of realized capital losses. When realized capital losses are larger than realized capital gains in a period, the remaining capital losses generate a tax rebate that can be immediately invested.*

Definition 1 is assumed in several papers that study portfolio choice with capital gain taxes (Constantinides (1983); Dammon, Spatt, and Zhang (2001a,b, 2004); Garlappi, Naik, and Slive (2001); Hur (2001); DeMiguel and Uppal (2005); Gallmeyer, Kaniel, and Tompaidis (2006)). In particular, it is always optimal for an investor to immediately realize a capital loss to capture the resulting tax rebate.

Given most tax codes restrict the use of capital losses, our alternative form of realized capital gain taxation is referred to as the *limited use of capital losses* (LUL) case.

Definition 2 (Limited Use of Capital Losses (LUL) Case)**.** *Under the limited use of capital losses (LUL) case, an investor can only use realized capital losses to offset current realized capital gains. Unused capital losses can be carried forward indefinitely to future trading dates.*

Under the LUL case, we assume that the investor immediately realizes all capital losses even if they are not used. The no-arbitrage analysis in Gallmeyer and Srivastava (2010) shows that an investor is indifferent between realizing an unused capital loss or carrying it forward.

For tractability, our definition of the limited use of capital losses does not include the ability to use capital losses to offset current taxable income. In the U.S. tax code, individual investors can only offset up to $3,000 of taxable income per year with realized capital losses. Additionally, our analysis does not distinguish between differential taxation of long and short-term capital gains since our investors trade at an annual frequency. For such an analysis, see Dammon and Spatt (1996).

Under both the FUL and the LUL cases, realized capital gains and losses are subject to a constant capital gain tax rate of τ_C. When investors reduce their outstanding stock positions by selling, they incur realized capital gains or losses subject to taxation. The tax basis used for computing these realized capital gains or losses is calculated as a weighted-average purchase price.[6] In the FUL case, realized capital losses are treated as tax rebates, or negative taxes, for the investor. Hence, they lead to an increase in financial wealth when the loss is realized. In the LUL case, realized capital losses can only be used to offset current or future capital gains.

When an investor dies, capital gain taxes are forgiven and the tax bases of the stocks held in the investor's portfolio reset to the current market price. This is consistent with the reset provision in the U.S. tax code. Dividend and interest taxes are still paid at the time of death. We also consider the case when capital gain taxes are not forgiven which is consistent with the Canadian and many European tax codes. While investors can "wash sell" to immediately realize capital losses, they are precluded from shorting the stock which eliminates a "shorting the box" transaction to avoid paying capital gain taxes.[7] An imperfect form of "shorting the box" that involves trading in highly correlated, but different assets, is quantitatively studied in Gallmeyer, Kaniel, and Tompaidis (2006).

2.3 Investor Problem

To finance consumption, the investor trades in the money market and the risky stocks. The setting we have in mind is one where a taxable investor trades individual stocks or exchange traded funds (ETFs).[8] Given an initial equity endowment, a consumption and security trading policy is an *admissible trading strategy* if it is self-financing, involves no short selling of the stocks, and leads to nonnegative wealth over the investor's lifetime. The investor lives at most T periods and faces a positive probability of

[6]The U.S. tax code allows for a choice between the weighted-average price rule and the exact identification of the shares to be sold, while the Canadian and some European tax codes use the weighted-average price rule. While choosing to sell the shares with the smallest embedded gains using the exact identification rule is clearly most beneficial to the investor, solving for the optimal investment strategy becomes numerically intractable for a large number of trading periods given the dimension of the state variable increases with time (Dybvig and Koo, 1996; Hur, 2001; DeMiguel and Uppal, 2005). Furthermore, for parameterizations similar to those in this paper, DeMiguel and Uppal (2005) numerically show that the certainty-equivalent wealth loss using the weighted-average price basis rule as compared to the exact identification rule is small.

[7]We permit wash sales in our analysis as highly correlated substitute securities typically exist in most stock markets allowing an investor to re-establish a position with a similar risk-return profile after a capital loss. For an analysis of possible portfolio effects of wash sales when adequate substitute securities do not exist, see Jensen and Marekwica (2010).

[8]To isolate the role of the LUL assumption, we abstract away from investing in mutual funds where unrealized capital gain concerns can also be important. For a recent empirical study addressing such tax effects, see Ivković and Weisbenner (2009). Like mutual funds, ETFs must pass unrealized capital gains onto investors generated by portfolio rebalancing. However, many ETFs substantially reduce and in some cases eliminate unrealized capital gains to investors. This is achieved through a "redemption in kind" process described in Poterba and Shoven (2002).

death each period. The probability that an investor lives up to period $t < T$ is given by a survival function, calibrated to the 1990 U.S. Life Table, compiled by the National Center for Health Statistics where we assume period $t = 0$ corresponds to age 20 and period $T = 80$ corresponds to age 100. At period $T = 80$, the investor exits the economy with certainty.

The investor's objective is to maximize his expected utility of real lifetime consumption and a time of death bequest motive by choosing an admissible consumption-trading strategy given an initial endowment. The utility function for consumption and wealth is of the constant relative risk aversion form with a coefficient of relative risk aversion of γ. Using the principle of dynamic programming, the Bellman equation for the investor's optimization problem, derived in Appendix A, can be solved numerically by backward induction starting at time T. Given we solve a consumption and investment problem with multiple stocks and several endogenous state variables due to capital gain taxation under the LUL assumption, existing numerical solution approaches as described in Brandt et al. (2005), Gallmeyer, Kaniel, and Tompaidis (2006), and Garlappi and Skoulakis (2008) are ill-suited for our problem. Instead, we use a test region iterative contraction method. Additional details are provided in Yang (2010). The numerical solution of our problem is outlined in Appendix B.[9]

2.4 Scenarios Considered

Without capital gain taxation, rebalancing to the optimal risk-return trade off can be performed at no cost. However, under both the LUL and FUL assumptions, optimal portfolios will deviate from no capital gain tax benchmarks due to tax trading costs. Given a crucial part of our analysis is understanding how the LUL case influences portfolio choice across multiple stocks, we explore a two stock portfolio choice problem in addition to a one stock problem.

To disentangle the role of the LUL assumption on portfolio choice, we focus on two benchmark portfolio choice problems. One benchmark is the case when the investor faces no capital gain taxation, abbreviated NCGT. In this benchmark, the investor still pays dividend and interest taxes. Given the investment opportunity set is constant and the investor has CRRA preferences in this benchmark, the optimal trading strategy is to hold a constant fraction of wealth in each stock at all times. Second, we also use the FUL case as a benchmark to compare with the LUL case.

[9] The parallel computing code used to solve the portfolio choice problems is available from the authors. As a run-time benchmark based on our computing resources, the two asset LUL portfolio choice problem takes approximately 90 hours to solve using 100 CPUs in parallel.

In all our parameterizations, the investor begins investing at age 20 and can live to a maximum of 100 years. Hence, the maximum horizon for an investor is $T = 80$. The investor's constant relative risk aversion preferences are calibrated with a time discount parameter $\beta = 0.96$. The bequest motive is calibrated such that the investor plans to provide a perpetual real income stream to his heirs.

We trade off our desire to calibrate to realistic stock price returns and incorporate trading costs other than capital gain taxation with being able to easily disentangle the role of the LUL assumption on portfolio choice. Instead of calibrating to specific equity classes in our two stock problem, we parameterize to identically distributed, but not perfectly correlated, stocks using parameters that are consistent with a large capitalization U.S. exchange traded fund. We also abstract away from any other transaction costs than capital gain taxation given the magnitude of capital gain taxation is typically much larger than other trading costs and our desire to construct an NCGT benchmark free of the complications of a no-trade region induced by transaction costs. By parameterizing to identically distributed stocks, the benchmark NCGT two stock case leads to a setting with a 50 percent allocation of each stock in the risky portfolio. Any deviation from these weights is then driven only by capital gain taxation, making it easier to disentangle the effect of the LUL assumption on optimal portfolio choice.

We assume that the return dynamics of the aggregate stock market are as follows: the expected return due to capital gains is $\mu = 8\%$, the dividend yield is $\delta = 2\%$, and the volatility is $\sigma = 16\%$. We use these dynamics when we study a single stock portfolio choice problem. For all parameterizations, the money market's return is $r_f = 5\%$.

When we study a two stock portfolio choice problem, both stocks are assumed to have identical expected returns, dividend growths, and volatilities. We allow the return correlation to vary and report results for correlations $\rho = 0.4$, 0.8, and 0.9. To keep the pre-tax Sharpe ratio of an equally-weighted portfolio of these two stocks fixed across return correlations and equal to the aggregate stock market, we assume each stock's dynamics are $\mu_i = 8\%$, $\delta_i = 2\%$, and $\sigma_i = \frac{\sigma}{\sqrt{0.5(1+\rho)}}$.

Our base case choice of parameters, referred to throughout as the "Base Case," studies portfolio problems with one and two stocks using the security return parameters just described. For the two stock case, we assume $\rho = 0.8$. The tax rates used are set to roughly match those faced by a wealthy investor under the U.S. tax code. We assume that interest is taxed at the investor's marginal income rate $\tau_I = 35\%$. Dividends are taxed at $\tau_D = 15\%$. The capital gain tax rate is set to the long-term rate

$\tau_C = 20\%$.[10] To be consistent with the U.S. tax code, capital gain taxes are forgiven at the investor's death. The relative risk aversion coefficient is assumed to be $\gamma = 5$.

We also consider several variations of the Base Case parameters. An immediate way to increase the value of the FUL tax-loss selling option is to increase the capital gain tax rate. In the "Capital Gain Tax 30% Case," the capital gain tax rate is increased to $\tau_C = 30\%$ for both the one and two stock cases, roughly equal to the 28% rate imposed after the U.S. 1986 Tax Reform Act. This rate also provides a setting that is roughly consistent with the long-term capital gain tax rate paid in many European countries. For example, the capital gain tax rates in Finland, France, Sweden and Norway are currently 28%, 29%, 30%, and 28%, respectively. In 2009, Germany's individual capital gain tax rate rose to approximately 28% from 0%.[11]

The "Correlation 0.90 Case" and the "Correlation 0.40 Case" capture, in the two stock case, different diversification costs of not holding an equally-weighted risky stock portfolio. For space considerations, our other comparative statics are only reported for the one stock case. To capture a case where stock holdings decrease for the NCGT investor and hence the dollar value of tax-loss selling decreases for the FUL investor, the "Higher Risk Aversion Case" assumes that the relative risk aversion of the investor increases to $\gamma = 10$. Finally, given tax forgiveness at death is primarily a feature of only the U.S. tax code, the "No Tax Forgiveness at Death Case" assumes capital gain taxes are assessed when the investor dies, a feature consistent with Canadian and European tax codes.

3 A Two Date Example

Before numerically studying the consumption-portfolio problem outlined in Section 2, we analyze a two trading date example to highlight the role the limited use of capital losses plays in determining an investor's optimal trading strategy. Given the portfolio problem only lasts for two periods, this example conveniently allows us to follow the optimal trading path of the investor over time.

In this example which is a simplified version of the model in Section 2, the investor lives with

[10] The U.S. Tax Relief and Reform Act of 2003 changed several features of the tax code with respect to investments. In particular, the long-term capital gain tax rate dropped from $\tau_C = 20\%$ to $\tau_C = 15\%$ for most individuals. Dividend taxation switched from being linked to the investor's marginal income tax rate to a flat rate of $\tau_D = 15\%$. The 2006 Tax Reconciliation Act extended these rates to be effective until 2010. From 2011, these rates will generally revert to the rates effective before 2003 unless another tax law change is made. Given the high likelihood that the long-term capital gain tax rate will rise to $\tau_C = 20\%$ in 2011, we use that for our rate. For a comprehensive summary of U.S. capital gain tax rates through time, see Figure 1 in Sialm (2009).

[11] The German capital gain tax rate is 25% plus a church tax and tax to finance the five eastern states of Germany. The total tax rate is approximately 28%.

probability one until $T = 2$ and maximizes the expected utility of final period wealth over CRRA preferences with a coefficient of relative risk aversion equal to 5. The investor trades in one non-dividend paying stock and a riskless money market. Over the investor's lifetime, he pays taxes on the money market's interest payment as well as capital gain taxes on the stock. At time $T = 2$, the portfolio is liquidated and the investor consumes the after-tax wealth. To isolate the effect of the limited use of capital losses, no capital gain tax liabilities are forgiven at time $T = 2$.[12] The investor is initially endowed with one share of stock with a pre-existing tax basis-to-price ratio, $b(0)$, that is varied to capture different tax trading costs.[13] When the tax basis-to-price ratio is initially set lower (higher) than one, the investor has a capital gain (loss) in his endowed stock position.

Using the same notation as Section 2 and Appendix A, the price system parameters are $S_0(0) = S_1(0) = 1$, $r = 0.05$, $\mu = 0.08$, and $\sigma = 0.16$, where S_0 and S_1 denote the money market and stock prices respectively. For simplicity, the stock's price evolves as a binomial tree, so the investor will make a portfolio choice decision at $t = 0$ and $t = 1$ conditional on the stock going up or down in price. To map into a binomial distribution, the rate of appreciation (depreciation) of the stock over one time period is $e^{\sigma} = e^{0.16} = 1.174$ ($e^{-\sigma} = e^{-0.16} = 0.852$). The continuously-compounded expected stock return $\mu = 0.08$ determines the probabilities in the binomial tree. The tax rates are $\tau_I = 0.35$ and $\tau_C = 0.3$. The range for the investor's endowed basis-to-price ratio $b(0)$ is $[0.73, 1.38]$. This range corresponds to the lowest and highest stock price achievable at time $t = 2$. Choosing this range for the tax basis-to-price ratio allows us to capture a broad range of scenarios for how taxes are paid.

We examine the portfolio choice problem under the LUL case as well as under our two benchmarks — the FUL case and the NCGT case. Figure 1 summarizes the evolution of the optimal portfolio choice expressed as an equity-to-wealth ratio $\overline{\pi}$ (top three plots in the left panel) and the capital gain taxes paid Φ_{CG} (top three plots of the middle panel and all plots in the right panel). All plots are functions of the initial basis-to-price ratio $b(0)$. Portfolio choice decisions are made at times $t = 0$ and $t = 1$, while capital gain taxes are potentially paid at times $t = 0$, $t = 1$, and $t = 2$. In each plot, the solid line corresponds to the LUL case, the dashed line corresponds to the FUL case, and the dotted line corresponds to the NCGT case.

From the dotted lines in the equity-to-wealth plots of Figure 1, a benchmark NCGT investor always

[12] Capital gain tax forgiveness at death is considered in the long-dated portfolio problem studied in Sections 4 through 6.

[13] We use the tax basis-to-price ratio throughout our analysis given it conveniently summarizes the current state of tax trading costs in an investor's portfolio.

holds an equity-to-wealth ratio of approximately 0.43. To maintain this constant fraction, the investor trades the stock each period. At $t = 0$, the investor reduces his position from 1 share to 0.43 shares given the stock price is initially one; the proceeds of selling 0.57 shares are invested in the money market. At $t = 1$ when the stock price increases, the investor's fraction of wealth in equity rises above its optimal amount. The investor then reduces his equity-to-wealth ratio back to 0.43 by selling shares of stock and investing the proceeds in the money market. When the stock price decreases at $t = 1$, the investor is underexposed to equity and buys shares by selling part of the money market investment leading to an equity-to-wealth ratio of 0.43 again.

With capital gain taxes, the investor can no longer costlessly trade leading to significant deviations from the NCGT case. However, the LUL trading strategy is considerably more sensitive to tax trading costs relative to the FUL trading strategy as can be seen in the first three plots in the left panel of Figure 1. This greater sensitivity is driven by the lack of tax rebates in the LUL case which impacts the optimal trading strategy across a broad range of basis-to-price ratios.

For a large enough basis-to-price ratio ($b(0) \geq 1.15$), the LUL investor optimally trades as if he is the NCGT investor. In this region, realized capital losses at time $t = 0$ are large enough to cover any possible future capital gain taxes as can be seen in the tax plots of Figure 1. The optimal FUL trading strategy in this region is considerably different with the FUL investor holding even more equity than the NCGT case. This extra demand for equity is driven by the artificial tax rebate collected at $t = 0$ and possibly in the future if the stock price falls as can be seen in the tax plots. For the FUL investor, tax rebates act to truncate the down-side risk of holding equity elevating the demand.

As the basis-to-price ratio falls below 1.15, the LUL investor faces capital gain taxes when trading which greatly impacts his demand for equity. When the basis-to-price ratio $b(0)$ is between 1.07 and 1.15, the LUL investor still never pays any capital gain taxes over his lifetime, but only by significantly reducing his equity-to-wealth ratio relative to the NCGT case. When $b(0) = 1.07$, the LUL investor's optimal equity-to-wealth ratio falls to 0.27 from 0.43. As the basis-to-price ratio continues to fall toward 1.0, the LUL investor optimally holds more equity at $t = 0$, but still far below the NCGT benchmark. For the FUL investor, the ability to collect tax rebates through tax loss selling still highly skews his portfolio choice as his optimal equity-to-wealth ratio is still above the NCGT case. Additionally, the tax rebate artificially inflates his $t = 0$ wealth $W(0)$ as seen in the bottom left plot of Figure 1. Given the FUL investor's equity-to-wealth ratio is above the NCGT case and his wealth is elevated, his dollar investment in equity is also significantly higher than the NCGT case.

Tax trading costs at $t = 0$ matter for the LUL investor when the basis-to-price ratio falls below 1.0. Given the initial endowment is one share of stock, the LUL investor is grossly over-exposed to equity from a risk-return perspective. When the basis-to-price ratio $b(0)$ is close to one, the LUL investor trades to an equity position still significantly below the NCGT benchmark. Given he no longer has capital losses to shield future taxes, the after-tax benefit of holding stocks is still greatly reduced. As the basis-to-price ratio continues to fall, the tax cost of trading at time $t = 0$ begins to dominate the benefit of holding less stock due to a risk-return motive leading the LUL investor to sell less equity. For the FUL investor, the probability of collecting tax rebates in the future still significantly skews his equity allocation since he continues to hold an equity allocation larger than the NCGT benchmark. At the lowest initial basis-to-price ratio $b(0) = 0.73$, the FUL investor never can collect a tax rebate in the future. At this point, tax rebates can no longer skew the FUL investor's trading strategy implying the LUL and FUL strategies converge.

Overall, this simple three date example highlights that the LUL investor's optimal trading strategy is quite sensitive to tax trading costs as captured by the basis-to-price ratio. In particular, if current capital losses are large enough to offset all future capital gain taxes, the LUL investor can trade as if he is the NCGT investor. For small capital gains or losses embedded in the current portfolio, future taxes cannot be offset leading to a lower demand for equity than the NCGT investor. The FUL trading strategy masks this sensitivity since equity demand is artificially elevated due to tax rebates, skewing the after-tax risk-return trade off of holding equity.

4 The Conditional Structure of Optimal Portfolios

We now turn to the long-dated consumption-portfolio problem outlined in Section 2 to understand quantitatively how the LUL trading strategy behaves. To highlight the conditional nature of the trading strategy, we characterize the structure of optimal portfolios at a particular time and state.

We focus on presenting Base Case and Capital Gain Tax 30% Case results for both one and two stock portfolio choice problems. Given the Base Case capital gain tax rate is 20%, the Capital Gain Tax 30% Case captures the sensitivity of the optimal trading strategy to the tax rate. Additionally, this rate is similar to the rate of capital gain taxation in several European tax codes as mentioned earlier. The one stock results are summarized in Figures 2-4 and Table 1, whereas the two stock results are summarized in Figure 5 and Tables 3-4. The tables provide the same information as the figures for a subset of the state variables in a more convenient numerical form. We also consider several one and two stock comparative static cases summarized in Tables 2 and 5.[14]

For the one stock case, we present optimal equity-to-wealth ratios ($\overline{\pi}(t)$) conditional on the beginning period equity-to-wealth and basis-to-price ratios ($\underline{\pi}(t)$ and $b(t)$), for the FUL and the LUL cases at ages 20 and 80. To save space in the two stock case, we present the two optimal equity-to-wealth ratios ($\overline{\pi}_1(t)$ and $\overline{\pi}_2(t)$) conditional on the two basis-to-price ratios ($b_1(t)$ and $b_2(t)$) and a fixed beginning period equity-to-wealth ratio allocation of $\underline{\pi}_1(t) = 0.4$ and $\underline{\pi}_2(t) = 0.3$ at age 80. This beginning period stock allocation is chosen such that the investor is overexposed to equity. By varying the basis-to-price ratios, the tax cost of trading can be varied. In all LUL cases, we assume a zero carry-over loss. Cases with a positive carry over loss are well-captured by just examining trading strategies with basis-to-price ratios bigger than one entering the period. For the NCGT benchmark, the optimal portfolio choice is an overall equity-to-wealth ratio of 0.50 at all times for these parameters. In the two stock case, this implies an equity-to-wealth ratio of 0.25 in each stock.

