Appendix for "Interruptible Electricity Contracts from an Electricity Retailer's Point of View: Valuation and Optimal Interruption"

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A. Notation

Temperature model

T_t: actual temperature on day t

$\bar{T}_t$: average temperature for the t-th day in the year

Δ_t^T: difference between actual and average temperatures on day t, $\Delta_t^T = T_t - \bar{T}_t$

ρ_1^T: first order autocorrelation for temperature differences from the average temperature

ρ_2^T: second order autocorrelation for temperature differences from the average temperature

σ_t^T: magnitude of temperature fluctuations on day t

$\sigma_{(0)}^T$: fixed term of temperature fluctuations

$\sigma_{(1)}^T$: magnitude of seasonal term of temperature fluctuations

ϕ: day during the year on which temperature fluctuations are greatest

Load vs. Temperature Model

L_t: load at time t

α_L: load intercept

β_L: marginal expected increase of load per one degree Fahrenheit increase in temperature

σ_L: magnitude of fluctuations in the load-temperature model

Load vs. Price Model

$\beta_{S,l}$: Marginal increase in expected spot price per unit increase in load in the low demand regime

$\alpha_{S,l}$: intercept for the load-price relationship in the low demand regime

$\beta_{S,h}$: Marginal increase in expected spot price per unit increase in load in the high demand regime

$\alpha_{S,h}$: intercept for the load-price relationship in the high demand regime

σ_S: magnitude of fluctuations in the load-price relationship

S_b: supply level that marks the boundary between the low demand and the high demand regimes.

Prices

P_t: spot electricity price at time t

p_{retail}: fixed retail price, charged by the electricity retailer to its retail customers.

p_{reduced}: fixed retail price paid by customers that have signed a pay-in-advance interruptible contract

p_{fine}: fine per unit of interrupted load paid to the customers that have signed a pay-as-you-go interruptible contract

$p_{\text{generation}}$: unit cost per unit load available to the electricity retailer at a fixed price

Interruptible contracts

$L_{\text{advance, daily}}$: maximum amount available for interruption under a pay-in-advance contract, for one day

$L_{\text{advance, yearly}}$: total amount available for interruption under a pay-in-advance contract for one year

$L_{\text{advance, remaining}}$: total amount available for interruption, under a pay-in-advance contract, for the remaining period, $L_{\text{advance, remaining}} \leq L_{\text{advance, yearly}}$

l_{advance}: interrupted load under a pay-in-advance contract, in a particular day, $l_{\text{advance}} \leq L_{\text{advance, daily}}$

$L_{\text{under\_contract}}$: Total daily load of customers under a pay-in-advance interruptible contract

$L_{\text{pago, daily}}$: maximum amount available for interruption under a pay-as-you-go contract for one day

$L_{\text{pago, yearly}}$: total amount available for interruption under a pay-as-you-go contract for one year

$L_{\text{pago, remaining}}$: total amount available for interruption, under a pay-as-you-go contract, for the remaining period, $L_{\text{pago, remaining}} \leq L_{\text{pago, yearly}}$

l_{pago}: interrupted load under a pay-as-you-go contract, in a particular day, $l_{\text{pago}} \leq L_{\text{pago, daily}}$

$L_{\text{generation}}$: power available to the electricity retailer at a fixed price.

Competition

n: number of identical electricity retailers serving the same area.

$\pi^{(i)}$: profit function for the i^{th} retailer.

$l^{(i)}$: amount of load interrupted by the i^{th} retailer.

$L^{(i)}_{\text{generation}}$: power available to the i^{th} retailer at a fixed price.

$\bar{l}$: the total amount of load that can be interrupted in a single day by all the retailers.

$L_{\text{generation}}$: the total power available to all the electricity retailers at a fixed price.

B. Marginal Benefit of Interruption

In this appendix we calculate the marginal benefit from a unit of interruption in the case with an unlimited annual volume of interruption remaining.