4.1 Portfolio Choice with One Stock

Figure 2 presents the optimal portfolio choice strategy surfaces plotted as functions of the entering basis-to-price ratio and equity-to-wealth ratio for the LUL and FUL assumptions under the Base Case parameters. While these surfaces are instructive in understanding the basic tradeoffs between tax trading costs and the benefits of holding after-tax risk-return optimized portfolios, Figure 3 provides one dimensional slices of the portfolio choice surfaces by fixing different levels of the entering equity-to-wealth ratio. These slices, plotted against the basis-to-price ratio, make the differences between the LUL and FUL trading strategies more transparent. To easily see the impact of changing the tax rate, Figure 4 plots the optimal equity-to-wealth ratios for the Capital Gain Tax 30% Case. Table 1 provides the same information in a numerical form for a subset of the basis-to-price ratios for the Base Case (Panel A) and the Capital Gain Tax 30% Case (Panel B).

Figures 3 and 4 explore how the optimal trading strategy responds to tax trading costs when the investor enters the trading period holding an equity position either less than ($\underline{\pi} = 0.3$; top plots), equal to ($\underline{\pi} = 0.5$; middle plots), or greater than ($\underline{\pi} = 0.7$; bottom plots) the NCGT benchmark. These

[14]We present only a subset of the comparative statics analyzed. Several different scenarios including higher stock volatility cases are available from the authors.

three entering equity positions demonstrate a strong difference between the LUL and FUL trading strategies that is influenced by the current basis-to-price ratio.

The greatest difference between the LUL and FUL trading strategies occurs when the basis-to-price ratio is greater than or equal to one. The LUL investor's trading strategy behaves similar to the example presented in Section 3. At a basis-to-price ratio of one, the investor can trade the stock with no immediate tax consequences. Given the reduction in the desirability to hold equity due to the capital gain tax, the LUL investor optimally holds less equity than the NCGT benchmark. For example at age 20 in the Capital Gain Tax 30% Case, the LUL investor's optimal equity-to-wealth ratio is 0.45 from Table 1. As the basis-to-price ratio increases above one, the LUL investor realizes embedded capital losses to offset against future capital gain taxes. In both the Base Case and the Capital Gain Tax 30% Case, the optimal LUL equity-to-wealth ratio converges to the NCGT benchmark of 0.50 as the basis-to-price ratio approaches 1.5. Given the increasing embedded capital loss, the LUL investor trades as if he does not pay capital gain taxes.

The FUL investor's trading strategy is starkly different when the basis-to-price ratio is greater than or equal to one. In both the Base Case and Capital Gain Tax 30% Case, the equity-to-wealth ratio at a basis-to-price ratio of one ranges from 14% to 28% higher than the NCGT benchmark. This additional demand for equity is driven by the collection of the tax rebate. Under the FUL form of capital gain taxation, drops in equity prices are partially insured through tax rebates which has a first order effect on the investor's demand for equity. As the basis-to-price ratio increases above one, equity-to-wealth ratios grow even higher as the tax rebate induces an income effect leading to an even higher investment in equity. From Table 1, the FUL equity-to-wealth ratios are both increasing with age and the capital gain tax rate. Given tax forgiveness at death, the probability that an older investor will be locked into a large capital gain is reduced prompting a larger equity position today.

When the basis-to-price ratio falls below one, the entering equity-to-wealth ratio is more important in determining the optimal equity-to-wealth ratio for both the LUL and FUL investors as tax trading costs are more important. However, the potential for future tax rebates still imposes a wedge between the LUL and FUL optimal allocations as can be seen in Figures 3 and 4. When the entering equity-to-wealth ratio is $\underline{\pi} = 0.3$ (top panels), both LUL and FUL investors increase their equity positions, but the LUL investor is less aggressive. At $\underline{\pi} = 0.5$ (middle panels), both LUL and FUL investors choose not to trade for a low basis-to-price ratio. However, as the basis-to-price ratio approaches one, the two strategies diverge. The LUL investor can now reduce his equity position as the tax trading costs are lower. The FUL investor however amplifies his equity position as the probability of receiving tax rebates in the future increases as the embedded capital gains in the portfolio fall. When the investor enters the period overexposed to equity ($\underline{\pi} = 0.7$, bottom panels) with a low basis-to-price ratio, tax trading costs of selling dominate both the LUL and the FUL strategies. For example, at age 80 in the Capital Gain Tax 30% Case, both investor types choose not to trade. Given tax forgiveness at death, it is optimal for both investor types to be overexposed to equity to avoid paying capital gain taxes now. As the basis-to-price ratio approaches one, both investor types reduce their equity positions with the LUL investor selling more aggressively due to the lack of a potential tax rebate in the future.

Table 2 explores two comparative static cases — increasing the investor's risk aversion and imposing capital gain taxes at death. In Panel A, the investor's risk aversion is increased to $\gamma = 10$ to capture a scenario where equity is a less important component of the investor's portfolio. The NCGT equity-to-wealth allocation is now 0.25 as compared to 0.5 in the Base Case. Increasing the risk aversion leads to largely the same feature as in the Base Case except at lower allocations — the LUL optimal equity allocation is again significantly lower than the FUL optimal equity allocation when the FUL investor has a high probability of collecting tax rebates. Also, the FUL investor continues to hold an equity-to-wealth ratio greater than the NCGT benchmark when no capital gains are embedded in the existing portfolio. Given the U.S. tax code has the unique feature of capital gain taxation forgiveness at death, Panel B reports the optimal equity allocations when capital gain taxation is not forgiven at death. With no tax forgiveness, optimal equity allocations under the LUL and FUL cases no longer increase with age as can be seen by comparing the Base Case in Panel A of Table 1 with Panel B of Table 2. Importantly, the LUL equity allocation still is significantly lower than the FUL equity allocation when the FUL investor expects to collect tax rebates.

Summarizing, the lack of the tax rebates for the LUL investor leads to a significantly lower conditional equity allocation especially when the portfolio's basis-to-price ratio is greater than or equal to one. In particular, the LUL investor's conditional demand for equity endogenizes behavior that looks like increasing risk aversion in down markets that is typically captured by habit formation-based preferences. When the equity market's value increases, the LUL investor will tend to be overexposed to equity relative to the NCGT investor due to tax trading costs. When the equity market's value decreases, the LUL investor will hold an equity position lower than the NCGT investor due to the capital gain tax. The FUL investor's demand for equity does not display this feature due to tax rebates leading to higher equity allocations when the stock market falls.

4.2 Portfolio Choice with Two Stocks

In the one stock setting, the stock position can never simultaneously exhibit a capital gain and a capital loss. It is natural to ask how the wedge between the LUL and FUL investment strategies behaves when capital gains and losses can occur simultaneously. If enough realized capital gains are generated with multiple stocks, the tax rebates might have a smaller impact on the conditional trading strategies. For space considerations, we present results for the Base Case and the Capital Gain Tax 30% Case with two stocks for an age 80 investor who is overexposed to equity with $\underline{\pi}_1 = 0.3$ and $\underline{\pi}_2 = 0.4$. This choice of an initial stock position allows us to quantify the tradeoff between minimizing tax-induced trading costs and holding the optimal mix of equity and the money market. Given the two stocks are identically distributed with an 80% return correlation, the optimal NCGT equity allocation is an equity-to-wealth ratio of 0.25 in each stock.

Figure 5 presents the optimal equity-to-wealth ratio surfaces for each stock and the total stock allocation for different basis-to-price ratios under the LUL and FUL assumptions for the Base Case parameters. To aid in interpreting the main differences between the LUL and FUL strategies, Table 3 presents the Base Case results in a numerical form. To study the role of an increased capital gain tax rate, Table 4 presents the same optimal trading strategies for the Capital Gain Tax 30% Case.

When both stock positions have basis-to-price ratios greater than or equal to one, the optimal trading strategies are similar to the one stock case. The LUL investor chooses to hold equal positions in each stock with a total equity position never greater than the NCGT benchmark as seen in Panel A of Tables 3 and 4. The FUL investor however still trades more aggressively than the NCGT investor as seen in Panel B. For example, when the basis-to-price ratio is one for both stocks, the FUL investor trades to a total equity-to-wealth ratio of 0.60 in the Base Case and 0.67 in the Capital Gain Tax 30% Case. These quantities are 26% and 38% higher than the corresponding LUL strategies and 20% and 34% higher than the NCGT benchmark strategy. In particular, note that the higher capital gain tax rate leads to higher FUL equity-to-wealth ratios. As the tax rate increases, the FUL investor increases his equity position to amplify the tax rebates.

When both stock positions have low basis-to-price ratios and the investor is overinvested in equity, the LUL and FUL optimal trading strategies are similar. Given the stock portfolio has large embedded capital gains, the likelihood that the FUL investor will collect tax rebates in the future are small. Given tax forgiveness at death, the investor chooses to remain overexposed to equity.

The benefit of examining the two stock case is that we can examine how the optimal strategies behave when the investor simultaneously has an embedded capital gain and loss in the portfolio. Consider for example when the investor's equity positions have basis-to-price ratios of $b_1 = 1.2$ and $b_2 = 0.5$. Here the investor is overinvested in stock with a capital loss in stock 1 and a capital gain in stock 2. The LUL investor tax loss sells his position in stock 1 and reestablishes a position of $\overline{\pi}_1 = 0.19$ in the Base Case. Using his realized capital losses to offset the capital gain on stock 2, he reduces the stock 2 position to $\overline{\pi}_2 = 0.3$. The FUL investor however does not trade stock 2 retaining a position of $\overline{\pi}_2 = 0.4$, but does tax loss sell stock 1 to collect the tax rebate and retrade to a position of $\overline{\pi}_1 = 0.19$. By simply trading to collect the tax rebate, the FUL investor increases his wealth with a total equity-to-wealth ratio of 0.60. This behavior is quite prevalent for the FUL investor when one stock has a capital gain and the other a capital loss, especially in the Capital Gain Tax 30% Case. This again leads to a region where the FUL investor holds significantly more equity than the LUL investor due to the desire to capture tax rebates.

The two stock case allows us to examine the tradeoff between the tax cost of trading and the benefit of holding a well-diversified equity portfolio. In Table 5, the return correlation between the two stocks is changed to 0.90 and 0.40, respectively, to capture different diversification costs relative to the Base Case correlation of 0.80. As discussed in Section 2.4, the stock volatilities are changed when the correlation is changed to keep the pre-tax Sharpe ratio of an equally-weighted portfolio of these two stocks fixed across return correlations. When the return correlation increases, diversification benefits are less important implying the investor is willing to hold a less diversified position when it is costly to trade. This is evident, for example, by comparing the optimal stock 2 position as the stock 2 basis-to-price ratio varies, but the stock 1 basis-to-price ratio is fixed at 1. From a tax perspective, stock 1 can be traded at no cost; however, stock 2 is costly to trade if its basis-to-price ratio is less than 1. In this situation, the investor facing a return correlation of 0.4 is more willing to reduce the stock 2 position from 0.4 than the investor facing a return correlation of 0.9. More importantly, the return correlation does not have a large impact on the difference between the FUL and LUL strategies. When embedded capital gains are small in the portfolio, the FUL investor is still willing to hold a significantly higher total equity exposure relative to the LUL and NCGT investors.

While we have modeled the multiple stock setting with only two stocks, these results should generalize to portfolios with more than two stocks. For any stock with an embedded loss, it is always optimal to liquidate the entire position to generate a realized capital loss. For stocks that an investor is overexposed with embedded gains, any rebalancing will be small to minimize the capital gain taxes to be paid. Combining these two types of trades together, several states of the world will occur where the investor's realized losses are bigger than the realized gains. In the FUL case, this will lead to tax rebates that will increase optimal wealths and equity holdings relative to the LUL case.

5 The Lifetime Structure of Optimal Wealths and Portfolios

While examining optimal portfolio choice at a particular time and state is useful in understanding the conditional differences in the LUL and the FUL trading strategies, it provides no information about how the investor's wealth distribution or collected capital gain taxes behave given all quantities in the previous figures and tables are expressed as a fraction of wealth. Since tax rebates under the FUL case act as a state-dependent income process, this wealth impact is not captured in our previous results. To gain insights about the evolution of the optimal strategy including the wealth distribution over an investor's lifetime, we perform Monte Carlo simulations using our numerical solution of the optimal portfolio policies. The investor starts with no embedded stock gains at age 20 and an initial wealth of $100. We track the evolution of the investor's optimal portfolio over time conditional on the investor's survival. These results are reported in Tables 6 and 7 for one stock and two stock portfolio choice problems respectively. In each table, Panel A presents the Base Case, while Panel B presents the Capital Gain Tax 30% Case. All simulations are over $50,000$ paths.[15]

The tables report characteristics of the FUL and LUL portfolio choice problems at ages 40, 60, and 80. For each quantity reported, a selection of the percentiles of the distribution, the mean, and the standard deviation are reported. The column labeled "Wealth" gives the investor's current financial wealth expressed in dollars. The columns labeled "Equity-to-Wealth Ratio" and "Basis-to-Price Ratio" present the characteristics of the optimal equity position. For the two stock table, the "Stock 1 Equity-to-Wealth Ratio" ("Stock 2 Equity-to-Wealth Ratio") records the simulation characteristics for the

[15]For space considerations, we do not present the simulation results for all the comparative static cases we considered. These additional simulations are available from the authors.

smallest (largest) equity position.[16] The "Cumulative Capital Gain Tax-to-Wealth Ratio" column presents the undiscounted cumulative taxes paid from age 20 to the current age divided by the wealth at the current age. Finally, the column "LUL Carry Over Loss-to-Wealth Ratio" presents the carry over loss variable at the current age. Each mean estimate's standard error can be computed by dividing the Monte Carlo standard deviation given in the table by $\sqrt{50,000} = 223.6$.

The simulations demonstrate that the optimal wealths are significantly impacted by the ability of the FUL investor to collect tax rebates. From Tables 6 and 7, the FUL investor's wealth is higher at each percentile and age across all cases relative to the LUL investor's wealth. For example in the two stock Base Case at age 80, the mean FUL wealth is 14.8% higher than the LUL wealth. This mean wealth difference grows to 32.9% in the Capital Gain Tax 30% Case.

This increase in the FUL wealth distribution is driven by tax rebates directly and indirectly. The direct effect occurs when the FUL investor's wealth increases due to receiving tax rebates. This behavior is quite prevalent. Examining the FUL investor's cumulative capital gain tax-to-wealth ratio at age 80, we see that in both the one and the two stock Base Cases, over 10% of the wealth paths have accumulated negative undiscounted taxes or tax rebates. This percentage of paths out jumps to 25% for the one and the two stock Capital Gain Tax 30% Cases. In the Capital Gain Tax 30% Cases, these tax rebate paths are large enough to impact the mean cumulative capital gain tax-to-wealth ratios. For example in the one stock case, the mean cumulative capital gain tax-to-wealth ratio is negative implying more tax rebates are collected than capital gain taxes paid on average.

Tax rebates influence the FUL wealth distribution indirectly through higher equity holdings relative to the LUL investor as can be seen for example in the one stock Capital Gain Tax 30% Case. At age 20 in Table 1, the FUL investor's initial equity-to-wealth ratio is 0.61 as compared to the LUL investor's equity-to-wealth ratio of 0.45. The simulation results in Panel B of Table 6 show this difference in allocations persists. From the equity-to-wealth ratio column, the FUL equity holdings dominate the LUL equity holdings at all ages up to and including the 95th percentile leading to higher average equity holdings in the FUL case. This difference narrows as the investor ages. This decrease in the divergence is driven by both the FUL and LUL investors facing large embedded gains as can be seen in the mean basis-to-price ratios. The LUL investor's carry over loss-to-wealth ratio column also demonstrates this feature as the carry over loss variable is only nonzero earlier in the investor's life. Similar behavior

[16] Given the two stocks are ex ante identical, the stock characteristics are the same if they are recorded on a stock-by stock basis.

can be seen on the one stock Base Case as well as both two stock simulations.

The simulation results highlight that even though both the LUL and the FUL investors quickly hold portfolio positions with large embedded capital gains, the FUL investor's tax rebates available early in life greatly skew optimal wealths, collected taxes, and total dollar investment in equity. For ease in comparing our work with existing capital gain tax portfolio choice problems such as the one stock setting of Dammon, Spatt, and Zhang (2001b) and the two stock setting of Gallmeyer, Kaniel, and Tompaidis (2006), we have not incorporated economically-reasonable features that would further widen the wedge between optimal LUL and FUL portfolios. Several modifications to the current portfolio problem would lessen the capital gain lock-in effect by making low basis-to-price ratios less likely. Some examples include modeling a price system with mean-reverting dynamics, incorporating periodic liquidity shocks that force the investor to trade equity as in Constantinides (1983), and incorporating an income process that would lead to equity investment occurring through time.

6 The Economic Costs of the LUL and the FUL Cases

Table 8 quantifies the economic significance of capital gain taxes under the LUL and FUL assumptions. The table reports the wealth equivalent change of an age 20 NCGT investor due to imposing a capital gain tax. The wealth equivalent change is computed such that the investor's utility is the same from the NCGT case to the corresponding capital gain tax case with no initial embedded gains in the portfolio. A positive (negative) percentage wealth equivalent change denotes that the NCGT investor's welfare improves (worsens) by paying a capital gain tax. We present results for the one stock case (Panel A) and the two stock case (Panel B).[17] The left column presents the FUL wealth equivalent change, the middle column presents the LUL wealth equivalent change, and the right column computes the difference in wealth equivalent changes (FUL-LUL). A positive (negative) percentage for the difference denotes that the FUL investor is better (worse) off. Our measure of the cost of taxation is in contrast to most of the existing literature (Constantinides (1983); Dammon, Spatt, and Zhang (2001b); Garlappi, Naik, and Slive (2001)) as we do not measure tax costs relative to an accrual-based capital gain taxation system where all gains and losses are marked-to-market annually. Instead, our wealth equivalent change measure is meant to capture the change in an investor's welfare by imposing

[17]Due to different tolerances used in our numerical algorithm to manage the computing runtime of the two stock case, the one and two stock results are not directly comparable. However, the wealth equivalent changes presented are accurate to 0.1%.

a capital gain taxation scheme as compared to facing no capital gain taxation.

The wealth equivalent change analysis in Table 8 further iterates that tax rebates are an important driver of an FUL investor's optimal portfolio choice. For all one and two stock cases except the No Tax Forgiveness at Death Case, the FUL wealth equivalent changes are positive. In contrast, the LUL wealth equivalent changes are always negative. Hence, a NCGT investor is actually better off paying capital gain taxes under the FUL scenario than not being taxed. In both the one and two stock Base Cases, the NCGT investor's initial wealth is 2.2% higher by paying an FUL-based tax. When the capital gain tax rate increases to 30%, the benefit of paying taxes under the FUL assumption widens to 3.6% in the one stock case and 3.7% in the two stock case. The LUL investor who switches to a 30% capital gain tax, however, is worse off than in the 20% capital gain tax regime of the Base Case as the wealth changes become more negative. In the two stock case, we also consider varying the stock return correlation. Overall, the wealth difference between the FUL and the LUL cases are relatively insensitive to the return correlation change.

Not surprisingly, tax forgiveness at death plays an important role in making the FUL wealth equivalent change positive as demonstrated by the No Tax Forgiveness at Death Case given in the last line of Panel A. By removing an investor's ability to shield capital gains from taxation at death, the wealth equivalent change becomes negative. The LUL wealth equivalent changes are still more negative than the FUL wealth equivalent changes with a difference of 2.0%. This implies that even under no tax forgiveness at death, tax rebates still have an important role in mitigating the cost to the investor of the capital gain tax.

7 Conclusion

Our work has focused on the importance of integrating the limited use of losses into multiple stock portfolio selection problems with capital gain taxation. By requiring that capital losses can only be used to offset current or future realized gains, the after-tax risk-return tradeoff of holding equity is sharply impacted. With small embedded gains or losses in an existing portfolio, an investor facing the limited use of losses holds significantly less equity than an untaxed investor. If embedded capital losses are large enough, the taxed investor optimally can trade the untaxed investor's strategy. When embedded capital gains are large, the capital lock-in effect dominates making it difficult for an investor to trade out of a large equity position.

In contrast, without the limited use of losses, a taxed investor's trades are artificially impacted by tax rebates. These tax rebates act as an income process that pays off in down markets leading to a misleading higher demand for equity relative to an untaxed investor when capital gains are not too large in the existing portfolio. Through a simulation analysis, tax rebates greatly skew optimal wealths, collected taxes, and total dollar investment in equity over an investor's life. The motives for capturing tax rebates are strong enough to generate a counterfactual welfare result — a taxed investor who collects tax rebates actually prefers to pay capital gain taxes rather than being untaxed.

Our results in particular argue that taxable investors trading in down markets will seek significantly lower equity holdings than in up markets. This tax-induced time-varying demand for equity provides a simple mechanism that endogenizes behavior that looks like time-varying risk aversion as in Campbell and Cochrane (1999) or Chan and Kogan (2002). An interesting avenue for future research would be to empirically explore how capital gain taxation impacts the time-variation in asset prices since capital gain taxation has changed over time in the U.S. (see Sialm (2009)) and is heterogeneously imposed across countries. For example, one could consider a broader cross-country analysis of capital gain taxation and time-variation in stock prices similar to George and Hwang (2007) who study long-term return reversals in the U.S. and in Hong Kong, where capital gains are not taxed.

Finally, given the strong impact of the limited use of losses on the after-tax risk-return tradeoff of equity, several additional areas for future work are immediate. First, it would be useful to re-evaluate the advice given to taxable investors on the location of equity in taxable and tax-exempt accounts. Since the full use of losses inflates the value of tax-loss selling in taxable accounts, it would be beneficial to understand under what conditions the conventional wisdom of investing stocks in taxable accounts is still optimal. Second, the limited use of losses treatment of capital gain taxes overcomes long-standing problems of modeling capital gain taxes in equilibrium that has lead to few asset pricing results (Klein (1999, 2001); Viard (2000)). In particular, investors' demands no longer are driven by tax rebates which complicate market clearing. It would be useful to understand how large the general equilibrium impact on stock prices can be due to the time-varying demand for equity under the limited use of losses.

A Investor Consumption-Portfolio Problem Description

The mathematical description of the portfolio problem outlined in Section 2 is now presented. Our multiple risky stock model is based on the single stock setting of Dammon, Spatt, and Zhang (2001b) and the multiple stock setting of Gallmeyer, Kaniel, and Tompaidis (2006) where our notation and setup mainly follows from the latter. The major difference here relative to Gallmeyer, Kaniel, and Tompaidis (2006) is that our work incorporates the limited use of capital losses with no short selling.

A.1 Security Market

The economy is discrete-time with trading dates $t = 0, ..., T$. The investor trades each period in a riskless money market and N risky stocks. For simplicity, we consider a constant opportunity set.

The riskless money market has a time t price of $S_0(t)$ and pays a continuously compounded pre-tax interest rate r. The money market's price dynamics are given by

$$S_0(t + \Delta_t) = S_0(t) \exp\left(r\Delta_t\right), \tag{A.1}$$

where Δ_t is an arbitrary time interval.

Stock market investment opportunities are represented by N stocks each with a time t ex-dividend price $S_n(t)$ for $n = 1, \ldots, N$. Each stock pays a pre-tax dividend of $\delta_n S_n(t)$ at time t where δ_n is stock n's dividend yield. Stock n's pre-tax ex-dividend price follows a lognormal distribution with price dynamics over the time interval Δ_t given by

$$S_n(t + \Delta_t) = S_n(t) \exp\left(\left(\mu_n - \frac{1}{2}\sigma_n^2\right)\Delta_t + \sigma_n\sqrt{\Delta_t}\tilde{z}_n\right), \tag{A.2}$$

where $\tilde{z}_n$ is a standard normal distribution. The quantity μ_n is the instantaneous capital gain expected growth rate and σ_n is the instantaneous volatility of the stock. The shocks $\tilde{z}_n$ for $n = 1, \ldots, N$ have a variance-covariance matrix Σ inducing a correlation structure across stocks. To match the yearly trading interval of the investor in our economy, we assume that $\Delta_t = 1$ year.

A.2 Investor's Problem

Given a discrete-time economy with trading dates $t = 0, \ldots, T$, an investor endowed with initial wealth in the assets chooses an optimal consumption and investment policy in the presence of realized capital gain taxation. The investor lives for at most T periods and faces a positive probability of death each period. The probability that an investor lives up to period t, $0 < t < T$, is given by the survival function $H(t) = \exp\left(-\sum_{s=0}^{t}\lambda_s\right)$ where λ_s is the single-period hazard rate for period s where we assume $\lambda_s > 0$, $\forall s$, and $\lambda_T = \infty$. This implies $0 \leq H(t) < 1$, $\forall\, 0 \leq t < T$. At T, the investor exits the economy, implying $H(T) = 0$. We assume that the investor makes annual decisions starting at age 20 corresponding to $t = 0$ with certain exit from the economy at age 100 implying $T = 80$. The hazard rates λ_s are calibrated to the 1990 U.S. Life Tables compiled by the National Center for Health Statistics to compute the survival function $H(t)$ from ages 20 ($t = 0$) to 99 ($t = 79$).

The trading strategy from time t to $t+1$ in the money market and the stocks is given by $(\alpha(t), \theta(t))$ where $\alpha(t)$ denotes the shares of the money market held and $\theta(t) \equiv (\theta_1(t), \ldots, \theta_N(t))^\top$ denotes the vector of shares of stocks held where an individual element $\theta_n(t)$ denotes the holding of stock n.

A.2.1 Interest and Dividend Taxation

The investor faces three forms of taxation in our analysis: interest taxation, dividend taxation, and capital gain taxation. Interest income is taxed as ordinary income at the constant rate τ_I, while dividend income is taxed at the constant rate τ_D. The total taxes paid on interest and dividend income at time t are

$$\Phi_{I,D}(t) = \tau_I \, \alpha(t-1) \, S_0(t-1) \, (\exp(r) - 1) + \tau_D \sum_{n=1}^{N} \theta_n(t-1) S_n(t) \delta_n. \tag{A.3}$$

If the investor dies at time t, interest and dividend taxes are still paid.

A.2.2 Capital Gain Taxation

Using our two definitions of capital gain taxation, realized capital gains and losses in the stock are subject to a constant capital gain tax rate of τ_C. Computing the capital gain taxes due each period requires keeping track of the past purchase prices of each stock which forms that stock's tax basis. The tax basis for each stock is calculated as a weighted-average purchase price. Let $B_n(t)$ denote the nominal tax basis of stock n after trading at time t. The stock basis evolves as

$$B_n(t) = \begin{cases} S_n(t) & \text{if } \theta_n(t) = 0 \text{ or } \frac{B_n(t-1)}{S_n(t)} > 1, \\ \frac{B_n(t-1)\, \theta_n(t-1) + S_n(t)\, (\theta_n(t) - \theta_n(t-1))^+}{\theta_n(t-1) + (\theta_n(t) - \theta_n(t-1))^+} & \text{otherwise,} \end{cases} \tag{A.4}$$

where $x^+ \triangleq \max(x, 0)$. If $\theta_n(t) = 0$, the basis resets to the current stock price, $B_n(t) = S_n(t)$. Here we have assumed that the investor is precluded from short-selling stock n.

Under the FUL case, any realized capital gains or losses are subject to capital gain taxation. The capital gain taxes $\Phi_{CG}^{FUL}(t)$ at time t under the FUL case are

$$\Phi_{CG}^{FUL}(t) = \tau_C \left(\sum_{n=1}^{N} (S_n(t) - B_n(t-1))^+ (\theta_n(t-1) - \theta_n(t))^+ - \sum_{n=1}^{N} (B_n(t-1) - S_n(t))^+ \theta_n(t-1) \right), \tag{A.5}$$

where the first term calculates taxes from selling stocks with capital gains and the second term calculates reductions in taxes through capital losses from tax-loss selling. If death occurs at some time t', all capital gain taxes are forgiven implying $\Phi_{CG}^{FUL}(t') = 0$.

While the FUL case allows for negative taxes or a tax rebate when capital losses are realized, the LUL case eliminates all tax rebates. Realized capital losses can only be used to offset current or future realized capital gains. As a result, an additional state variable, the accumulated capital loss $L(t)$, is required. This state variable measures accumulated unused realized capital losses as of time t and evolves as

$$L(t) = \left(L(t-1) + \sum_{n=1}^{N} (B_n(t-1) - S_n(t))^+ \theta_n(t-1) - \sum_{n=1}^{N} (S_n(t) - B_n(t-1))^+ (\theta_n(t-1) - \theta_n(t))^+ \right)^+. \tag{A.6}$$

The accumulated capital loss $L(t)$ is modeled as a nonnegative state variable. A positive value is interpreted as unused realized capital losses. The first summation in (A.6) captures any increase in accumulated capital losses due to tax-loss selling. Based on Gallmeyer and Srivastava (2010), the investor is always weakly better off realizing all capital losses today even if he cannot use them immediately. This feature simplifies our analysis in that extra state variables are not needed that track capital losses still inside the portfolio. The second summation in (A.6) captures any decline in accumulated capital losses that are used to offset realized capital gains when shares are sold at time t. The max operator is applied to the entire expression as it is possible that realized sales with capital gains may extinguish all unused capital losses.

Under the LUL case, only realized capital gains are subject to capital gain taxation. Realized capital losses are used to offset future realized gains. The capital gain taxes $\Phi_{CG}^{LUL}(t)$ at time t under the LUL case are

$$\Phi_{CG}^{LUL}(t) = \tau_C \Bigg(\sum_{n=1}^{N} (S_n(t) - B_n(t-1))^+ (\theta_n(t-1) - \theta_n(t))^+ - \sum_{n=1}^{N} (B_n(t-1) - S_n(t))^+ \theta_n(t-1) - L(t-1) \Bigg)^+, \quad \text{(A.7)}$$

where capital gain taxes are paid when the investor realizes capital gains and does not have large enough accumulated capital losses $L(t-1)$ or current realized capital losses to offset that gain. If death occurs at some time t', all capital gain taxes are forgiven implying $\Phi_{CG}^{LUL}(t') = 0$.

A.2.3 Trading Strategies

We now define the set of admissible trading strategies when the investor can invest in the stock and the riskless money market. Again, we assume that the investor is prohibited from shorting any security.

The quantity $W(t+1)$ denotes the time $t+1$ wealth before portfolio rebalancing and any capital gain taxes are paid, but after dividend and interest taxes are paid. It is given by

$$W(t+1) = \alpha(t)\, S_0(t)\, ((1-\tau_I)\exp(r) + \tau_I) + \sum_{n=1}^{N} S_n(t+1)\, (1 + \delta_n(1-\tau_D))\theta_n(t), \quad \text{(A.8)}$$

where (A.3) has been substituted. Given that no resources are lost when rebalancing the portfolio at time t, $W(t)$ is given by

$$W(t) = \alpha(t) S_0(t) + \sum_{n=1}^{N} S_n(t)\theta_n(t) + C(t) + \Phi_C^j(t), \quad j \in \{FUL, LUL\}, \quad \text{(A.9)}$$

where $C(t) > 0$ is the time t consumption.

Substituting (A.9) into (A.8) gives the dynamic after-tax wealth evolution of the investor,

$$W(t+1) = \left(W(t) - \sum_{i=1}^{N} S_n(t)\theta_n(t) - C(t) - \Phi_C^j(t) \right) ((1-\tau_I)\exp(r) + \tau_I) + \sum_{n=1}^{N} S_n(t+1)\, (1 + \delta_n(1-\tau_D))\theta_n(t), \quad j \in \{FUL, LUL\}. \quad \text{(A.10)}$$

Additionally, the investor faces a margin constraint modeled as in Gallmeyer, Kaniel, and Tompaidis (2006). The margin constraint imposes a lower bound on the dollar amount of borrowing in the money market,

$$\alpha(t)\, S_0(t) \geq -(1-m_+) \sum_{n=1}^{N} S_n(t)\theta_n(t), \quad \text{(A.11)}$$

where $1 - m_+$ denotes the fraction of equity that is marginable. Throughout, we use $m_+ = 0.5$ which is consistent with Federal Reserve Regulation T for initial margins.

An *admissible trading strategy* is a consumption and a security trading policy (C, α, θ) such that for all t, $C(t) \geq 0$, $W(t) \geq 0$, $\theta(t) \geq 0$, and (A.10)-(A.11) are satisfied. The set of admissible trading strategies is denoted $\mathcal{A}$.

A.2.4 Investor's Objective

The investor's objective is to maximize his discounted expected utility of real lifetime consumption and final-period wealth at the time of death by choosing an admissible trading strategy given an initial endowment. If death occurs on date t, the investor's assets totaling $W(t)$ are liquidated and used to purchase a perpetuity that pays to his heirs a constant real after-tax cash flow of $R^* W(t)$ each period starting on date $t+1$. The quantity R^* is the one-period after-tax real riskless interest rate computed using simple compounding. In terms of the instantaneous nominal riskless money market rate r and the instantaneous inflation rate i, R^* is defined by

$$R^* = ((1 - \tau_D)\exp(r) + \tau_D)\exp(-i) - 1.$$

Under the assumption that the investor and his heirs have identical preferences of the constant relative risk aversion (CRRA) form with a coefficient of relative risk aversion of γ and a common time preference parameter β, the investor's optimization problem is given by

$$\max_{(C,\alpha,\theta)\in\mathcal{A}} E\left[\sum_{t=0}^{T} \beta^t \left\{ \frac{H(t)}{1-\gamma} \left(\exp(-it)C(t)\right)^{1-\gamma} + \frac{H(t-1) - H(t)}{1-\gamma} \sum_{s=t+1}^{\infty} \beta^{s-t} \left(\exp(-it)R^* W(t)\right)^{1-\gamma} \right\}\right]. \tag{A.12}$$

The objective function captures the expected utility of future real consumption as well as the bequest motive to the investor's heirs.

Since $\sum_{s=t+1}^{\infty} \beta^{s-t} = \frac{\beta}{1-\beta}$, the investor's objective function simplifies, leading to the optimization problem

$$\max_{(C,\alpha,\theta)\in\mathcal{A}} E\left[\sum_{t=0}^{T} \beta^t \left\{ \frac{H(t)}{1-\gamma} \left(\exp(-it)C(t)\right)^{1-\gamma} + \frac{H(t-1) - H(t)}{1-\gamma} \frac{\beta}{1-\beta} \left(\exp(-it)R^* W(t)\right)^{1-\gamma} \right\}\right]. \tag{A.13}$$

A.3 Change of Variables

As in a no-tax portfolio choice problem with CRRA preferences, the optimization problem (A.13) is homogeneous in wealth, and thus independent of the investor's initial wealth. To show that wealth is not needed as a state variable when solving (A.13), we express the optimization problem's controls as being proportional to time t wealth $W(t)$ before security trading but after the payment of taxes on dividends and interest. We define

$$\underline{\pi}_n(t) \triangleq \frac{S_n(t)\theta_n(t-1)}{W(t)}, \qquad \overline{\pi}_n(t) \triangleq \frac{S_n(t)\theta_n(t)}{W(t)}, \tag{A.14}$$

where $\underline{\pi}_n(t)$ and $\overline{\pi}_n(t)$ are the proportions of stock n owned entering and leaving period t, with respect to time t wealth $W(t)$. Note that the investor will never choose a trading strategy that leads to a non-positive wealth at any time given our utility function choice, the bequest motive, and the positive probability of death over each period. Hence, portfolio weights are well-defined as $W(t) > 0$ for all t.

Using (A.14), it is useful to express each stock's basis $B_n(t)$ as a basis-price ratio $b_n(t+1) \triangleq \frac{B_n(t)}{S_n(t+1)}$.

Using (A.4), the basis-price ratio evolves as

$$b_n(t+1) = \begin{cases} \frac{S_n(t)}{S_n(t+1)} & \text{if } \underline{\pi}_n(t) = 0 \text{ or } b_n(t) > 1, \\ \frac{b_n(t)\underline{\pi}_n(t) + (\overline{\pi}_n(t) - \underline{\pi}_n(t))^+}{\frac{S_n(t+1)}{S_n(t)}(\underline{\pi}_n(t) + (\overline{\pi}_n(t) - \underline{\pi}_n(t))^+)} & \text{otherwise.} \end{cases} \quad \text{(A.15)}$$

If $\overline{\pi}_n(t) = 0$, the basis-price ratio $b_n(t+1)$ resets to the ratio of the time t and $t+1$ stock n price, $b_n(t+1) = \frac{S_n(t)}{S_n(t+1)}$. The basis-price ratio at time $t+1$ can be expressed as a function of the capital gain of stock n over one period $\frac{S_n(t+1)}{S_n(t)}$, the previous period's basis-price ratio $b_n(t)$, and the equity proportions $\overline{\pi}_n(t)$ and $\underline{\pi}_n(t)$.

For the LUL case, the accumulated loss state variable $L(t)$ must also be expressed proportional to $W(t)$. Similar to the stock position, we define

$$\underline{l}(t) \triangleq \frac{L(t-1)}{W(t)}, \qquad \bar{l}(t) \triangleq \frac{L(t)}{W(t)}, \quad \text{(A.16)}$$

where $\underline{l}(t)$ and $\bar{l}(t)$ are the proportions of accumulated capital losses entering and leaving period t, with respect to time t wealth $W(t)$.

Using (A.6), the proportional accumulated capital losses evolve as

$$\bar{l}(t) = \left(\underline{l}(t) + \sum_{n=1}^{N}(b_n(t)-1)^+\underline{\pi}_n(t) - \sum_{n=1}^{N}(1-b_n(t))^+(\underline{\pi}_n(t) - \overline{\pi}_n(t))^+\right)^+. \quad \text{(A.17)}$$

Note that this quantity is independent of wealth $W(t)$.

Using the equity proportions, the basis-price ratios, and the proportional accumulated capital losses, the total capital gain taxes paid at time t, $\Phi^i_{CG}(t)$, can be written proportional to $W(t)$. Expressing $\Phi^i_{CG}(t) = W(t)\,\phi^i_{CG}(t)$, where $i \in \{FUL, LUL\}$, we obtain that $\phi^i_{CG}(t)$ is independent of $W(t)$. For the FUL case,

$$\phi^{FUL}_{CG}(t) = \tau_C\left(\sum_{n=1}^{N}(1-b_n(t))^+(\underline{\pi}_n(t) - \overline{\pi}_n(t))^+ - \sum_{n=1}^{N}(b_n(t)-1)^+\underline{\pi}_n(t)\right). \quad \text{(A.18)}$$

For the LUL case,

$$\phi^{LUL}_{CG}(t) = \tau_C\left(\sum_{n=1}^{N}(1-b_n(t))^+(\underline{\pi}_n(t) - \overline{\pi}_n(t))^+ - \sum_{n=1}^{N}(b_n(t)-1)^+\underline{\pi}_n(t) - \underline{l}(t)\right)^+. \quad \text{(A.19)}$$

Given that no resources are lost when portfolio rebalancing and paying taxes, equation (A.9) implies that the money market investment $\alpha(t)\,S_0(t)$ can be written proportional to $W(t)$ as

$$\alpha(t)\,S_0(t) = W(t)\left(1 - \sum_{n=1}^{N}\overline{\pi}_n(t) - c(t) - \phi^i_{CG}(t)\right), \quad i \in \{FUL, LUL\}, \quad \text{(A.20)}$$

where $c(t) \triangleq \frac{C(t)}{W(t)}$. Using (A.20), the margin constraint can also be written independent of wealth:

$$1 - c(t) - \phi^i_{CG}(t) \geq m_+ \sum_{n=1}^{N}\overline{\pi}_n(t). \quad \text{(A.21)}$$

The wealth evolution equation (A.10) can also be written proportional to $W(t)$ implying

$$\frac{W(t+1)}{W(t)} = \frac{1}{1 - \sum_{n=1}^{N} \underline{\pi}_n(t+1)(1+\delta_n(1-\tau_D))} \times \left[((1-\tau_D)\exp(r) + \tau_D) \left(1 - \sum_{n=1}^{N} \overline{\pi}_n(t) - c(t) - \phi_{CG}^i(t) \right) \right], \quad i \in \{FUL, LUL\}. \quad \text{(A.22)}$$

Additionally, the stock proportion evolution and the accumulated capital loss evolution are given by

$$\underline{\pi}_n(t+1) = \frac{\frac{S_n(t+1)}{S_n(t)} \overline{\pi}_n(t)}{\frac{W(t+1)}{W(t)}}, \quad \underline{l}(t+1) = \frac{\bar{l}(t)}{\frac{W(t+1)}{W(t)}}, \quad \text{(A.23)}$$

where both quantities are independent of time t wealth. This evolution is needed in the dynamic programming formulation of the investor's problem. In particular, $\underline{\pi}_n$ is a state variable and $\overline{\pi}_n$ is a control variable.

Using the principle of dynamic programming and substituting out $W(t)$, the Bellman equation for the investor's optimization problem (A.13) in the FUL case is summarized by $2 \times N + 1$ state variables where we have two state variables for each stock and a state variable for time. After this change of variables, the Bellman equation is

$$V(t, \underline{\pi}(t), b(t)) = \max_{c(t), \overline{\pi}(t)} \frac{e^{-\lambda_t} c(t)^{1-\gamma}}{1-\gamma} + \frac{(1-e^{-\lambda_t})\beta (R^*)^{1-\gamma}}{(1-\beta)(1-\gamma)} + e^{-\lambda_t} \beta E_t \left[\left(\frac{e^{-i} W(t+1)}{W(t)} \right)^{(1-\gamma)} V(t+1, \underline{\pi}(t+1), b(t+1)) \right], \quad \text{(A.24)}$$

for $t = 0, 1, \ldots, T-1$ subject to the wealth evolution equation (A.22), the margin constraint (A.21), and the stock proportion dynamics (A.23). In the LUL case, an additional state variable is needed, $\underline{l}$, the accumulated capital losses. The Bellman equation for this investor's problem is given by

$$V(t, \underline{\pi}(t), b(t), \underline{l}(t)) = \max_{c(t), \overline{\pi}(t)} \frac{e^{-\lambda_t} c(t)^{1-\gamma}}{1-\gamma} + \frac{(1-e^{-\lambda_t})\beta (R^*)^{1-\gamma}}{(1-\beta)(1-\gamma)} + e^{-\lambda_t} \beta E_t \left[\left(\frac{e^{-i} W(t+1)}{W(t)} \right)^{(1-\gamma)} V(t+1, \underline{\pi}(t+1), b(t+1), \underline{l}(t+1)) \right], \quad \text{(A.25)}$$

for $t = 0, 1, \ldots, T-1$ subject to the wealth evolution equation (A.22), the margin constraint (A.21), and the stock/capital loss proportion dynamics (A.23). Note that $\underline{\pi}(t)$, $\overline{\pi}(t)$, and $b(t)$ are vectors of length N to capture the trading position and tax basis for each stock.