From Equation (4) from the paper we have that the net profit from interrupting amounts $l_{\text{advance}}, l_{\text{pago}}$ from the pay-in-advance and the pay-as-you-go interruptible contracts on day $t-1$, is given by $\mathbb{E}(\Delta\pi_t)$, where:

$$\begin{aligned}\frac{\Delta\pi_t(L_t, p_{\text{spot},t}, l_{\text{advance}}, l_{\text{pago}})}{16} =& (L_t - L_{\text{under\_contract}} - l_{\text{pago}})p_{\text{retail}} \\ &+ (L_{\text{under\_contract}} - l_{\text{advance}})p_{\text{reduced}} \\ &- L_{\text{generation}}p_{\text{generation}} \\ &- l_{\text{pago}}p_{\text{fine}} \\ &- (L_t - l_{\text{advance}} - l_{\text{pago}} - L_{\text{generation}})p_{\text{spot},t}.\end{aligned}$$

From this equation we have that the marginal cost of interrupting the pay-in-advance interruptible contract is given by p_{reduced}, while the marginal cost of interrupting the pay-as-you-go interruptible contract is given by $p_{\text{retail}} + p_{\text{fine}}$. The marginal benefit is the same for both types of contracts, and is given by:

$$\frac{\partial}{\partial l}\mathbb{E}\left((L_t - l - L_{\text{generation}})p_{\text{spot}}\right).$$

To calculate the marginal benefit, we define the function:

$$\text{Benefit}(y) = \mathbb{E}\left(p_{\text{spot}}(y + \beta_L\sigma_T\varepsilon_T + \sigma_L\varepsilon_L - L_{\text{generation}})\right),$$

where we have dropped the t subscript, set $\sigma_T = \sigma_t^T$, and where y_t is the expected load on date t, after interruption l,

$$y_t = \beta_L\left(\bar{T}_t + \bar{\Delta}_t^T\right) + \alpha_L - l,$$

where $\bar{\Delta}_t^T$ is the expected temperature deviation from the historical average temperature on date t:

$$\Delta_t^T = \mathbb{E}(\Delta_t^T) = \rho_1^T\Delta_{t-1}^T + \rho_2^T\Delta_{t-1}^T.$$

From Equations (2), (3) from the paper, we have that:

$$\begin{aligned}p_{\text{spot}} =& \beta_{S,l}\left(y+\beta_L\sigma_T\varepsilon_T+\sigma_L\varepsilon_L+\sigma_S\varepsilon_S\right)+\alpha_{S,l}\\&+\Theta\left(y+\beta_L\sigma_T\varepsilon_T+\sigma_L\varepsilon_L+\sigma_S\varepsilon_S-S_b\right)\left[\left(\beta_{S,h}-\beta_{S,l}\right)\left(y+\beta_L\sigma_T\varepsilon_T+\sigma_L\varepsilon_L+\sigma_S\varepsilon_S\right)+\alpha_{S,h}-\alpha_{S,l}\right],\end{aligned}$$

where Θ is the step function, with $\Theta(x)=0$, if $x\leq 0$, and $\Theta(x)=1$, if $x>0$.

The benefit from interruption is then equal to:

$$\begin{aligned}\text{Benefit}(y) =& (\beta_{S,l}y+\alpha_{S,l})(y-L_{\text{generation}})+\beta_{S,l}\left(\beta_L^2(\sigma_T)^2+\sigma_L^2\right)\\&+\mathbb{E}\left[\Theta\left(y+\beta_L\sigma_T\varepsilon_T+\sigma_L\varepsilon_L+\sigma_S\varepsilon_S-S_b\right)\right.\\&\times\left(\left(\beta_{S,h}-\beta_{S,l}\right)\left(y+\beta_L\sigma_L\sigma_T\varepsilon_T+\sigma_L\varepsilon_L+\sigma_S\varepsilon_S\right)+\alpha_{S,h}-\alpha_{S,l}\right)\\&\left.\times\left(y+\beta_L\sigma_T\varepsilon_T+\sigma_L\varepsilon_L-L_{\text{generation}}\right)\right],\end{aligned}\tag{1}$$