B Numerical Optimization

To numerically solve the Bellman equations (A.24) and (A.25), we extend the methodology of Brandt et al. (2005) and Garlappi and Skoulakis (2008) to incorporate endogenous state variables and constraints on portfolio weights. In addition, since the state variable evolution is given by functions that are piecewise linear, the Bellman equation corresponds to a singular stochastic control problem that we solve employing a domain decomposition of the state space. We first briefly sketch the algorithm before providing additional details. A full description can be found in Yang (2010).

B.1 Sketch of Algorithm

Step 1 - Domain Decomposition

a. The state space is decomposed into degenerate and non-degenerate regions. The degenerate region corresponds to when a stock's basis-price ratio is above 1. The solution at a point in the degenerate region is mapped to a solution at a point in the non-degenerate region.

b. For a point in the non-degenerate region, the choice space is decomposed into partitions in such a way that, in each partition, the evolution of all state variables is differentiable (and linear).

Step 2 - Dynamic Programming

a. For each time step, starting at the terminal time and working backward, a quasi-random grid is constructed in the non-degenerate region of the state space. For each point on the grid, the value function, the optimal consumption, and the optimal portfolio decisions are computed.

b. The value function is approximated using a set of basis functions, consisting of radial basis functions and low order polynomials. This approximation is used in earlier time steps to compute conditional expectations of the value function.

Step 3 - Karush-Kuhn-Tucker (KKT) Conditions

To solve the Bellman equation for each point on the quasi-random grid in the non-degenerate region and for each partition in the choice space, the following steps are performed.

a. A Lagrangian function is constructed for the value function using the portfolio position constraints, the corresponding Lagrange multipliers, and the state variable evolution.

b. For each partition in the choice space, the system of first order conditions (KKT conditions) are constructed from the Lagrangian function.

c. The optimal solution of the KKT conditions is found using a double iterative process:

   i. An approximate optimal portfolio is chosen and the corresponding approximate optimal consumption is computed.

   ii. Given the approximate optimal consumption, the corresponding approximate optimal portfolio is updated by solving the system of KKT conditions. The solution is computed by approximating the conditional expectations in the derivatives of the Lagrangian function using a cross-test-solution regression:

      1. A quasi-random set of feasible allocations and consumptions is chosen.
      2. For each feasible choice, the required conditional expectations are computed using the approximate value function from the next time step that was already computed.
      3. For each feasible choice, the computed conditional expectations are projected on a set of basis functions of the choice variables. The basis functions are chosen such that the KKT system of equations is linear in the choice variables.
      4. The resulting linear system of equations is then solved.

   iii. The consumption choice is then updated to the choice corresponding to the new approximate optimal portfolio.

   iv. Step (ii) is repeated using a smaller region in which feasible portfolio choices are drawn. The region is chosen based on the location of the previously computed approximate optimal portfolio. This is the *test region contraction* step.

v. These steps are repeated until the consumption and portfolio choices converge.

We now provide a more detailed description of each step for the limited use of losses case. The full use of losses case is similar. As a reminder, the optimization problem being solved is equation (A.25):

$$V(t,\underline{\pi}(t),b(t),\underline{l}(t)) = \max_{c(t),\overline{\pi}(t)} \frac{e^{-\lambda_t}c(t)^{1-\gamma}}{1-\gamma} + \frac{(1-e^{-\lambda_t})\beta(R^*)^{1-\gamma}}{(1-\beta)(1-\gamma)} + e^{-\lambda_t}\beta E_t\Bigg[\left(\frac{e^{-i}W(t+1)}{W(t)}\right)^{(1-\gamma)} V(t+1,\underline{\pi}(t+1),b(t+1),\underline{l}(t+1))\Bigg],$$

for $t = 0, 1, \ldots, T-1$ subject to the wealth evolution equation (A.22), the margin constraint (A.21), the stock/capital loss proportion dynamics (A.23), the basis-price evolution (A.15), the accumulated capital loss evolution (A.17), and the capital gain taxes (A.19).

B.2 Algorithm Step 1 - Domain Decomposition

The first step in solving the optimization problem is to decompose the state space into a degenerate and a non-degenerate region. The solution at any point in the degenerate region can be mapped to the solution at a point in the non-degenerate region, and the problem solved only over the non-degenerate region. The degeneracy arises when the basis-price ratio of a stock is above 1, in which case it is optimal to immediately liquidate the position and add the realized capital loss to the accumulated loss state variable.

Take as given a point in the state space $\left(\mathring{\underline{\pi}}(t) \in \mathbb{R}^N, \mathring{b}(t) \in \mathbb{R}^N_+, \mathring{l}(t) \in \mathbb{R}_+\right)$. We define the following sets:

the index set of all risky assets: $I = \{1, \ldots, N\}$,

the index set of degenerate assets: $I_t^D = \left\{i = 1, \ldots, N : \mathring{b}_i(t) > 1\right\}$,

the index set of non-degenerate assets: $I_t^{\bar{D}} = \left\{i = 1, \ldots, N : \mathring{b}_i(t) \leq 1\right\}$.

The set $\left(I_t^D, I_t^{\bar{D}}\right)$ forms a partition of I. Given any point $\left(\mathring{\underline{\pi}}(t), \mathring{b}(t), \mathring{l}(t)\right)$ in the state space, there exists an equivalent point $(\underline{\pi}(t), b(t), l(t))$ in the non-degenerate region of the state space, such that

$$\begin{aligned} V\left(t,\underline{\pi}(t),b(t),l(t)\right) &= V\left(t,\mathring{\underline{\pi}}(t),\mathring{b}(t),\mathring{l}(t)\right) \\ \overline{\pi}^*\left(t,\underline{\pi}(t),b(t),l(t)\right) &= \overline{\pi}^*\left(t,\mathring{\underline{\pi}}(t),\mathring{b}(t),\mathring{l}(t)\right) \\ c^*\left(t,\underline{\pi}(t),b(t),l(t)\right) &= c^*\left(t,\mathring{\underline{\pi}}(t),\mathring{b}(t),\mathring{l}(t)\right) \end{aligned}$$

where

$$\underline{\pi}_i(t) = \begin{cases} 0 & \text{if } i \in I_t^D \\ \mathring{\underline{\pi}}_i(t) & \text{if } i \in I_t^{\bar{D}} \end{cases}, \quad b_i(t) = \begin{cases} 1 & \text{if } i \in I_t^D \\ \mathring{b}_i(t) & \text{if } i \in I_t^{\bar{D}} \end{cases}, \quad l(t) = \mathring{l}(t) + \sum_{i \in I_t^D}\left(\mathring{b}_i(t) - 1\right)\mathring{\underline{\pi}}_i(t).$$

The second step employed in the domain decomposition is to decompose the choice space for each point in the non-degenerate region into partitions such that, in each partition, the piecewise linear constraints of the optimization problem become linear. This is achieved by choosing the following partitions:

Index set of stock positions when stock n's position reduced: $I_t^{RP} = \left\{ n \in I_t^{\bar{D}} : \overline{\pi}_n(t) \leq \underline{\pi}_n(t) \right\}$.

Index set of stock positions when stock n's position increased: $I_t^{IP} = \left\{ n \in I_t^{\bar{D}} : \overline{\pi}_n(t) > \underline{\pi}_n(t) \right\}$.

To find the optimal solution for each point in the non-degenerate part of the state space, we solve for each partition in the choice space and choose the solution with the higher value of the value function.

B.3 Algorithm Step 2 - Dynamic Programming

Given the structure of the non-degenerate region of the state space, and to ensure that we solve the optimization problem in a sufficiently dense set of points in the non-degenerate region, we further decompose the non-degenerate region into cases where assets are either held in non-zero, or in zero, amounts. The number of cases is equal to 2^N and the cases are enumerated below. In each region we generate a quasi-random grid on which we solve the optimization problem. The dimension of the grid in each region is twice the number of stocks which are held in non-zero positions, corresponding to the initial stock position and the basis-price ratio. An additional dimension is added to all grids, corresponding to the level of the carry-over loss.

	Asset 1	$\cdots$	Asset $N-1$	Asset N	Dimensions
Case 1	Long	$\cdots$	Long	Long	$2N$
Case 2	Long	$\cdots$	Long	Zero	$2(N-1)$
Case 3	Long	$\cdots$	Zero	Long	$2(N-1)$
Case 4	Long	$\cdots$	Zero	Zero	$2(N-2)$
$\vdots$	$\vdots$		$\vdots$	$\vdots$	$\vdots$
Case 2^N	Zero	$\cdots$	Zero	Zero	0

Once the optimal strategy and the value function levels are computed for all points in the quasi-random grid at a particular time, the value function for any point in the state space is approximated by projecting the values on a set of basis functions. Some form of approximation is necessary, since it is necessary to estimate the value function at arbitrary points in the state space in order to compute the conditional expectations that arise naturally when the optimization problem is solved at grid points in the previous time slice. In the literature different approximations have been used, including a linear rule (see Gallmeyer, Kaniel, and Tompaidis (2006)) and projection on polynomials of the state variables (see Brandt et al. (2005)). We choose an approximation scheme that proceeds in two steps. First, we project the value function on a set of low order polynomials of the state variables. Second, we approximate the residuals with a set of radial basis functions. Each radial basis function is defined by its weight, center, and width. We adjust the number of centers, the location of each center, the corresponding widths, and the corresponding weights to achieve a good approximation of the value function. Additional details of the radial basis function approximation are in Yang (2010).

B.4 Algorithm Step 3 - Karush-Kuhn-Tucker Conditions

To solve the optimization problem at each grid point in the non-degenerate region of the state space, we construct, as in Yang (2010), a Lagrangian function that combines the value function at time t with the constraints on the choice variables. The Lagrangian, given a point in the state space, is a function of the choice variables and the Lagrange multipliers.

To easily express the constraints (A.17) and (A.19), define the wealth-proportional realized capital gains or losses as

$$g(t) = \sum_{n=1}^{N}(1-b_n(t))^+(\underline{\pi}_n(t)-\overline{\pi}_n(t))^+ - \sum_{n=1}^{N}(b_n(t)-1)^+\underline{\pi}_n(t). \tag{B.1}$$

Then, equations (A.17) and (A.19) can be written as

$$\bar{l}(t) = \Big(\underline{l}(t)-g(t)\Big)^+, \quad \phi_{CG}^{LUL}(t) = \tau_C\Big(g(t)-\underline{l}(t)\Big)^+.$$

Since the terms $\Big(\underline{l}(t)-g(t)\Big)^+$ and $\Big(g(t)-\underline{l}(t)\Big)^+$ are non-differentiable when $\underline{l}(t) = g(t)$, it is necessary to write two versions of the Lagrangian and solve them separately depending on whether $g(t) \geq \underline{l}(t)$ or $g(t) \leq \underline{l}(t)$. Assuming $g(t) \geq \underline{l}(t)$, the Lagrangian at $(\underline{\pi}(t), b(t), \underline{l}(t))$ is

$$\begin{aligned}\mathcal{L}\left(\overline{\pi}(t), c(t), \lambda_t^C, \lambda_t^m, \lambda_t^{RP}, \lambda_t^{IP}\right) &= e^{-\lambda_t}\frac{c(t)^{1-\gamma}}{1-\gamma} \\ &+ e^{-\lambda_t}\beta E_t\left[\left(\frac{e^{-i}W(t+1)}{W(t)}\right)^{1-\gamma} V\left(t+1, \underline{\pi}(t+1), b(t+1), \underline{l}(t+1)\right)\right] + \lambda_t^C(g(t)-\underline{l}(t)) \\ &+ \lambda_t^m\left[1-c(t)-\phi_C(t)-m^+\sum_{n=1}^{N}\overline{\pi}_n(t)\right] + \sum_{i\in I_t^{RP}}\lambda_t^{RP,i}\left[\underline{\pi}_i(t)-\overline{\pi}_i(t)\right] + \sum_{i\in I_t^{IP}}\lambda_t^{IP,i}\left[\overline{\pi}_i(t)-\underline{\pi}_i(t)\right],\end{aligned}$$

where λ_t^C is the Lagrange multiplier corresponding to the constraint that the carry-over loss, after taxes are paid or returned, cannot be negative; λ_t^m is the Lagrange multiplier corresponding to the margin constraint; and $\lambda_t^{RP}, \lambda_t^{IP}$ are the Lagrange multipliers corresponding to the partitioning of the choice variable space.

Assuming $g(t) \leq \underline{l}(t)$, the Lagrangian at $(\underline{\pi}(t), b(t), \underline{l}(t))$ is

$$\begin{aligned}\mathcal{L}\left(\overline{\pi}(t), c(t), \lambda_t^C, \lambda_t^m, \lambda_t^{RP}, \lambda_t^{IP}\right) &= e^{-\lambda_t}\frac{c(t)^{1-\gamma}}{1-\gamma} \\ &+ e^{-\lambda_t}\beta E_t\left[\left(\frac{e^{-i}W(t+1)}{W(t)}\right)^{1-\gamma} V\left(t+1, \underline{\pi}(t+1), b(t+1), \underline{l}(t+1)\right)\right] - \lambda_t^C(g(t)-\underline{l}(t)) \\ &+ \lambda_t^m\left[1-c(t)-\phi_C(t)-m^+\sum_{n=1}^{N}\overline{\pi}_n(t)\right] + \sum_{i\in I_t^{RP}}\lambda_t^{RP,i}\left[\underline{\pi}_i(t)-\overline{\pi}_i(t)\right] + \sum_{i\in I_t^{IP}}\lambda_t^{IP,i}\left[\overline{\pi}_i(t)-\underline{\pi}_i(t)\right],\end{aligned}$$

where λ_t^C is the Lagrange multiplier corresponding to the constraint that the capital gain tax paid cannot be negative. All other Lagrange multipliers are the same as in the previous case.

The KKT conditions are derived by differentiating the Lagrangian with respect to the choice variables and Lagrange multipliers. The following conditional expectations of the value function at time $t+1$ are estimated:

$$\begin{aligned}E_t&\left[\frac{\partial}{\partial\overline{\pi}_i(t)}\left[\left(\frac{W(t+1)}{W(t)}\right)^{1-\gamma} V\left(t+1, \underline{\pi}(t+1), b(t+1), \underline{l}(t+1)\right)\right]\middle|\, \underline{\pi}(t), b(t), \underline{l}(t), \overline{\pi}(t), c(t)\right], \\ E_t&\left[\frac{\partial}{\partial c(t)}\left[\left(\frac{W(t+1)}{W(t)}\right)^{1-\gamma} V\left(t+1, \underline{\pi}(t+1), b(t+1), \underline{l}(t+1)\right)\right]\middle|\, \underline{\pi}(t), b(t), \underline{l}(t), \overline{\pi}(t), c(t)\right].\end{aligned}$$

To estimate the conditional expectations, for each point in the state space $(\underline{\pi}(t+1),\ b(t+1),\ \underline{l}(t+1))$, we generate a set of test values for the choice variables, $\left(\overline{\pi}^{(j)}(t), c^{(j)}(t)\right)_{j=1}^{n_t}$, and calculate the conditional expectation:

$$E_t\left[\left(\frac{W(t+1)}{W(t)}\right)^{1-\gamma} V\left(t+1,\underline{\pi}(t+1),b(t+1),\underline{l}(t+1)\right)\middle|\,\underline{\pi}(t),b(t),\underline{l}(t),\overline{\pi}^{(j)}(t),c^{(j)}(t)\right].$$

The test solutions need to be generated consistently with the partition of the choice space in which the problem is solved. Given the n_t values of the conditional expectation, we approximate, for each value of $(\underline{\pi}(t+1),b(t+1),\underline{l}(t+1))$, the conditional expectation at any value of the choice variables by projecting onto a set of basis functions $(f_k)_{k=1}^{n_b}$:

$$\begin{aligned}E_t\left[\left(\frac{W(t+1)}{W(t)}\right)^{1-\gamma} V\left(t+1,\underline{\pi}(t+1),b(t+1),\underline{l}(t+1)\right)\middle|\,\underline{\pi}(t),b(t),\underline{l}(t),\overline{\pi}^{(j)}(t),c^{(j)}(t)\right]\\ \approx\sum_{j=1}^{n_b}\omega_j\left(\underline{\pi}(t),b(t),\underline{l}(t)\right)f_j\left(\overline{\pi}(t),c(t)\right).\end{aligned}$$

We use basis functions f_k that are polynomials of the choice variables $(\overline{\pi}(t),c(t))$ up to order two. Once the conditional expectation is approximated, we approximate its derivatives by

$$\begin{aligned}E_t\left[\frac{\partial}{\partial\overline{\pi}_i(t)}\left[\left(\frac{W(t+1)}{W(t)}\right)^{1-\gamma} V\left(t+1,\underline{\pi}(t+1),b(t+1),\underline{l}(t+1)\right)\right]\middle|\,\underline{\pi}(t),b(t),\underline{l}(t),\overline{\pi}(t),c(t)\right]\\ \approx\sum_{j=1}^{n_b}\omega_j\left(\underline{\pi}(t),b(t),\underline{l}(t)\right)\frac{\partial}{\partial\overline{\pi}_i(t)}f_j\left(\overline{\pi}(t),c(t)\right),\\ E_t\left[\frac{\partial}{\partial c(t)}\left[\left(\frac{W(t+1)}{W(t)}\right)^{1-\gamma} V\left(t+1,\underline{\pi}(t+1),b(t+1),\underline{l}(t+1)\right)\right]\middle|\,\underline{\pi}(t),b(t),\underline{l}(t),\overline{\pi}(t),c(t)\right]\\ \approx\sum_{j=1}^{n_b}\omega_j\left(\underline{\pi}(t),b(t),\underline{l}(t)\right)\frac{\partial}{\partial c(t)}f_j\left(\overline{\pi}(t),c(t)\right).\end{aligned}$$

Given our choice of polynomials of order two, the KKT system of equations for each point in the non-degenerate part of the state space becomes a system of linear equations in terms of the optimal portfolios. To account for the inaccuracy in approximating conditional expectations with quadratic functions, we use an iterative scheme, where we successively reduce the size of the region from which the test solutions are drawn. Details of this procedure, termed the "Test Region Iterative Contraction (TRIC)," are provided in Yang (2010).

A final detail in solving the KKT system of equations, is that, given a guess for the optimal portfolio, $\overline{\pi}^a(t)$, we first solve for the optimal consumption by solving the equation

$$0=e^{-\lambda_t}c(t)^{-\gamma}+e^{-\lambda_t}e^{-i(1-\gamma)}\beta\sum_{j=1}^{n_b}\omega_j\left(\underline{\pi}(t),b(t),\underline{l}(t)\right)\frac{\partial}{\partial c(t)}f_j\left(\overline{\pi}^a(t),c(t)\right)-\lambda_t^m.$$

Once the approximate optimal consumption is calculated, we solve the remaining, linear, system of KKT equations for the optimal portofolio. This step involves solving the system of KKT equations in all the possible partitions of the choice space, and choosing the solution that maximizes the value function. In the next iteration, the region from which test solutions for the portfolio positions are drawn is contracted around the computed solution. The approximate portfolio is also used to update the approximation to the optimal consumption. The iteration is repeated until the difference between successive solutions is sufficiently small.

References

Bergstresser, D. and J. Pontiff (2010). Investment taxation and portfolio performance. Boston College.

Brandt, M. W., A. Goyal, P. Santa-Clara, and J. R. Stroud (2005). A simulation approach to dynamic portfolio choice with an application to learning about return predictability. *Review of Financial Studies 18*(3), 831–873.

Cadenillas, A. and S. R. Pliska (1999). Optimal trading of a security when there are taxes and transaction costs. *Finance and Stochastics 3*(2), 137–165.

Campbell, J. Y. and J. Cochrane (1999). Force of habit: A consumption-based explanation of aggregate stock market behavior. *Journal of Political Economy 107*(2), 205–251.

Chan, Y. L. and L. Kogan (2002). Catching up with the Joneses: Heterogeneous preferences and the dynamics of asset prices. *Journal of Political Economy 110*(6), 1255–1285.

Clark, W. S. (2006). *OECD Tax Policy Studies No. 14 - Taxation of Capital Gains of Individuals: Policy Considerations and Approaches.* Organization for Economic Co-operation and Development.

Constantinides, G. (1983). Capital market equilibrium with personal tax. *Econometrica 51*, 611–636.

Dai, Z., E. Maydew, D. A. Shackelford, and H. H. Zhang (2008). Capital gains taxes and asset prices: Capitalization or lock-in? *Journal of Finance 63*(2), 709–742.

Dammon, R. and C. Spatt (1996). The optimal trading and pricing of securities with asymmetric capital gains taxes and transaction costs. *Review of Financial Studies 9*, 921–952.

Dammon, R., C. Spatt, and H. Zhang (2001a). Diversification and capital gains taxes with multiple risky assets. Carnegie Mellon University.

Dammon, R., C. Spatt, and H. Zhang (2001b). Optimal Consumption and Investment with Capital Gains Tax. *Review of Financial Studies 14*(3), 583–616.

Dammon, R., C. Spatt, and H. Zhang (2004). Optimal asset location and allocation with taxable and tax-deferred investing. *Journal of Finance 59*(3), 999–1037.

DeMiguel, V. and R. Uppal (2005). Portfolio investment with the exact tax basis via nonlinear programming. *Management Science 55*(2), 277–290.

Dybvig, P. and H. K. Koo (1996). Investment with taxes. Washington University.

Gallmeyer, M., R. Kaniel, and S. Tompaidis (2006). Tax management strategies with multiple risky assets. *Journal of Financial Economics 80*, 243–291.

Gallmeyer, M. and S. Srivastava (2010). Arbitrage and the tax code. University of Virginia.

Garlappi, L. and J. Huang (2006). Are stocks desirable in tax-deferred accounts? *Journal of Public Economics 90*, 2257–2283.

Garlappi, L., L. V. Naik, and J. Slive (2001). Portfolio selection with multiple assets and capital gains taxes. University of British Columbia.

Garlappi, L. and G. Skoulakis (2008). A state-variable decomposition approach for solving portfolio choice problems. University of British Columbia.

George, T. J. and C.-Y. Hwang (2007). Long-term return reversals: Overreaction or taxes? *Journal of Finance 62*(6), 2865–2896.

Huang, J. (2008). Taxable and tax-deferred investing: A tax-arbitrage approach. *Review of Financial Studies 21*(5), 2171–2207.

Hur, S.-K. (2001). Optimal portfolio selection with personal tax. University of Chicago.

Ivković, Z. and S. Weisbenner (2009). Individual investor mutual fund flows. *Journal of Financial Economics 92*(2), 223–237.

Jensen, B. A. and M. Marekwica (2010). Optimal portfolio choice with wash sale constraints. Copenhagen Business School.

Jouini, E., P.-F. Koehl, and N. Touzi (2000). Optimal investment with taxes: An existence result. *Journal of Mathematical Economics 33*(4), 373–388.

Klein, P. (1999). The capital gain lock-in effect and equilibrium returns. *Journal of Public Economics 71*(3), 355–378.

Klein, P. (2001). The capital gain lock-in effect and long-horizon return reversal. *Journal of Financial Economics 59*(1), 33–62.

Marekwica, M. (2009). Optimal tax-timing and asset allocation when tax rebates on capital losses are limited. Copenhagen Business School.

Poterba, J. M. (2002). Taxation, risk-taking, and household portfolio behavior. In A. J. Auerbach and M. Feldstein (Eds.), *Handbook of Public Economics*, Volume 3 of *Handbook of Public Economics*, Chapter 17, pp. 1109–1171. Elsevier.

Poterba, J. M. and J. B. Shoven (2002). Exchange-traded funds: A new investment option for taxable investors. *American Economic Review 92*(2), 422–427.

Sialm, C. (2009). Tax changes and asset pricing. *American Economic Review 99*, 1356–1383.

Viard, A. D. (2000). Dynamic asset pricing effects and incidence of realization-based capital gains taxes. *Journal of Monetary Economics 46*(2), 465–488.

Yang, C. (2010). *Functional Approximation Methods for Solving Stochastic Control Problems in Finance*. Ph. D. thesis, University of Texas at Austin.

Figure 1: **Example.** The figure reports properties of the optimal trading strategy for an LUL, an FUL, and an NCGT investor as a function of the investor's basis-to-price ratio at time $t = 0$, $b(0)$, when the investor initially owns one share of stock and no bond position. The left panel summarizes the after-tax optimal portfolio choice as a fraction of wealth $\overline{\pi}$ and the time $t = 0$ wealth $W(0)$. The middle panel summarizes the capital gain taxes paid Φ_{CG} at $t = 0$ and $t = 1$ as well as the investor's expected utility at $t = 0$. The right panel summarizes the capital gain taxes paid at $t = 2$ when the investor consumes. 'Up' and 'Dn' denote up and down moves through the binomial tree. The parameters used are given at the beginning of Section 3.

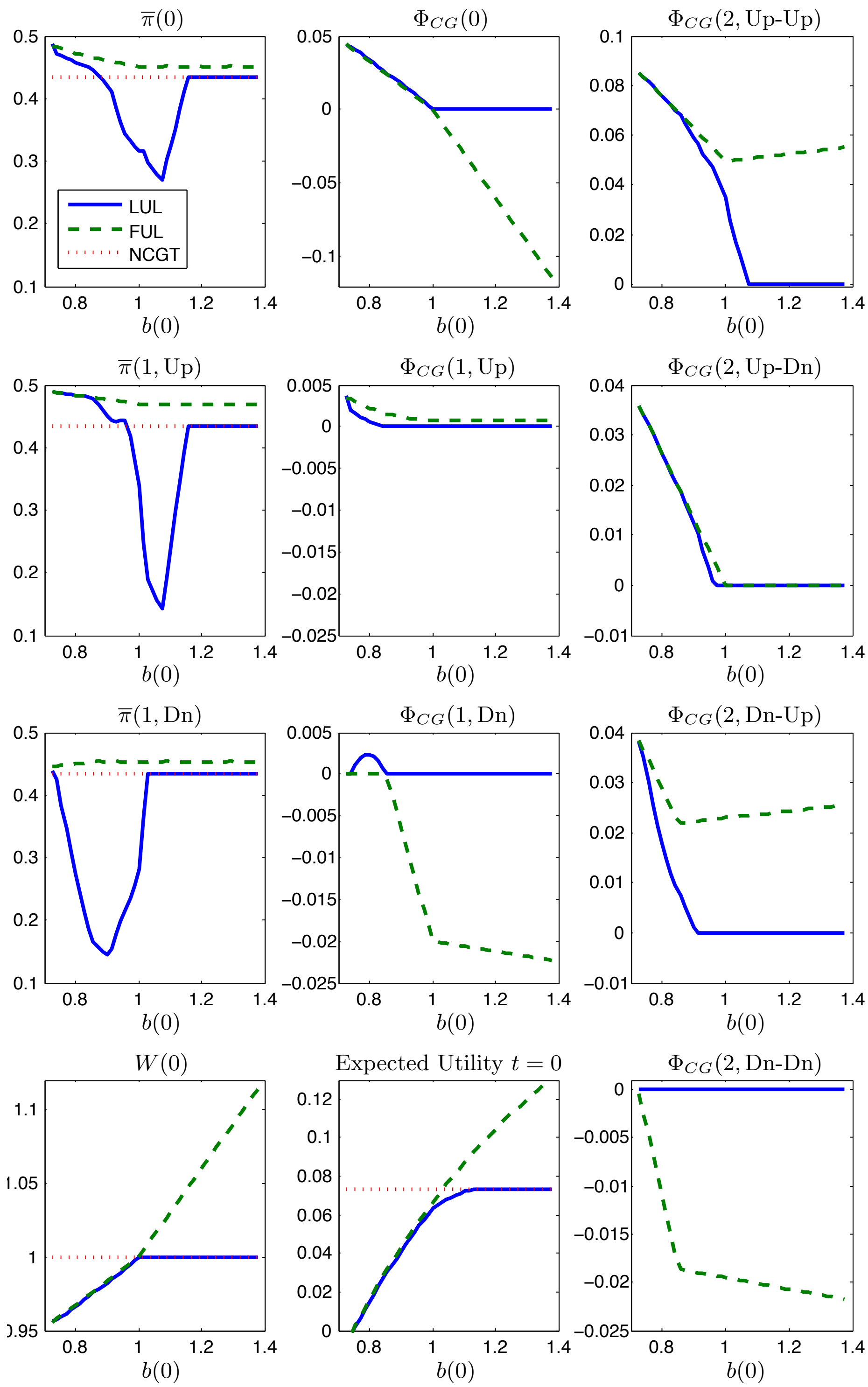

Figure 2: **Base Case Optimal Strategies - One Stock.** The left (right) panels summarize the optimal equity-to-wealth ratio $\overline{\pi}$ as a function of the equity-to-wealth ratio $\underline{\pi}$ and the basis-to-price ratio b entering the trading period for the LUL (FUL) case when one stock is traded. The top (bottom) panels present the equity-to-wealth ratio at age 20 (80). The LUL plots have a zero carry-over loss entering the trading period. The parameters used are the one stock Base Case parameters summarized in Section 2.4.

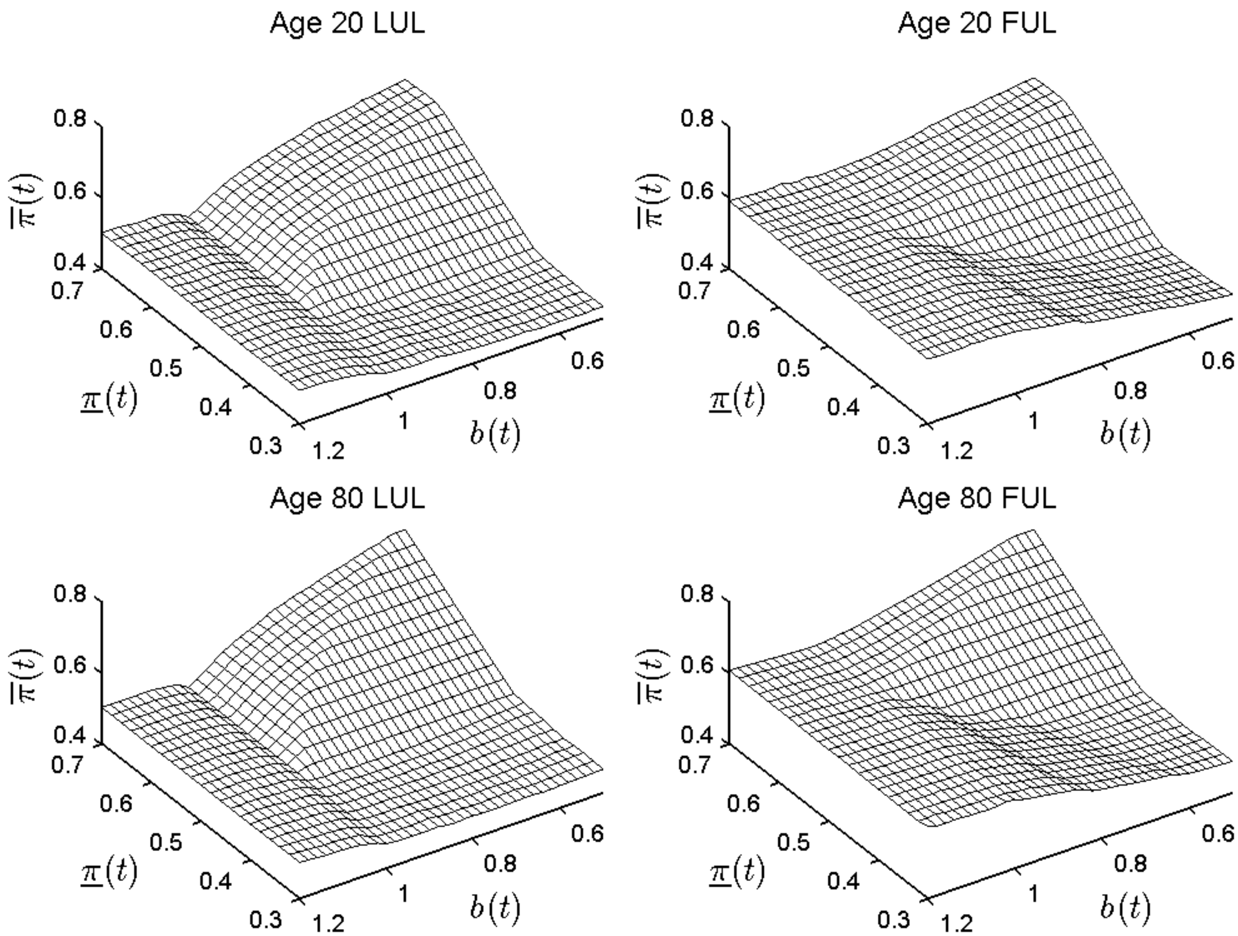

Figure 3: **Base Case Optimal Strategies Conditional on the Basis-to-Price Ratio - One Stock.** The left (right) panels summarize the optimal equity-to-wealth ratio $\overline{\pi}$ as a function of the basis-to-price ratio b at age 20 (80). The LUL (FUL) trading strategy is represented by a solid (dashed) line. The top panel is conditional on the entering equity-to-wealth ratio $\underline{\pi}$ equaling 0.3, the middle panel is conditional on the entering equity-to-wealth ratio equaling 0.5, and the bottom panel is conditional on the entering equity-to-wealth ratio equaling 0.7. The LUL plots have a zero carry-over loss entering the trading period. The parameters used are the one stock Base Case parameters summarized in Section 2.4.

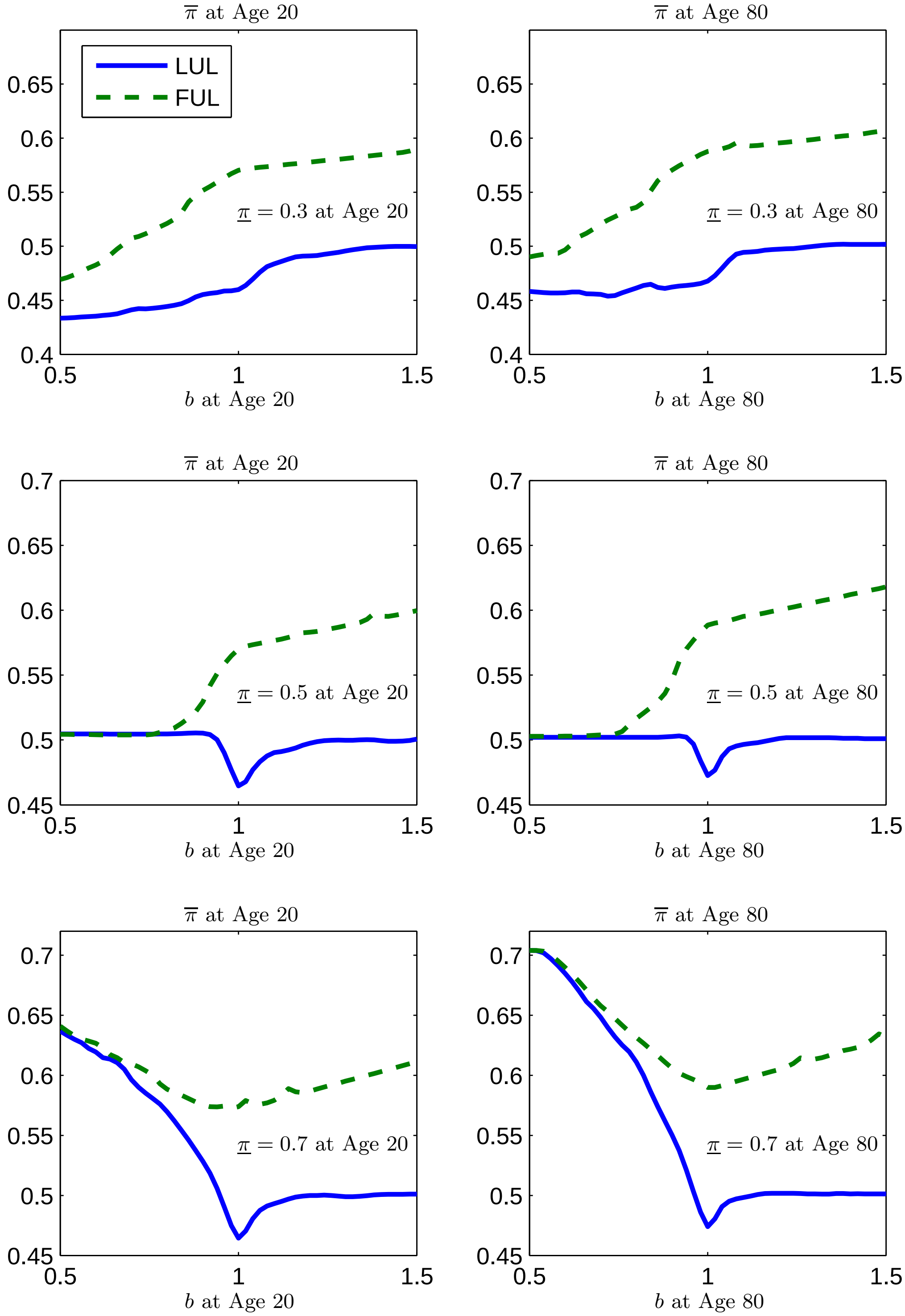

Figure 4: **Capital Gain Tax 30% Case Optimal Strategies Conditional on the Basis-to-Price Ratio - One Stock.** The left (right) panels summarize the optimal equity-to-wealth ratio $\overline{\pi}$ as a function of the basis-to-price ratio b at age 20 (80). The LUL (FUL) trading strategy is represented by a solid (dashed) line. The top panel is conditional on the entering equity-to-wealth ratio $\underline{\pi}$ equaling 0.3, the middle panel is conditional on the entering equity-to-wealth ratio equaling 0.5, and the bottom panel is conditional on the entering equity-to-wealth ratio equaling 0.7. The LUL plots have a zero carry-over loss entering the trading period. The parameters used are the one stock Capital Gain Tax 30% Case parameters summarized in Section 2.4.

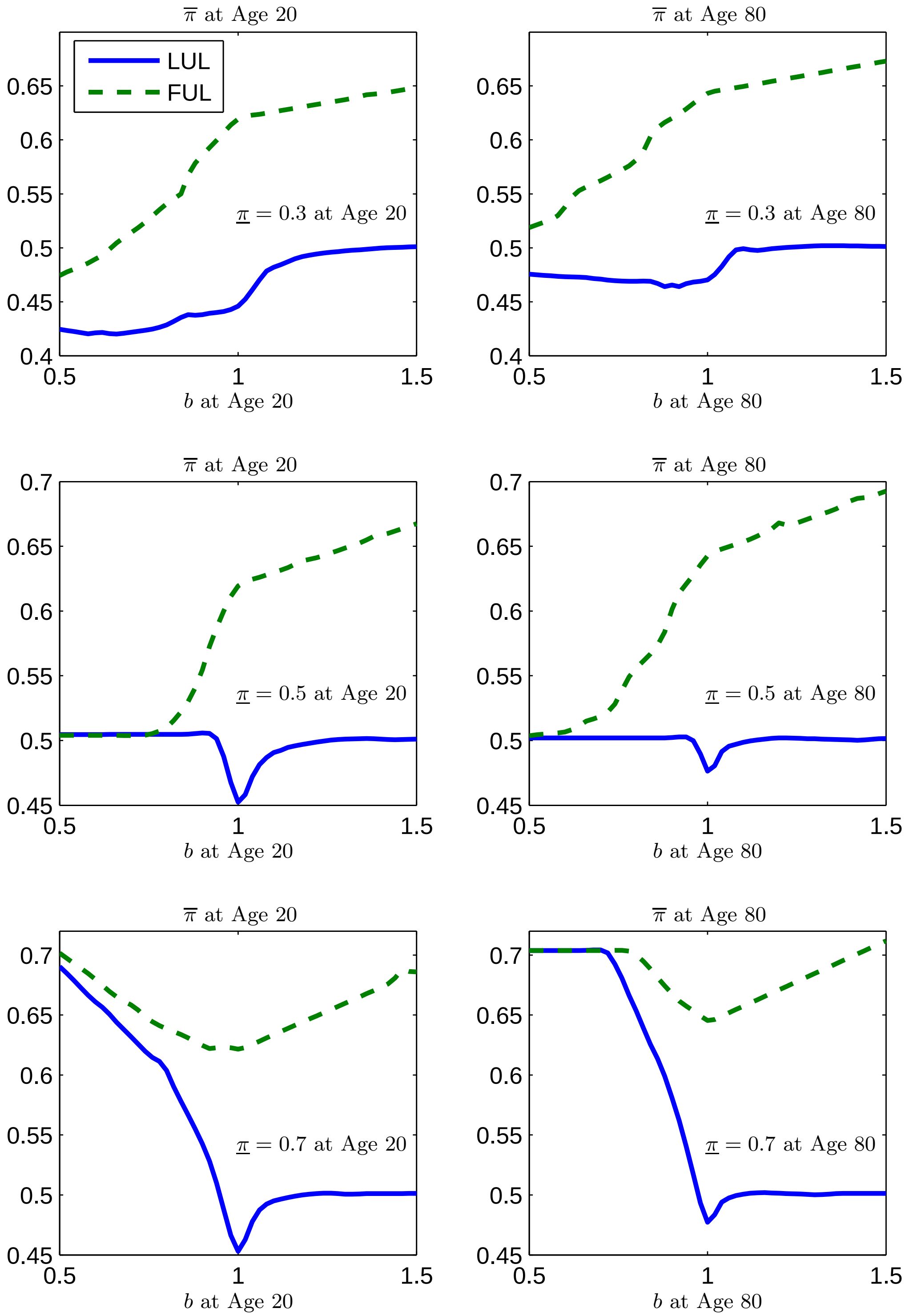

Figure 5: **Base Case Optimal Strategies - Two Stocks - Age 80.** The left (right) panels summarize optimal portfolio choice as a function of the basis-to-price ratios of each stock entering the trading period (b_1 and b_2) for the LUL (FUL) case when two stocks are traded. The top, middle, and bottom panels present the equity-to-wealth ratios for stock 1 ($\overline{\pi}_1$), stock 2 ($\overline{\pi}_2$), and the total equity allocation ($\overline{\pi}_1 + \overline{\pi}_2$) at age 80. The LUL plots have a zero carry-over loss entering the trading period. The investor enters the period with a stock 1 (stock 2) position of $\underline{\pi}_1 = 0.3$ ($\underline{\pi}_2 = 0.4$). The parameters used are the two stock Base Case parameters summarized in Section 2.4.