where we used that:

$$\begin{aligned}\mathbb{E}\left(\varepsilon_T^2\right)=\mathbb{E}(\varepsilon_L^2)=1,\\\mathbb{E}(\varepsilon_T\varepsilon_L)=\mathbb{E}(\varepsilon_T\varepsilon_S)=\mathbb{E}(\varepsilon_L\varepsilon_S)=0.\end{aligned}$$

Calculating the expected value in Equation (1) above we have:

$$\begin{aligned}
\text{Benefit}(y) =& (\beta_{S,l} y + \alpha_{S,l})(y - L_{\text{generation}}) + \beta_{S,l}\left(\beta_L^2 \sigma_T^2 + \sigma_L^2\right) \\
&+ \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} d\varepsilon_T d\varepsilon_L \frac{e^{-\frac{\varepsilon_T^2+\varepsilon_L^2}{2}}}{2\pi} \int_{-\infty}^{\infty} d\varepsilon_S \frac{e^{-\frac{\varepsilon_S^2}{2}}}{\sqrt{2\pi}} \\
&\quad \times \Theta\left(y + \beta_L \sigma_T \varepsilon_T + \sigma_L \varepsilon_L + \sigma_S \varepsilon_S - S_b\right) \\
&\quad \times \left(\left(\beta_{S,h} - \beta_{S,l}\right)\left(y + \beta_L \sigma_L \sigma_T \varepsilon_T + \sigma_L \varepsilon_L + \sigma_S \varepsilon_S\right) + \alpha_{S,h} - \alpha_{S,l}\right) \\
&\quad \times \left(y + \beta_L \sigma_T \varepsilon_T + \sigma_L \varepsilon_L - L_{\text{generation}}\right), \\
=& (\beta_{S,l} y + \alpha_{S,l})(y - L_{\text{generation}}) + \beta_{S,l}\left(\beta_L^2 \sigma_T^2 + \sigma_L^2\right) \\
&+ \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} d\varepsilon_T d\varepsilon_L \frac{e^{-\frac{\varepsilon_T^2+\varepsilon_L^2}{2}}}{2\pi} \\
&\quad \times \frac{y + \beta_L \sigma_T \varepsilon_T + \sigma_L \varepsilon_L - L_{\text{generation}}}{2} \\
&\quad \times \Bigg[\sqrt{\frac{2}{\pi}} e^{-\frac{\left(y+\beta_L\sigma_T\varepsilon_T+\sigma_L\varepsilon_L-S_b\right)^2}{2\sigma_S^2}} (\beta_{S,h} - \beta_{S,l})\sigma_S \\
&\qquad + \left(\alpha_{S,h} - \alpha_{S,l} + (\beta_{S,h} - \beta_{S,l})\left(y + \beta_L \sigma_T \varepsilon_T + \sigma_L \varepsilon_L\right)\right) \\
&\qquad \times \left(\text{erf}\left(\frac{y + \beta_L \sigma_T \varepsilon_T + \sigma_L \varepsilon_L - S_b}{\sqrt{2}\sigma_S}\right) + 1\right)\Bigg], \\
=& \frac{\beta_{S,h} + \beta_{S,l}}{2}\left(y(y - L_{\text{generation}}) + \beta_L^2 \sigma_T^2 + \sigma_L^2\right) + \frac{\alpha_{S,h} + \alpha_{S,l}}{2}\left(y - L_{\text{generation}}\right) \\
&+ \frac{(\beta_{S,h} - \beta_{S,l})\sigma_S^2\left((y - L_{\text{generation}})\sigma_S^2 + (S_b - L_{\text{generation}})\left(\beta_L^2 \sigma_T^2 + \sigma_L^2\right)\right)}{\sqrt{2\pi}\left(\sigma_S^2 + \sigma_L^2 + \beta_L^2 \sigma_T^2\right)^{3/2}} e^{-\frac{(y-S_b)^2}{2\left(\sigma_S^2+\sigma_L^2+\beta_l^2\sigma_T^2\right)}} \\
&+ \int_{-\infty}^{\infty} \frac{(y - L_{\text{generation}} + \varepsilon)\left(\alpha_{S,h} - \alpha_{S,l} + (\beta_{S,h} - \beta_{S,l})(y + \varepsilon)\right)}{2\sqrt{\beta_L^2 \sigma_T^2 + \sigma_L^2}} \text{erf}\left(\frac{y - S_b + \varepsilon}{\sqrt{2}\sigma_S}\right) \frac{e^{-\frac{\varepsilon^2}{2\left(\beta_L^2\sigma_T^2+\sigma_L^2\right)}}}{\sqrt{2\pi}} d\varepsilon.
\end{aligned}$$