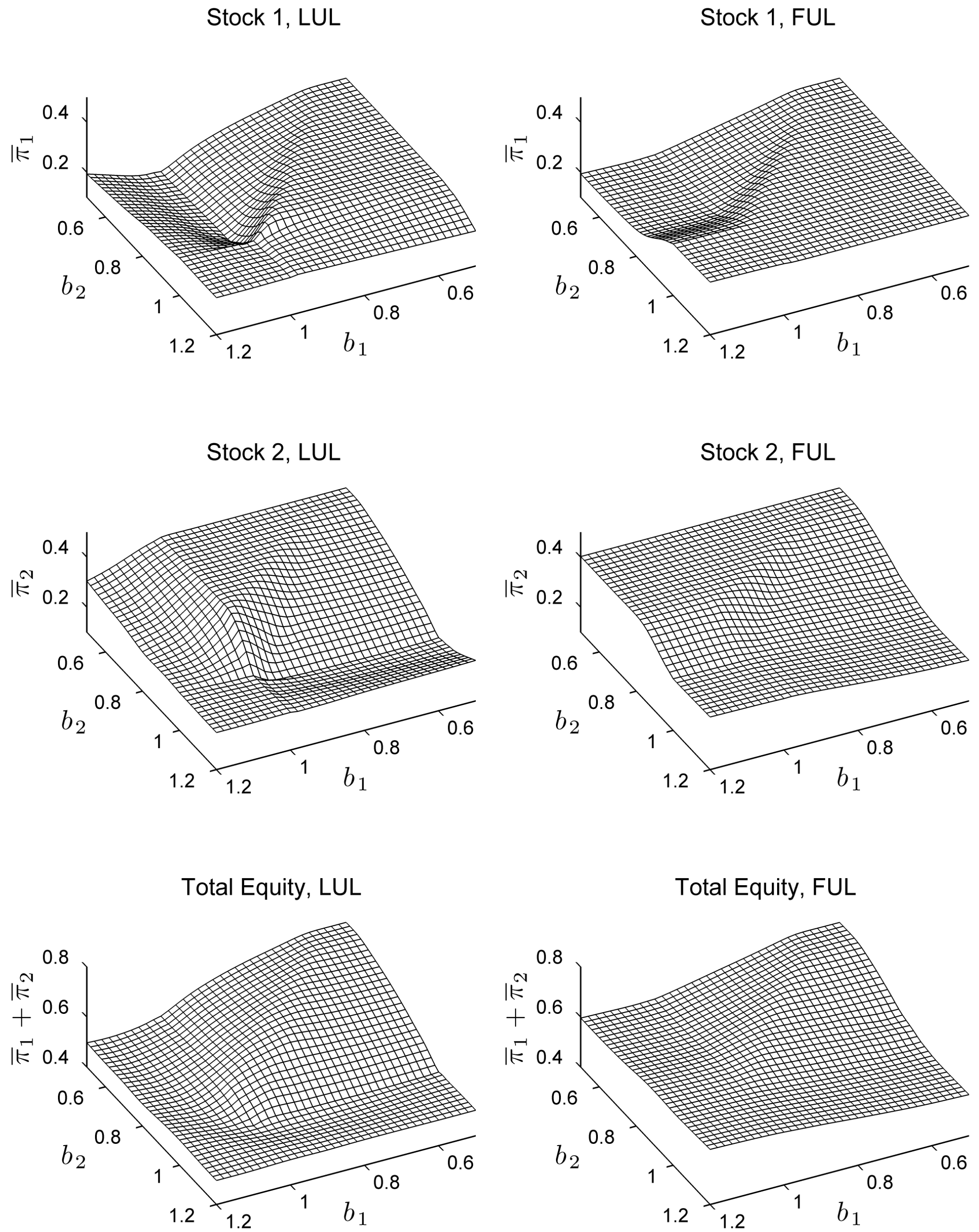

Table 1: **Base Case and Capital Gain Tax 30% Case Optimal Strategies - One Stock.** Panel A (Panel B) summarizes optimal portfolio choice as a function of the equity-to-wealth ratio and the basis-to-price ratio entering the trading period for the LUL and FUL cases for the Base Case (Capital Gain Tax 30% Case). In each panel, the equity-to-wealth ratios at age 20 and 80 are presented. The right panels compute the percentage increase in the FUL equity-to-wealth ratio relative to the LUL equity-to-wealth ratio. The LUL plots have a zero carry-over loss entering the trading period. The parameters used are the one stock Base Case parameters and the Capital Gain Tax 30% Case parameters summarized in Section 2.4.

Panel A — Base Case

LUL Exiting Equity-to-Wealth Ratio - Age 20

Entering Equity-to-Wealth Ratio \ Entering Basis-to-Price Ratio	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2
0.30	0.43	0.43	0.44	0.44	0.45	0.46	0.49	0.49
0.40	0.43	0.43	0.43	0.44	0.45	0.46	0.49	0.49
0.50	0.50	0.50	0.50	0.50	0.51	0.46	0.49	0.50
0.60	0.61	0.61	0.60	0.57	0.53	0.46	0.49	0.50
0.70	0.63	0.62	0.60	0.57	0.53	0.46	0.49	0.50

FUL Exiting Equity-to-Wealth Ratio - Age 20

Entering Equity-to-Wealth Ratio \ Entering Basis-to-Price Ratio	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2
0.30	0.47	0.48	0.50	0.52	0.55	0.57	0.58	0.58
0.40	0.45	0.46	0.48	0.51	0.54	0.57	0.58	0.58
0.50	0.50	0.50	0.50	0.50	0.53	0.57	0.58	0.59
0.60	0.61	0.61	0.60	0.59	0.58	0.57	0.59	0.59
0.70	0.64	0.63	0.61	0.59	0.58	0.57	0.58	0.59

% Change ((FUL-LUL)/LUL) - Age 20

Entering Equity-to-Wealth Ratio \ Entering Basis-to-Price Ratio	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2
0.30	7.9%	11.4%	15.0%	18.0%	21.8%	25.2%	19.9%	17.7%
0.40	3.6%	7.2%	11.4%	16.6%	21.1%	25.2%	18.2%	18.2%
0.50	0.0%	0.0%	0.0%	0.0%	4.8%	24.8%	17.7%	17.7%
0.60	0.0%	0.0%	1.4%	4.1%	8.6%	24.6%	18.7%	17.5%
0.70	0.9%	1.2%	2.1%	4.3%	9.6%	24.7%	18.4%	18.3%

LUL Exiting Equity-to-Wealth Ratio - Age 80

Entering Equity-to-Wealth Ratio \ Entering Basis-to-Price Ratio	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2
0.30	0.47	0.46	0.46	0.46	0.47	0.47	0.49	0.50
0.40	0.47	0.47	0.46	0.46	0.47	0.47	0.50	0.50
0.50	0.50	0.50	0.50	0.50	0.50	0.47	0.50	0.50
0.60	0.60	0.60	0.60	0.60	0.55	0.47	0.50	0.50
0.70	0.70	0.68	0.64	0.61	0.55	0.47	0.50	0.50

FUL Exiting Equity-to-Wealth Ratio - Age 80

Entering Equity-to-Wealth Ratio \ Entering Basis-to-Price Ratio	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2
0.30	0.49	0.50	0.52	0.54	0.57	0.59	0.59	0.60
0.40	0.48	0.49	0.50	0.52	0.56	0.59	0.59	0.60
0.50	0.50	0.50	0.50	0.51	0.55	0.59	0.59	0.60
0.60	0.60	0.60	0.60	0.60	0.60	0.59	0.60	0.60
0.70	0.70	0.69	0.66	0.63	0.61	0.59	0.60	0.61

% Change ((FUL-LUL)/LUL) - Age 80

Entering Equity-to-Wealth Ratio \ Entering Basis-to-Price Ratio	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2
0.30	4.9%	7.5%	12.3%	16.2%	22.3%	26.3%	19.7%	20.2%
0.40	2.5%	4.3%	7.9%	14.1%	21.0%	25.2%	19.7%	19.6%
0.50	0.0%	0.0%	0.0%	2.1%	8.8%	24.7%	19.4%	19.8%
0.60	0.0%	0.0%	0.0%	0.3%	8.6%	24.5%	18.9%	20.5%
0.70	0.0%	1.3%	2.1%	3.8%	9.8%	24.8%	19.1%	20.7%

Panel B — Capital Gain Tax 30% Case

LUL Exiting Equity-to-Wealth Ratio - Age 20

Entering Equity-to-Wealth Ratio \ Entering Basis-to-Price Ratio	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2
0.30	0.44	0.43	0.43	0.43	0.44	0.45	0.48	0.49
0.40	0.44	0.44	0.43	0.43	0.44	0.45	0.49	0.49
0.50	0.50	0.50	0.50	0.50	0.50	0.45	0.49	0.50
0.60	0.61	0.61	0.61	0.59	0.54	0.45	0.49	0.50
0.70	0.68	0.65	0.62	0.60	0.54	0.45	0.49	0.50

FUL Exiting Equity-to-Wealth Ratio - Age 20

Entering Equity-to-Wealth Ratio \ Entering Basis-to-Price Ratio	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2
0.30	0.48	0.49	0.52	0.54	0.58	0.61	0.62	0.62
0.40	0.46	0.47	0.49	0.53	0.57	0.61	0.62	0.63
0.50	0.50	0.50	0.50	0.51	0.55	0.61	0.62	0.63
0.60	0.61	0.61	0.61	0.61	0.61	0.61	0.62	0.64
0.70	0.70	0.68	0.66	0.63	0.62	0.61	0.63	0.64

% Change ((FUL-LUL)/LUL) - Age 20

Entering Equity-to-Wealth Ratio \ Entering Basis-to-Price Ratio	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2
0.30	10.0%	14.5%	19.8%	25.1%	31.4%	36.8%	27.9%	27.1%
0.40	4.6%	8.9%	15.7%	22.3%	30.2%	36.9%	27.0%	27.4%
0.50	0.0%	0.0%	0.0%	1.6%	9.4%	35.9%	26.7%	26.5%
0.60	0.0%	0.0%	0.0%	2.1%	11.9%	35.9%	27.2%	27.1%
0.70	2.7%	3.9%	5.3%	6.1%	14.8%	35.9%	27.4%	27.8%

LUL Exiting Equity-to-Wealth Ratio - Age 80

Entering Equity-to-Wealth Ratio \ Entering Basis-to-Price Ratio	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2
0.30	0.47	0.46	0.46	0.46	0.46	0.47	0.50	0.50
0.40	0.48	0.47	0.46	0.46	0.46	0.47	0.50	0.50
0.50	0.50	0.50	0.50	0.50	0.50	0.47	0.50	0.50
0.60	0.60	0.60	0.60	0.60	0.59	0.47	0.50	0.50
0.70	0.70	0.70	0.70	0.66	0.59	0.47	0.50	0.50

FUL Exiting Equity-to-Wealth Ratio - Age 80

Entering Equity-to-Wealth Ratio \ Entering Basis-to-Price Ratio	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2
0.30	0.52	0.54	0.56	0.58	0.62	0.64	0.65	0.66
0.40	0.50	0.51	0.54	0.57	0.61	0.65	0.65	0.66
0.50	0.50	0.50	0.52	0.56	0.60	0.64	0.65	0.66
0.60	0.60	0.60	0.60	0.60	0.60	0.64	0.66	0.67
0.70	0.70	0.70	0.70	0.70	0.67	0.65	0.66	0.67

% Change ((FUL-LUL)/LUL) - Age 80

Entering Equity-to-Wealth Ratio \ Entering Basis-to-Price Ratio	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2
0.30	11.0%	16.5%	21.1%	25.9%	34.3%	37.6%	30.8%	31.5%
0.40	5.0%	9.2%	16.8%	23.2%	32.7%	37.8%	31.5%	31.4%
0.50	0.0%	0.4%	3.5%	10.8%	19.7%	36.3%	31.3%	32.5%
0.60	0.0%	0.0%	0.0%	0.0%	2.1%	36.4%	31.3%	33.3%
0.70	0.0%	0.0%	0.0%	5.3%	12.9%	36.7%	31.1%	33.9%

Table 2: **Higher Risk Aversion and No Tax Forgiveness at Death - One Stock.** Panel A (Panel B) summarizes optimal portfolio choice as a function of the equity-to-wealth ratio and the basis-to-price ratio entering the trading period for the LUL and FUL cases for the Higher Risk Aversion Case (No Tax Forgiveness at Death Case). In each panel, the equity-to-wealth ratios at age 20 and 80 are presented. The right panels compute the percentage increase in the FUL equity-to-wealth ratio relative to the LUL equity-to-wealth ratio. The LUL plots have a zero carry-over loss entering the trading period. The parameters used are the one stock Higher Risk Aversion Case parameters and the No Tax Forgiveness at Death Case parameters summarized in Section 2.4.

Panel A — Higher Risk Aversion Case

LUL Exiting Equity-to-Wealth Ratio - Age 20

Entering Equity-to-Wealth Ratio	Entering Basis-to-Price Ratio							
	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2
0.05	0.22	0.22	0.22	0.22	0.23	0.23	0.23	0.24
0.15	0.20	0.20	0.21	0.21	0.22	0.23	0.24	0.24
0.25	0.25	0.25	0.25	0.25	0.25	0.23	0.24	0.25
0.35	0.35	0.33	0.31	0.30	0.27	0.23	0.24	0.25
0.45	0.35	0.33	0.31	0.29	0.27	0.23	0.25	0.25

FUL Exiting Equity-to-Wealth Ratio - Age 20

Entering Equity-to-Wealth Ratio	Entering Basis-to-Price Ratio							
	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2
0.05	0.27	0.28	0.28	0.29	0.29	0.29	0.29	0.30
0.15	0.22	0.23	0.25	0.27	0.28	0.29	0.29	0.29
0.25	0.25	0.25	0.25	0.25	0.27	0.29	0.29	0.29
0.35	0.35	0.34	0.32	0.31	0.30	0.29	0.29	0.29
0.45	0.35	0.34	0.32	0.31	0.30	0.29	0.29	0.30

% Change ((FUL-LUL)/LUL) - Age 20

Entering Equity-to-Wealth Ratio	Entering Basis-to-Price Ratio							
	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2
0.05	25.6%	22.9%	25.3%	26.9%	25.7%	25.7%	26.0%	25.0%
0.15	8.4%	15.4%	19.2%	24.2%	25.7%	25.2%	21.5%	19.8%
0.25	0.0%	0.0%	0.0%	0.0%	6.7%	24.9%	20.3%	18.8%
0.35	0.9%	2.1%	2.6%	4.4%	8.1%	24.6%	19.4%	18.4%
0.45	0.9%	2.9%	2.9%	4.5%	8.8%	25.6%	18.6%	18.1%

LUL Exiting Equity-to-Wealth Ratio - Age 80

Entering Equity-to-Wealth Ratio	Entering Basis-to-Price Ratio							
	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2
0.05	0.22	0.22	0.22	0.22	0.23	0.23	0.23	0.23
0.15	0.20	0.20	0.21	0.22	0.22	0.23	0.24	0.24
0.25	0.25	0.25	0.25	0.25	0.25	0.23	0.24	0.25
0.35	0.35	0.35	0.34	0.31	0.28	0.23	0.24	0.25
0.45	0.38	0.36	0.34	0.31	0.28	0.23	0.25	0.25

FUL Exiting Equity-to-Wealth Ratio - Age 80

Entering Equity-to-Wealth Ratio	Entering Basis-to-Price Ratio							
	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2
0.05	0.26	0.28	0.28	0.29	0.29	0.29	0.29	0.30
0.15	0.22	0.23	0.25	0.26	0.28	0.29	0.30	0.30
0.25	0.25	0.25	0.25	0.25	0.26	0.29	0.29	0.30
0.35	0.35	0.35	0.34	0.32	0.30	0.29	0.30	0.30
0.45	0.38	0.36	0.34	0.32	0.30	0.29	0.30	0.30

% Change ((FUL-LUL)/LUL) - Age 80

Entering Equity-to-Wealth Ratio	Entering Basis-to-Price Ratio							
	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2
0.05	19.3%	25.8%	27.1%	27.7%	28.5%	28.7%	27.5%	29.0%
0.15	10.1%	13.3%	17.9%	20.0%	27.4%	29.0%	24.9%	21.6%
0.25	0.0%	0.0%	0.0%	0.0%	5.4%	28.0%	21.7%	20.6%
0.35	0.0%	0.1%	0.0%	1.6%	9.1%	28.7%	21.4%	19.4%
0.45	0.6%	0.0%	1.3%	4.1%	9.4%	28.5%	19.6%	19.7%

Panel B — No Tax Forgiveness at Death Case

LUL Exiting Equity-to-Wealth Ratio - Age 20

Entering Equity-to-Wealth Ratio	Entering Basis-to-Price Ratio							
	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2
0.30	0.46	0.46	0.47	0.47	0.47	0.47	0.49	0.50
0.40	0.46	0.46	0.46	0.47	0.47	0.47	0.50	0.50
0.50	0.50	0.50	0.50	0.50	0.50	0.48	0.50	0.50
0.60	0.61	0.60	0.59	0.57	0.53	0.48	0.50	0.50
0.70	0.62	0.61	0.59	0.57	0.53	0.48	0.50	0.50

FUL Exiting Equity-to-Wealth Ratio - Age 20

Entering Equity-to-Wealth Ratio	Entering Basis-to-Price Ratio							
	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2
0.30	0.49	0.51	0.53	0.54	0.56	0.58	0.58	0.59
0.40	0.48	0.49	0.51	0.53	0.56	0.58	0.58	0.59
0.50	0.50	0.50	0.50	0.52	0.55	0.58	0.59	0.59
0.60	0.61	0.61	0.60	0.59	0.58	0.58	0.59	0.59
0.70	0.62	0.61	0.60	0.59	0.58	0.58	0.59	0.60

% Change ((FUL-LUL)/LUL) - Age 20

Entering Equity-to-Wealth Ratio	Entering Basis-to-Price Ratio							
	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2
0.30	6.9%	9.5%	12.7%	15.5%	19.2%	22.0%	17.7%	17.0%
0.40	4.2%	6.6%	9.7%	13.8%	18.9%	22.1%	17.7%	17.1%
0.50	0.0%	0.0%	0.0%	2.8%	8.2%	21.4%	17.3%	18.2%
0.60	0.0%	0.2%	1.3%	3.5%	7.7%	21.4%	17.5%	17.7%
0.70	0.2%	0.3%	1.7%	3.8%	8.2%	21.4%	17.4%	19.1%

LUL Exiting Equity-to-Wealth Ratio - Age 80

Entering Equity-to-Wealth Ratio	Entering Basis-to-Price Ratio							
	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2
0.30	0.50	0.50	0.49	0.48	0.48	0.47	0.48	0.49
0.40	0.50	0.50	0.49	0.49	0.48	0.47	0.48	0.48
0.50	0.50	0.50	0.50	0.50	0.50	0.48	0.48	0.49
0.60	0.60	0.60	0.60	0.58	0.55	0.48	0.49	0.48
0.70	0.62	0.61	0.60	0.58	0.54	0.48	0.49	0.49

FUL Exiting Equity-to-Wealth Ratio - Age 80

Entering Equity-to-Wealth Ratio	Entering Basis-to-Price Ratio							
	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2
0.30	0.53	0.54	0.55	0.56	0.57	0.58	0.58	0.59
0.40	0.52	0.53	0.54	0.56	0.57	0.58	0.58	0.59
0.50	0.50	0.52	0.53	0.55	0.56	0.58	0.59	0.59
0.60	0.60	0.60	0.60	0.59	0.58	0.58	0.59	0.59
0.70	0.62	0.61	0.60	0.59	0.58	0.58	0.59	0.60

% Change ((FUL-LUL)/LUL) - Age 80

Entering Equity-to-Wealth Ratio	Entering Basis-to-Price Ratio							
	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2
0.30	7.0%	9.7%	13.5%	16.1%	20.0%	22.6%	20.4%	20.8%
0.40	3.5%	6.4%	10.0%	14.5%	18.6%	22.6%	21.1%	22.3%
0.50	0.0%	2.5%	5.9%	9.5%	12.4%	22.2%	21.1%	22.0%
0.60	0.0%	0.0%	0.0%	1.5%	7.0%	22.3%	21.2%	22.6%
0.70	0.0%	0.0%	0.0%	1.5%	7.3%	22.4%	21.3%	22.4%

Table 3: **Base Case Optimal Strategies - Two Stocks - Age 80.** The table summarizes optimal portfolio choice as a function of the basis-to-price ratios entering the trading period for the LUL and FUL cases. The top and middle panels present results for the LUL and FUL cases respectively. The bottom panels compute the percentage increase in the FUL equity-to-wealth ratio relative to the LUL equity-to-wealth ratio for each stock as well as the total equity allocation. The LUL plots have a zero carry-over loss entering the trading period. The investor enters the period with a stock 1 (stock 2) position of $\underline{\pi}_1 = 0.3$ ($\underline{\pi}_2 = 0.4$). The parameters used are the two stock Base Case parameters summarized in Section 2.4.

Panel A – LUL

Basis-to-Price Ratio Stock 1	Equity-to-Wealth Ratio Stock 1 — Basis-to-Price Ratio Stock 2: 0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2
0.5	0.30	0.30	0.30	0.30	0.30	0.30	0.29	0.24
0.6	0.30	0.30	0.30	0.30	0.30	0.30	0.28	0.24
0.7	0.27	0.28	0.30	0.30	0.30	0.30	0.28	0.25
0.8	0.24	0.24	0.25	0.30	0.30	0.30	0.27	0.25
0.9	0.19	0.20	0.20	0.21	0.29	0.30	0.26	0.25
1.0	0.12	0.13	0.13	0.14	0.15	0.24	0.25	0.25
1.1	0.15	0.16	0.17	0.19	0.21	0.24	0.25	0.25
1.2	0.19	0.20	0.21	0.23	0.24	0.25	0.25	0.25

Basis-to-Price Ratio Stock 1	Equity-to-Wealth Ratio Stock 2 — Basis-to-Price Ratio Stock 2: 0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2
0.5	0.40	0.39	0.36	0.32	0.26	0.20	0.22	0.26
0.6	0.40	0.40	0.36	0.32	0.26	0.19	0.22	0.26
0.7	0.40	0.40	0.37	0.32	0.26	0.19	0.22	0.26
0.8	0.40	0.40	0.40	0.34	0.27	0.19	0.22	0.25
0.9	0.40	0.40	0.40	0.40	0.28	0.20	0.23	0.25
1.0	0.40	0.40	0.40	0.40	0.38	0.24	0.25	0.25
1.1	0.35	0.34	0.32	0.30	0.27	0.24	0.25	0.25
1.2	0.30	0.29	0.28	0.26	0.26	0.25	0.25	0.25

Basis-to-Price Ratio Stock 1	Total Equity-to-Wealth Ratio — Basis-to-Price Ratio Stock 2: 0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2
0.5	0.70	0.69	0.66	0.62	0.56	0.50	0.50	0.50
0.6	0.70	0.70	0.66	0.62	0.56	0.49	0.50	0.50
0.7	0.68	0.68	0.67	0.63	0.56	0.49	0.49	0.50
0.8	0.64	0.65	0.66	0.64	0.57	0.49	0.49	0.50
0.9	0.60	0.60	0.60	0.61	0.57	0.49	0.49	0.50
1.0	0.53	0.53	0.53	0.54	0.53	0.48	0.49	0.50
1.1	0.50	0.49	0.49	0.48	0.48	0.49	0.50	0.50
1.2	0.50	0.49	0.49	0.49	0.50	0.50	0.50	0.50

Panel B – FUL

Basis-to-Price Ratio Stock 1	Equity-to-Wealth Ratio Stock 1 — Basis-to-Price Ratio Stock 2: 0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2
0.5	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30
0.6	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30
0.7	0.28	0.29	0.30	0.30	0.30	0.30	0.30	0.30
0.8	0.25	0.25	0.26	0.30	0.30	0.30	0.30	0.30
0.9	0.22	0.22	0.23	0.25	0.30	0.30	0.30	0.30
1.0	0.19	0.19	0.20	0.20	0.25	0.30	0.30	0.31
1.1	0.19	0.19	0.19	0.20	0.25	0.30	0.31	0.31
1.2	0.19	0.19	0.20	0.21	0.25	0.31	0.31	0.31

Basis-to-Price Ratio Stock 1	Equity-to-Wealth Ratio Stock 2 — Basis-to-Price Ratio Stock 2: 0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2
0.5	0.40	0.40	0.37	0.32	0.28	0.26	0.26	0.26
0.6	0.40	0.40	0.37	0.33	0.29	0.26	0.27	0.27
0.7	0.40	0.40	0.37	0.33	0.30	0.27	0.27	0.28
0.8	0.40	0.40	0.40	0.34	0.31	0.28	0.28	0.29
0.9	0.40	0.40	0.40	0.39	0.32	0.29	0.29	0.30
1.0	0.40	0.40	0.40	0.40	0.36	0.30	0.30	0.31
1.1	0.40	0.40	0.40	0.40	0.36	0.30	0.31	0.31
1.2	0.40	0.40	0.40	0.40	0.36	0.31	0.31	0.31

Basis-to-Price Ratio Stock 1	Total Equity-to-Wealth Ratio — Basis-to-Price Ratio Stock 2: 0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2
0.5	0.70	0.70	0.67	0.62	0.58	0.56	0.56	0.56
0.6	0.70	0.70	0.67	0.63	0.59	0.57	0.57	0.57
0.7	0.68	0.69	0.68	0.64	0.60	0.57	0.57	0.58
0.8	0.65	0.65	0.66	0.65	0.61	0.58	0.58	0.59
0.9	0.62	0.62	0.63	0.64	0.62	0.59	0.60	0.60
1.0	0.59	0.59	0.60	0.61	0.61	0.60	0.61	0.61
1.1	0.59	0.59	0.60	0.61	0.61	0.61	0.61	0.62
1.2	0.60	0.60	0.60	0.61	0.62	0.61	0.61	0.62

Panel C – (FUL-LUL)/LUL

Basis-to-Price Ratio Stock 1	Equity-to-Wealth Ratio Stock 1 — Basis-to-Price Ratio Stock 2: 0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2
0.5	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	5.2%	25.6%
0.6	0.4%	0.0%	0.0%	0.0%	0.0%	0.4%	7.0%	24.0%
0.7	1.6%	1.9%	0.1%	0.0%	0.0%	0.8%	9.3%	22.5%
0.8	3.3%	3.3%	3.9%	1.5%	0.0%	1.1%	11.7%	21.1%
0.9	12.7%	12.2%	12.9%	17.7%	5.5%	1.8%	14.3%	19.6%
1.0	53.6%	50.7%	48.5%	50.2%	69.1%	26.1%	22.6%	22.0%
1.1	29.3%	20.6%	13.2%	8.0%	17.8%	24.5%	22.5%	24.2%
1.2	0.4%	-4.5%	-8.1%	-10.8%	4.3%	22.1%	23.5%	25.0%

Basis-to-Price Ratio Stock 1	Equity-to-Wealth Ratio Stock 2 — Basis-to-Price Ratio Stock 2: 0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2
0.5	0.0%	1.7%	1.7%	0.5%	7.7%	33.2%	20.7%	0.3%
0.6	0.0%	1.1%	1.8%	2.7%	11.0%	36.9%	22.5%	3.5%
0.7	0.0%	0.0%	1.4%	3.3%	13.9%	41.2%	25.6%	7.7%
0.8	0.0%	0.0%	0.0%	1.5%	16.5%	45.1%	28.6%	13.3%
0.9	0.0%	0.0%	0.0%	-2.4%	13.6%	48.4%	30.1%	20.6%
1.0	0.3%	0.2%	0.2%	0.4%	-6.3%	26.1%	22.6%	22.0%
1.1	14.7%	19.6%	26.5%	36.1%	34.2%	24.5%	22.5%	24.2%
1.2	32.2%	38.5%	44.5%	56.7%	42.4%	22.1%	23.5%	25.0%

Basis-to-Price Ratio Stock 1	Total Equity-to-Wealth Ratio — Basis-to-Price Ratio Stock 2: 0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2
0.5	0.0%	0.9%	0.9%	0.3%	3.6%	12.9%	11.8%	12.4%
0.6	0.2%	0.6%	1.0%	1.4%	5.1%	14.7%	13.7%	13.4%
0.7	0.6%	0.7%	0.8%	1.7%	6.5%	16.6%	16.5%	15.0%
0.8	1.2%	1.3%	1.4%	1.5%	7.7%	18.5%	19.3%	17.2%
0.9	4.1%	4.0%	4.3%	4.6%	9.5%	20.4%	21.6%	20.1%
1.0	13.0%	12.3%	12.2%	13.1%	15.1%	26.1%	22.6%	22.0%
1.1	19.0%	19.9%	21.9%	25.2%	27.0%	24.4%	22.5%	24.2%
1.2	19.9%	20.8%	21.6%	24.8%	24.0%	22.2%	23.5%	25.1%

Table 4: **Capital Gain Tax 30% Case Optimal Strategies - Two Stocks - Age 80.** The table summarizes optimal portfolio choice as a function of the basis-to-price ratios entering the trading period for the LUL and FUL cases. The top and middle panels present results for the LUL and FUL cases respectively. The bottom panels compute the percentage increase in the FUL equity-to-wealth ratio relative to the LUL equity-to-wealth ratio for each stock as well as the total equity allocation. The LUL plots have a zero carry-over loss entering the trading period. The investor enters the period with a stock 1 (stock 2) position of $\underline{\pi}_1 = 0.3$ ($\underline{\pi}_2 = 0.4$). The parameters used are the two stock capital Gain Tax 30% Case parameters summarized in Section 2.4.

Panel A – LUL

	Equity-to-Wealth Ratio Stock 1								Equity-to-Wealth Ratio Stock 2								Total Equity-to-Wealth Ratio							
	Basis-to-Price Ratio Stock 2								Basis-to-Price Ratio Stock 2								Basis-to-Price Ratio Stock 2							
Basis 1	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2
0.5	0.30	0.30	0.30	0.30	0.30	0.30	0.28	0.23	0.40	0.40	0.40	0.37	0.29	0.20	0.22	0.27	0.70	0.70	0.70	0.67	0.60	0.50	0.50	0.50
0.6	0.30	0.30	0.30	0.30	0.30	0.30	0.28	0.23	0.40	0.40	0.40	0.37	0.29	0.20	0.22	0.27	0.70	0.70	0.70	0.67	0.59	0.50	0.50	0.50
0.7	0.30	0.30	0.30	0.30	0.30	0.30	0.27	0.23	0.40	0.40	0.40	0.37	0.29	0.20	0.22	0.26	0.70	0.70	0.70	0.67	0.60	0.50	0.50	0.50
0.8	0.29	0.29	0.30	0.30	0.30	0.30	0.26	0.24	0.40	0.40	0.40	0.38	0.30	0.20	0.23	0.26	0.69	0.69	0.70	0.69	0.60	0.50	0.49	0.50
0.9	0.23	0.23	0.23	0.24	0.30	0.29	0.26	0.24	0.40	0.40	0.40	0.40	0.32	0.20	0.23	0.26	0.63	0.63	0.63	0.64	0.61	0.50	0.49	0.50
1.0	0.13	0.13	0.13	0.13	0.13	0.24	0.25	0.25	0.40	0.40	0.40	0.40	0.40	0.24	0.25	0.25	0.53	0.53	0.53	0.53	0.53	0.48	0.49	0.50
1.1	0.15	0.16	0.17	0.19	0.22	0.24	0.25	0.25	0.35	0.34	0.32	0.29	0.26	0.24	0.25	0.25	0.50	0.49	0.49	0.48	0.48	0.49	0.50	0.50
1.2	0.19	0.20	0.22	0.24	0.25	0.25	0.25	0.25	0.33	0.31	0.28	0.26	0.24	0.25	0.25	0.25	0.52	0.51	0.50	0.49	0.50	0.50	0.50	0.50

Panel B – FUL

	Equity-to-Wealth Ratio Stock 1								Equity-to-Wealth Ratio Stock 2								Total Equity-to-Wealth Ratio							
	Basis-to-Price Ratio Stock 2								Basis-to-Price Ratio Stock 2								Basis-to-Price Ratio Stock 2							
Basis 1	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2
0.5	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.40	0.40	0.40	0.40	0.37	0.32	0.32	0.33	0.70	0.70	0.70	0.70	0.67	0.62	0.62	0.63
0.6	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.40	0.40	0.40	0.40	0.37	0.31	0.32	0.33	0.70	0.70	0.70	0.70	0.67	0.62	0.62	0.63
0.7	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.40	0.40	0.40	0.40	0.37	0.32	0.32	0.34	0.70	0.70	0.70	0.70	0.68	0.62	0.62	0.64
0.8	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.40	0.40	0.40	0.40	0.38	0.34	0.34	0.35	0.70	0.70	0.70	0.70	0.68	0.64	0.64	0.65
0.9	0.26	0.26	0.27	0.28	0.30	0.30	0.30	0.30	0.40	0.40	0.40	0.40	0.39	0.35	0.35	0.36	0.66	0.66	0.67	0.68	0.69	0.65	0.65	0.66
1.0	0.22	0.22	0.23	0.24	0.26	0.33	0.34	0.34	0.40	0.40	0.40	0.40	0.40	0.33	0.34	0.34	0.63	0.63	0.63	0.64	0.66	0.67	0.67	0.68
1.1	0.22	0.22	0.23	0.24	0.25	0.34	0.34	0.35	0.40	0.40	0.40	0.40	0.40	0.34	0.34	0.35	0.62	0.62	0.63	0.64	0.66	0.67	0.68	0.69
1.2	0.23	0.23	0.23	0.24	0.26	0.34	0.34	0.35	0.40	0.40	0.40	0.40	0.40	0.34	0.34	0.35	0.63	0.63	0.63	0.64	0.66	0.68	0.69	0.70

Panel C – (FUL-LUL)/LUL

	Equity-to-Wealth Ratio Stock 1								Equity-to-Wealth Ratio Stock 2								Total Equity-to-Wealth Ratio							
	Basis-to-Price Ratio Stock 2								Basis-to-Price Ratio Stock 2								Basis-to-Price Ratio Stock 2							
Basis 1	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2
0.5	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	6.7%	32.5%	0.0%	0.0%	0.0%	8.5%	25.1%	57.5%	44.4%	21.2%	0.0%	0.0%	0.0%	4.7%	12.4%	23.0%	23.2%	26.4%
0.6	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	8.7%	30.4%	0.0%	0.0%	0.0%	9.4%	27.7%	57.9%	41.9%	23.2%	0.0%	0.0%	0.0%	5.2%	13.6%	22.9%	23.4%	26.6%
0.7	0.0%	0.0%	0.0%	0.0%	0.0%	0.9%	11.2%	28.5%	0.0%	0.0%	0.0%	8.8%	27.3%	61.4%	43.6%	28.3%	0.0%	0.0%	0.0%	4.9%	13.5%	24.9%	25.8%	28.4%
0.8	2.9%	3.0%	1.2%	0.0%	0.0%	1.5%	14.2%	26.7%	0.0%	0.0%	0.0%	4.9%	26.1%	68.8%	49.6%	34.7%	1.2%	1.3%	0.5%	2.6%	13.1%	28.5%	30.5%	30.9%
0.9	14.8%	15.7%	17.9%	19.7%	1.6%	2.3%	17.0%	24.4%	0.0%	0.0%	0.0%	0.0%	22.9%	73.8%	52.3%	41.3%	5.4%	5.6%	6.4%	7.1%	12.6%	31.4%	33.7%	33.1%
1.0	69.1%	71.0%	75.9%	80.3%	92.3%	38.7%	36.4%	36.0%	0.3%	0.2%	0.2%	0.3%	0.4%	38.7%	36.4%	36.0%	17.4%	17.5%	18.6%	20.2%	23.4%	38.7%	36.4%	36.0%
1.1	50.7%	42.0%	33.5%	22.2%	16.2%	37.9%	36.8%	37.9%	13.8%	19.1%	26.4%	39.9%	55.3%	37.9%	36.8%	37.9%	24.7%	26.3%	28.9%	32.8%	37.5%	37.9%	36.8%	37.9%
1.2	19.9%	11.3%	4.0%	2.8%	2.4%	36.4%	37.4%	39.3%	22.8%	31.8%	43.6%	55.4%	65.7%	36.4%	37.4%	39.3%	21.8%	23.6%	26.0%	30.4%	33.4%	36.4%	37.4%	39.3%

Table 5: **Different Return Correlations - Two Stock.** The table summarizes optimal portfolio choice as a function of the basis-to-price ratios entering the trading period for the FUL and LUL cases for the Correlation 0.9 Case (Panel A) and the Correlation 0.4 Case (Panel B). In each Panel, the top and middle tables present results for the LUL and FUL cases respectively. The bottom tables in each Panel compute the percentage increase in the FUL equity-to-wealth ratio relative to the LUL equity-to-wealth ratio for each stock as well as the total equity allocation. The LUL cases have a zero carry-over loss entering the trading period. The investor enters the period with a stock 1 (stock 2) position of $\underline{\pi}_1 = 0.3$ ($\underline{\pi}_2 = 0.4$). The parameters used are the two stock Correlation 0.90 Case parameters and the two stock Correlation 0.40 Case parameters summarized in Section 2.4.

Panel A – Correlation 0.90 Case

Basis-to-Price Ratio Stock 1	**LUL Equity-to-Wealth Ratio Stock 1** Basis-to-Price Ratio Stock 2				**LUL Equity-to-Wealth Ratio Stock 2** Basis-to-Price Ratio Stock 2				**LUL Total Equity-to-Wealth Ratio** Basis-to-Price Ratio Stock 2			
	0.6	0.8	1.0	1.2	0.6	0.8	1.0	1.2	0.6	0.8	1.0	1.2
0.6	0.30	0.30	0.30	0.26	0.40	0.32	0.19	0.24	0.70	0.62	0.49	0.50
0.8	0.23	0.29	0.30	0.26	0.40	0.35	0.19	0.24	0.64	0.64	0.49	0.50
1.0	0.10	0.11	0.24	0.25	0.40	0.40	0.24	0.25	0.50	0.51	0.48	0.50
1.2	0.18	0.22	0.25	0.25	0.32	0.27	0.25	0.25	0.51	0.50	0.50	0.50

Basis-to-Price Ratio Stock 1	**FUL Equity-to-Wealth Ratio Stock 1** Basis-to-Price Ratio Stock 2				**FUL Equity-to-Wealth Ratio Stock 2** Basis-to-Price Ratio Stock 2				**FUL Total Equity-to-Wealth Ratio** Basis-to-Price Ratio Stock 2			
	0.6	0.8	1.0	1.2	0.6	0.8	1.0	1.2	0.6	0.8	1.0	1.2
0.6	0.30	0.30	0.30	0.30	0.40	0.33	0.26	0.27	0.70	0.63	0.56	0.57
0.8	0.24	0.30	0.30	0.30	0.40	0.35	0.26	0.27	0.64	0.65	0.57	0.58
1.0	0.17	0.18	0.30	0.30	0.40	0.40	0.30	0.30	0.57	0.58	0.59	0.60
1.2	0.17	0.18	0.30	0.31	0.40	0.40	0.30	0.31	0.57	0.58	0.60	0.61

Basis-to-Price Ratio Stock 1	**% Change Stock 1 Equity-to-Wealth Ratio** Basis-to-Price Ratio Stock 2				**% Change Stock 2 Equity-to-Wealth Ratio** Basis-to-Price Ratio Stock 2				**% Change Total Equity-to-Wealth Ratio** Basis-to-Price Ratio Stock 2			
	0.6	0.8	1.0	1.2	0.6	0.8	1.0	1.2	0.6	0.8	1.0	1.2
0.6	0%	0%	0%	17%	0%	4%	38%	9%	0%	2%	15%	13%
0.8	1%	1%	0%	17%	0%	2%	43%	13%	0%	2%	17%	15%
1.0	61%	65%	24%	20%	0%	0%	24%	20%	13%	14%	24%	20%
1.2	-8%	-19%	22%	23%	24%	47%	22%	23%	13%	17%	22%	23%

Panel B - Correlation 0.40 Case

Basis-to-Price Ratio Stock 1	**LUL Equity-to-Wealth Ratio Stock 1** Basis-to-Price Ratio Stock 2				**LUL Equity-to-Wealth Ratio Stock 2** Basis-to-Price Ratio Stock 2				**LUL Total Equity-to-Wealth Ratio** Basis-to-Price Ratio Stock 2			
	0.6	0.8	1.0	1.2	0.6	0.8	1.0	1.2	0.6	0.8	1.0	1.2
0.6	0.30	0.30	0.30	0.25	0.37	0.31	0.22	0.25	0.67	0.61	0.52	0.50
0.8	0.28	0.30	0.30	0.25	0.39	0.32	0.22	0.25	0.68	0.62	0.52	0.50
1.0	0.20	0.20	0.24	0.25	0.40	0.37	0.24	0.25	0.60	0.57	0.48	0.50
1.2	0.22	0.24	0.25	0.25	0.27	0.25	0.25	0.25	0.48	0.49	0.50	0.50

Basis-to-Price Ratio Stock 1	**FUL Equity-to-Wealth Ratio Stock 1** Basis-to-Price Ratio Stock 2				**FUL Equity-to-Wealth Ratio Stock 2** Basis-to-Price Ratio Stock 2				**FUL Total Equity-to-Wealth Ratio** Basis-to-Price Ratio Stock 2			
	0.6	0.8	1.0	1.2	0.6	0.8	1.0	1.2	0.6	0.8	1.0	1.2
0.6	0.30	0.30	0.30	0.30	0.38	0.33	0.29	0.29	0.68	0.63	0.59	0.60
0.8	0.30	0.30	0.30	0.30	0.38	0.33	0.29	0.30	0.68	0.63	0.60	0.60
1.0	0.25	0.28	0.30	0.30	0.40	0.35	0.30	0.30	0.65	0.62	0.60	0.61
1.2	0.25	0.28	0.30	0.31	0.40	0.35	0.30	0.31	0.65	0.63	0.61	0.62

Basis-to-Price Ratio Stock 1	**% Change Stock 1 Equity-to-Wealth Ratio** Basis-to-Price Ratio Stock 2				**% Change Stock 2 Equity-to-Wealth Ratio** Basis-to-Price Ratio Stock 2				**% Change Total Equity-to-Wealth Ratio** Basis-to-Price Ratio Stock 2			
	0.6	0.8	1.0	1.2	0.6	0.8	1.0	1.2	0.6	0.8	1.0	1.2
0.6	0%	0%	1%	19%	2%	5%	33%	19%	1%	3%	14%	19%
0.8	5%	0%	1%	20%	-2%	3%	33%	20%	1%	2%	15%	20%
1.0	25%	35%	25%	22%	0%	-7%	25%	22%	8%	8%	25%	22%
1.2	15%	18%	22%	24%	51%	40%	22%	24%	35%	29%	22%	24%

Table 6: **One Stock Simulations.** This table presents simulation results for portfolio characteristics under the LUL and the FUL cases at ages 40, 60, and 80 over 50,000 paths. The investor starts at age 20 with no embedded capital gains and zero carry-over loss (LUL cases). The parameters used are the one stock Base Case and Capital Gain Tax 30% parameters summarized in Section 2.4.