The last integral can be expressed in terms of the following functions:

$$\bar{f}_0(\mu,\sigma) = \int_{-\infty}^{\infty} \frac{e^{-\frac{x^2}{2}}}{\sqrt{2\pi}} \operatorname{erf}(\mu+\sigma x)dx, \quad f_0(\mu) = \bar{f}_0(\mu,1),$$
$$\bar{f}_1(\mu,\sigma) = \int_{-\infty}^{\infty} \frac{e^{-\frac{x^2}{2}}}{\sqrt{2\pi}} x \operatorname{erf}(\mu+\sigma x)dx, \quad f_1(\mu) = \bar{f}_1(\mu,1),$$
$$\bar{f}_2(\mu,\sigma) = \int_{-\infty}^{\infty} \frac{e^{-\frac{x^2}{2}}}{\sqrt{2\pi}} x^2 \operatorname{erf}(\mu+\sigma x)dx, \quad f_2(\mu) = \bar{f}_2(\mu,1) - f_0(\mu).$$

It can be shown that:

$$\bar{f}_0(\mu,\sigma) = f_0\left(\sqrt{\frac{3}{2}}\frac{\mu}{\sqrt{\sigma^2+1/2}}\right),$$
$$\bar{f}_1(\mu,\sigma) = \sqrt{\frac{3}{2}}\frac{\sigma^2}{\sqrt{\sigma^2+1/2}} f_1\left(\sqrt{\frac{3}{2}}\frac{\mu}{\sqrt{\sigma^2+1/2}}\right),$$
$$\bar{f}_2(\mu,\sigma) = \frac{3}{2}\frac{\sigma^4}{\sigma^2+1/2} f_2\left(\sqrt{\frac{3}{2}}\frac{\mu}{\sqrt{\sigma^2+1/2}}\right) + (\sigma^2+1) f_0\left(\sqrt{\frac{3}{2}}\frac{\mu}{\sqrt{\sigma^2+1/2}}\right).$$

In Table 1 we approximate the functions f_0, f_1, f_2 pointwise by rational functions of the absolute value of the argument. The approximation is of the form

$$f_i(x) = \begin{cases} \frac{\sum_{j=0}^{n_i} a^i_j |x|^j}{\sum_{j=0}^{n_i} b^i_j |x|^j} \operatorname{sign}(x), & \text{if } |x| < 7 \\ 1, & \text{otherwise} \end{cases}, \qquad i = 0,1,2, \quad n_0 = 7, n_1 = n_2 = 8$$

The coefficients were chosen using the `MiniMaxApproximation` function in Mathematica and were found to be pointwise accurate with an error smaller than 10^{-7}.