Panel A: One Stock Base Case

		Wealth ($)		Equity-to-Wealth Ratio		Basis-to-Price Ratio		Cumulative Capital Gain Tax-to-Wealth Ratio		LUL Carry Over Loss-to-Wealth Ratio
	Percentile	LUL	FUL	LUL	FUL	LUL	FUL	LUL	FUL	
Age 40	5%	160.5	161.3	46.0%	52.3%	7.8%	7.8%	0.00%	-2.03%	0.00%
	10%	178.8	183.4	48.5%	55.3%	10.0%	10.0%	0.00%	-1.12%	0.00%
	25%	220.1	233.3	52.1%	60.3%	15.3%	15.3%	0.00%	-0.11%	0.00%
	50%	290.9	316.5	61.1%	66.0%	23.8%	23.8%	0.00%	0.38%	0.00%
	75%	403.5	437.9	68.4%	70.4%	37.5%	36.5%	0.33%	1.01%	0.00%
	90%	568.0	610.1	72.4%	72.6%	56.6%	54.5%	1.15%	1.65%	1.48%
	95%	647.9	707.4	73.2%	73.4%	69.7%	67.7%	1.60%	2.02%	4.93%
	Mean	337.7	364.6	60.5%	64.8%	29.1%	28.7%	0.29%	0.27%	0.83%
	Std. Dev.	172.0	187.7	8.9%	6.6%	19.2%	18.6%	0.56%	1.38%	3.73%
Age 60	5%	356.3	375.9	48.2%	51.5%	1.2%	1.2%	0.00%	-0.56%	0.00%
	10%	429.7	465.3	51.6%	55.3%	1.8%	1.8%	0.00%	-0.18%	0.00%
	25%	615.0	684.0	59.4%	62.3%	3.1%	3.1%	0.00%	0.14%	0.00%
	50%	988.5	1,109.8	68.1%	69.5%	6.2%	6.1%	0.19%	0.59%	0.00%
	75%	1,631.4	1,833.4	74.4%	74.9%	12.7%	12.1%	0.91%	1.19%	0.00%
	90%	2,614.2	2,847.1	77.9%	77.9%	24.6%	22.6%	1.55%	1.75%	0.00%
	95%	3,420.4	3,841.1	78.3%	78.8%	36.3%	32.6%	1.90%	2.09%	0.00%
	Mean	1,346.3	1,479.9	66.3%	67.9%	10.6%	10.0%	0.52%	0.65%	0.10%
	Std. Dev.	1,182.6	1,276.5	9.5%	8.4%	12.6%	11.6%	0.67%	0.91%	1.41%
Age 80	5%	904.6	983.7	51.0%	52.2%	0.2%	0.2%	0.00%	-0.15%	0.00%
	10%	1,182.1	1,311.7	55.2%	56.4%	0.3%	0.3%	0.00%	-0.03%	0.00%
	25%	1,928.6	2,177.4	63.2%	64.3%	0.7%	0.7%	0.00%	0.07%	0.00%
	50%	3,621.8	4,083.6	71.6%	72.4%	1.7%	1.6%	0.14%	0.26%	0.00%
	75%	7,051.4	7,864.5	78.1%	78.5%	4.1%	3.9%	0.45%	0.54%	0.00%
	90%	12,928.9	14,155.2	82.3%	82.6%	11.1%	9.7%	0.79%	0.86%	0.00%
	95%	18,514.4	20,245.4	84.3%	84.5%	19.5%	16.7%	1.01%	1.07%	0.00%
	Mean	5,990.0	6,618.7	70.1%	70.8%	4.5%	4.1%	0.28%	0.33%	0.01%
	Std. Dev.	7,776.4	8,452.5	10.1%	9.8%	8.4%	7.6%	0.36%	0.44%	0.53%

Panel B: One Stock Capital Gain Tax 30% Case

		Wealth ($)		Equity-to-Wealth Ratio		Basis-to-Price Ratio		Cumulative Capital Gain Tax-to-Wealth Ratio		LUL Carry Over Loss-to-Wealth Ratio
	Percentile	LUL	FUL	LUL	FUL	LUL	FUL	LUL	FUL	
Age 40	5%	160.8	162.0	45.6%	56.8%	7.8%	7.8%	0.00%	-3.49%	0.00%
	10%	179.4	187.1	48.0%	60.0%	10.0%	10.0%	0.00%	-2.09%	0.00%
	25%	219.2	240.2	51.7%	65.5%	15.4%	15.3%	0.00%	-0.62%	0.00%
	50%	287.8	334.1	60.8%	71.9%	23.8%	23.8%	0.00%	0.10%	0.00%
	75%	398.5	477.6	69.8%	77.2%	37.9%	36.5%	0.00%	0.44%	0.00%
	90%	570.5	691.7	76.4%	80.6%	57.0%	53.8%	0.00%	0.95%	0.00%
	95%	669.1	821.3	79.7%	81.9%	70.0%	67.1%	0.00%	1.31%	4.59%
	Mean	339.6	397.9	61.3%	70.9%	29.2%	28.6%	0.03%	-0.37%	0.74%
	Std. Dev.	185.1	231.0	10.7%	7.8%	19.3%	18.5%	0.18%	1.82%	3.64%
Age 60	5%	355.5	385.1	49.2%	55.4%	1.2%	1.2%	0.00%	-1.20%	0.00%
	10%	429.8	483.2	52.5%	59.8%	1.8%	1.8%	0.00%	-0.63%	0.00%
	25%	612.6	731.7	60.8%	67.6%	3.1%	3.1%	0.00%	-0.14%	0.00%
	50%	986.9	1,251.2	70.3%	75.2%	6.2%	6.0%	0.00%	0.05%	0.00%
	75%	1,697.0	2,206.1	77.8%	80.9%	12.9%	11.8%	0.00%	0.24%	0.00%
	90%	2,871.6	3,627.1	82.9%	84.7%	25.1%	21.2%	0.05%	0.58%	0.00%
	95%	3,970.0	5,139.5	85.1%	86.4%	37.8%	30.8%	0.39%	0.84%	0.00%
	Mean	1,447.6	1,807.9	69.0%	73.6%	10.8%	9.8%	0.05%	-0.04%	0.09%
	Std. Dev.	1,504.9	1,860.9	11.0%	9.4%	12.9%	11.2%	0.20%	0.88%	1.40%
Age 80	5%	904.6	1,030.0	51.8%	54.8%	0.2%	0.2%	0.00%	-0.37%	0.00%
	10%	1,181.7	1,387.4	56.3%	59.6%	0.3%	0.3%	0.00%	-0.18%	0.00%
	25%	1,930.5	2,406.7	64.9%	68.0%	0.7%	0.7%	0.00%	-0.03%	0.00%
	50%	3,695.4	4,799.4	73.6%	75.7%	1.7%	1.6%	0.00%	0.01%	0.00%
	75%	7,644.6	9,971.7	80.2%	81.6%	3.9%	3.6%	0.00%	0.07%	0.00%
	90%	14,878.9	19,228.0	84.6%	85.4%	10.4%	8.2%	0.01%	0.17%	0.00%
	95%	22,829.6	29,064.7	86.6%	87.2%	18.4%	14.1%	0.10%	0.27%	0.00%
	Mean	6,907.8	8,828.3	71.9%	74.0%	4.3%	3.7%	0.02%	-0.02%	0.01%
	Std. Dev.	11,131.1	13,881.4	10.5%	9.8%	8.2%	6.7%	0.07%	0.36%	0.54%

Table 7: **Two Stock Simulations.** This table presents simulation results for portfolio characteristics under the LUL and the FUL cases at ages 40, 60, and 80 over 50,000 paths. The investor starts at age 20 with no embedded capital gains and zero carry-over loss (LUL cases). The parameters used are the two stock Base Case and Capital Gain Tax 30% parameters summarized in Section 2.4.

Panel A: Two Stock Base Case

		Wealth ($)		Stock 1 Equity-to-Wealth Ratio		Stock 2 Equity-to-Wealth Ratio		Total Equity-to-Wealth Ratio		Stock 1 Basis-to-Price Ratio		Stock 2 Basis-to-Price Ratio		Cumulative Capital Gain Tax-to-Wealth Ratio		LUL Carry Over Loss-to-Wealth Ratio
	Percentile	LUL	FUL	LUL	FUL	LUL	FUL	LUL	FUL	LUL	FUL	LUL	FUL	LUL	FUL	
Age 40	5%	158.9	159.9	18.7%	20.1%	24.8%	29.8%	46.4%	54.9%	9.1%	9.1%	6.5%	6.5%	0.00%	-2.36%	0.00%
	10%	179.5	184.8	20.1%	22.0%	25.9%	31.6%	48.5%	57.2%	11.9%	11.9%	8.2%	8.2%	0.00%	-1.38%	0.00%
	25%	221.7	237.7	22.6%	25.1%	29.0%	34.8%	52.3%	62.2%	17.9%	17.9%	12.8%	12.7%	0.00%	-0.30%	0.00%
	50%	293.3	326.6	25.6%	28.7%	34.7%	38.1%	61.3%	67.7%	28.7%	28.7%	20.4%	20.3%	0.00%	0.29%	0.00%
	75%	409.8	459.2	29.9%	32.3%	39.3%	41.2%	69.1%	71.8%	44.5%	44.4%	32.2%	31.8%	0.19%	0.94%	0.00%
	90%	562.1	629.1	33.4%	34.9%	43.1%	44.4%	73.2%	74.2%	62.7%	63.3%	49.2%	47.5%	1.04%	1.55%	2.16%
	95%	683.4	761.1	35.2%	36.1%	45.4%	46.3%	74.5%	75.1%	74.9%	75.8%	65.3%	59.6%	1.49%	1.91%	6.37%
	Mean	341.6	376.5	26.2%	28.5%	34.6%	38.1%	60.8%	66.6%	33.5%	33.6%	25.5%	24.9%	0.25%	0.13%	0.97%
	Std. Dev.	178.3	201.6	5.0%	4.9%	6.5%	4.9%	9.3%	6.4%	20.4%	20.5%	18.6%	17.4%	0.52%	1.45%	3.88%
Age 60	5%	361.4	386.2	16.2%	16.9%	26.0%	29.8%	47.1%	52.4%	1.5%	1.6%	1.0%	1.0%	0.00%	-0.71%	0.00%
	10%	432.8	477.3	18.4%	19.2%	28.7%	32.3%	50.7%	56.5%	2.3%	2.3%	1.4%	1.4%	0.00%	-0.28%	0.00%
	25%	619.5	709.7	22.0%	23.2%	34.2%	36.7%	59.2%	63.6%	4.2%	4.2%	2.5%	2.5%	0.00%	0.10%	0.00%
	50%	996.9	1,148.1	26.7%	28.1%	39.6%	41.1%	67.9%	70.4%	8.2%	8.2%	5.0%	5.0%	0.16%	0.59%	0.00%
	75%	1,679.1	1,914.2	31.6%	32.5%	44.2%	45.6%	74.2%	75.3%	16.0%	15.7%	9.8%	9.7%	0.94%	1.22%	0.00%
	90%	2,706.5	3,072.3	35.2%	35.2%	49.2%	49.7%	77.6%	78.1%	29.0%	27.7%	18.3%	17.3%	1.57%	1.78%	0.00%
	95%	3,590.8	4,045.3	37.0%	36.4%	52.2%	52.4%	78.6%	78.8%	41.6%	37.9%	27.4%	25.4%	1.92%	2.11%	0.00%
	Mean	1,368.0	1,553.8	26.7%	27.6%	39.3%	41.1%	66.0%	68.6%	12.9%	12.4%	8.4%	8.2%	0.52%	0.64%	0.11%
	Std. Dev.	1,224.8	1,372.7	6.4%	6.1%	7.7%	6.8%	9.9%	8.3%	13.8%	12.8%	11.0%	10.4%	0.68%	0.95%	1.23%
Age 80	5%	906.4	1,014.5	14.1%	14.4%	28.2%	29.8%	49.3%	52.0%	0.3%	0.3%	0.2%	0.2%	0.00%	-0.19%	0.00%
	10%	1,160.5	1,332.9	16.7%	17.0%	31.2%	32.8%	53.7%	56.3%	0.5%	0.5%	0.3%	0.3%	0.00%	-0.05%	0.00%
	25%	1,922.5	2,256.2	21.0%	21.6%	36.9%	38.0%	62.4%	64.3%	1.0%	1.0%	0.6%	0.6%	0.00%	0.08%	0.00%
	50%	3,641.5	4,248.8	26.2%	26.8%	42.8%	43.6%	71.1%	72.3%	2.4%	2.4%	1.2%	1.2%	0.18%	0.31%	0.00%
	75%	7,076.9	8,151.6	31.6%	32.0%	49.1%	49.9%	77.6%	78.3%	5.6%	5.7%	3.0%	2.9%	0.55%	0.63%	0.00%
	90%	13,048.2	14,898.2	35.6%	35.9%	55.5%	56.0%	81.7%	82.2%	13.8%	12.8%	6.9%	6.5%	0.93%	0.99%	0.00%
	95%	18,666.4	21,183.1	37.6%	37.9%	59.3%	59.5%	83.5%	84.0%	24.6%	20.8%	13.2%	11.3%	1.17%	1.23%	0.00%
	Mean	5,993.5	6,882.7	26.2%	26.6%	43.2%	44.1%	69.3%	70.7%	5.8%	5.3%	3.2%	3.2%	0.33%	0.38%	0.01%
	Std. Dev.	7,641.2	8,640.4	7.2%	7.1%	9.2%	8.9%	10.4%	9.7%	10.2%	8.6%	6.6%	7.0%	0.42%	0.49%	0.40%

Panel B: Two Stock Capital Gain Tax 30% Case

		Wealth ($)		Stock 1 Equity-to-Wealth Ratio		Stock 2 Equity-to-Wealth Ratio		Total Equity-to-Wealth Ratio		Stock 1 Basis-to-Price Ratio		Stock 2 Basis-to-Price Ratio		Cumulative Capital Gain Tax-to-Wealth Ratio		LUL Carry Over Loss-to-Wealth Ratio
	Percentile	LUL	FUL	LUL	FUL	LUL	FUL	LUL	FUL	LUL	FUL	LUL	FUL	LUL	FUL	
Age 40	5%	159.4	161.9	18.0%	20.5%	24.4%	32.4%	45.3%	59.1%	9.2%	9.1%	6.5%	6.5%	0.00%	-3.95%	0.00%
	10%	179.7	187.6	19.5%	22.6%	25.5%	34.2%	47.6%	61.6%	11.9%	11.9%	8.2%	8.2%	0.00%	-2.44%	0.00%
	25%	220.5	245.0	22.0%	26.2%	28.5%	37.8%	51.4%	67.0%	17.9%	17.9%	12.7%	12.7%	0.00%	-0.88%	0.00%
	50%	289.8	344.4	25.0%	30.1%	34.7%	42.0%	60.6%	73.1%	28.7%	28.7%	20.4%	20.3%	0.00%	0.02%	0.00%
	75%	403.4	499.0	29.4%	33.9%	40.8%	46.4%	69.7%	78.1%	44.5%	44.4%	32.2%	31.8%	0.00%	0.36%	0.00%
	90%	561.3	703.3	33.3%	36.9%	46.1%	50.5%	76.4%	81.1%	62.4%	63.5%	49.3%	47.5%	0.00%	1.00%	1.88%
	95%	692.8	865.8	35.5%	38.4%	49.4%	53.0%	79.4%	82.4%	74.1%	76.1%	66.8%	58.9%	0.11%	1.41%	5.33%
	Mean	340.9	408.0	25.8%	29.9%	35.3%	42.3%	61.1%	72.1%	33.4%	33.6%	25.6%	24.8%	0.05%	-0.53%	0.89%
	Std. Dev.	185.6	241.2	5.3%	5.4%	8.0%	6.3%	10.8%	7.3%	20.1%	20.5%	18.9%	17.2%	0.25%	1.96%	3.78%
Age 60	5%	360.5	397.5	15.5%	16.3%	26.0%	32.3%	47.1%	56.6%	1.6%	1.6%	1.0%	1.0%	0.00%	-1.37%	0.00%
	10%	430.2	495.6	17.8%	18.9%	28.8%	35.1%	50.7%	60.9%	2.3%	2.3%	1.4%	1.4%	0.00%	-0.76%	0.00%
	25%	611.0	757.4	21.6%	23.4%	34.8%	39.8%	59.7%	68.3%	4.2%	4.2%	2.5%	2.5%	0.00%	-0.20%	0.00%
	50%	983.0	1,279.0	26.2%	28.7%	41.4%	44.9%	69.6%	75.5%	8.2%	8.2%	5.0%	5.0%	0.00%	0.03%	0.00%
	75%	1,706.7	2,253.1	31.3%	33.6%	47.9%	51.1%	77.4%	80.9%	16.1%	15.6%	9.8%	9.7%	0.00%	0.35%	0.00%
	90%	2,914.2	3,814.9	35.6%	37.2%	54.5%	57.2%	82.3%	84.5%	29.7%	27.7%	18.5%	17.3%	0.41%	0.84%	0.00%
	95%	4,012.7	5,164.4	37.7%	39.0%	58.5%	60.7%	84.4%	86.0%	43.0%	37.6%	28.0%	24.7%	0.87%	1.19%	0.00%
	Mean	1,436.2	1,854.7	26.4%	28.3%	41.6%	45.6%	68.1%	73.9%	13.1%	12.4%	8.4%	8.0%	0.11%	-0.01%	0.10%
	Std. Dev.	1,452.6	1,881.2	6.7%	6.9%	9.7%	8.5%	11.5%	9.0%	14.2%	12.6%	10.8%	9.9%	0.34%	0.97%	1.17%
Age 80	5%	903.6	1,063.3	13.4%	13.2%	29.0%	31.9%	50.9%	55.4%	0.3%	0.3%	0.2%	0.2%	0.00%	-0.44%	0.00%
	10%	1,155.3	1,416.9	16.1%	16.0%	32.2%	35.2%	55.3%	59.8%	0.5%	0.5%	0.3%	0.3%	0.00%	-0.22%	0.00%
	25%	1,899.7	2,489.2	20.7%	21.1%	38.1%	40.5%	64.0%	68.0%	1.0%	1.0%	0.6%	0.5%	0.00%	-0.05%	0.00%
	50%	3,656.2	4,953.7	25.9%	26.8%	44.6%	46.6%	73.0%	75.7%	2.4%	2.3%	1.2%	1.2%	0.00%	0.01%	0.00%
	75%	7,480.5	10,100.8	31.4%	32.4%	52.2%	54.1%	79.8%	81.5%	5.7%	5.3%	3.0%	2.8%	0.00%	0.10%	0.00%
	90%	14,819.2	19,688.6	35.8%	36.6%	59.7%	61.3%	84.2%	85.2%	15.7%	12.0%	7.4%	6.1%	0.13%	0.28%	0.00%
	95%	22,268.4	29,507.8	38.0%	38.6%	64.0%	65.4%	86.3%	87.1%	28.0%	20.0%	14.2%	10.1%	0.30%	0.44%	0.00%
	Mean	6,707.8	8,912.5	25.9%	26.5%	45.4%	47.5%	71.3%	74.1%	6.2%	5.1%	3.3%	3.0%	0.04%	0.00%	0.01%
	Std. Dev.	10,060.0	13,186.9	7.4%	7.7%	10.4%	10.0%	10.7%	9.6%	10.8%	8.3%	6.8%	6.8%	0.14%	0.40%	0.39%

Table 8: **Economic Cost of Taxation.** This table reports the wealth equivalent change in percent of an age 20 NCGT investor due to imposing a capital gain tax. The investor is assumed to initially have no embedded capital gains or losses in his portfolio. The wealth equivalent change is computed such that the investor's utility is the same from the NCGT case to the corresponding capital gain tax case. A positive percentage wealth equivalent change denotes the NCGT investor's welfare improves by paying a capital gain tax. Results are reported for the FUL and LUL cases. The last column computes the difference between these two cases. A positive percentage difference denotes that the FUL investor is better off. Panel A reports results for one stock, while Panel B reports results for two stocks. All parameters are summarized in Section 2.4.

Panel A — One Stock Cases

	FUL	**LUL**	**FUL-LUL**
Base Case	2.2%	-0.4%	2.6%
Capital Gain Tax 30% Case	3.6%	-0.5%	4.1%
Higher Risk Aversion Case	0.7%	-0.4%	1.1%
No Tax Forgiveness at Death Case	-1.4%	-3.5%	2.0%

Panel B — Two Stock Cases

	FUL	**LUL**	**FUL-LUL**
Base Case	2.2%	-0.6%	2.8%
Capital Gain Tax 30% Case	3.7%	-0.7%	4.4%
Two Stock Correlation 0.90 Case	2.2%	-0.6%	2.8%
Two Stock Correlation 0.40 Case	2.2%	-1.0%	3.2%