Combining all the previous formulas, we have:

$$
\begin{aligned}
\text{Benefit}(y) = & \frac{\beta_{S,h}+\beta_{S,l}}{2}\left(y(y-L_{\text{generation}})+\beta_L^2\sigma_T^2+\sigma_L^2\right)+\frac{\alpha_{S,h}+\alpha_{S,l}}{2}\left(y-L_{\text{generation}}\right) \\
& + \frac{(\beta_{S,h}-\beta_{S,l})\sigma_S^2\left((y-L_{\text{generation}})\sigma_S^2+(S_b-L_{\text{generation}})\left(\beta_L^2\sigma_T^2+\sigma_L^2\right)\right)}{\sqrt{2\pi}\left(\sigma_S^2+\sigma_L^2+\beta_L^2\sigma_T^2\right)^{3/2}} e^{-\frac{(y-S_b)^2}{2\left(\sigma_S^2+\sigma_L^2+\beta_l^2\sigma_T^2\right)}} \\
& + \frac{1}{2}\left((\beta_{S,h}-\beta_{S,l})\left(y(y-L_{\text{generation}})+\beta_L^2\sigma_T^2+\sigma_L^2\right)+(\alpha_{S,h}-\alpha_{S,l})(y-L_{\text{generation}})\right) \\
& \qquad \times f_0\left(\sqrt{\frac{3}{2}}\frac{y-S_b}{\sqrt{\sigma_S^2+\sigma_L^2+\beta_L^2\sigma_T^2}}\right) \\
& + \sqrt{\frac{3}{8}}\frac{\left(\alpha_{S,h}-\alpha_{S,l}+(\beta_{S,h}-\beta_{S,l})(2y-L_{\text{generation}})\right)\left(\beta_L^2\sigma_T^2+\sigma_L^2\right)}{\sqrt{\sigma_S^2+\sigma_L^2+\beta_L^2\sigma_T^2}} \\
& \qquad \times f_1\left(\sqrt{\frac{3}{2}}\frac{y-S_b}{\sqrt{\sigma_S^2+\sigma_L^2+\beta_L^2\sigma_T^2}}\right) \\
& + \frac{3}{4}\frac{(\beta_{S,h}-\beta_{S,l})\left(\beta_L^2\sigma_T^2+\sigma_L^2\right)^2}{\sigma_S^2+\sigma_L^2+\beta_L^2\sigma_T^2} \\
& \qquad \times f_2\left(\sqrt{\frac{3}{2}}\frac{y-S_b}{\sqrt{\sigma_S^2+\sigma_L^2+\beta_L^2\sigma_T^2}}\right).
\end{aligned}
$$

To calculate the marginal benefit, we need to differentiate the above expression with respect to the interruption amount. The final answer can be calculated in closed form, using the following formulas:

$$
\begin{aligned}
\frac{df_0(\mu)}{d\mu} &= \frac{2e^{-\mu^2/3}}{\sqrt{3\pi}}, \\
\frac{df_1(\mu)}{d\mu} &= -\frac{4e^{-\mu^2/3}\mu}{3\sqrt{3\pi}}, \\
\frac{df_2(\mu)}{d\mu} &= \frac{4e^{-\mu^2/3}\left(2\mu^2-3\right)}{9\sqrt{3\pi}}.
\end{aligned}
$$

C. Solution of Case with Competing Retailers

To solve Equation (10) from the paper, we introduce the function

$$\text{Benefit}^{(i)}(n,l^*,l^{(i)}) = \mathbb{E}\left(p_{\text{spot}}(n,l^*,l^{(i)})\left(\frac{\beta_L\left(\bar{T}+\bar{\Delta}^T\right)+\alpha_L+\beta_L\sigma_T\varepsilon_T+\sigma_L\varepsilon_L}{n}-L^{(i)}_{\text{generation}}-l^{(i)}\right)\right),$$

where we have dropped the t subscript, set $\sigma_T=\sigma_t^T$, and where $\bar{\Delta}^T$ is the expected temperature deviation from the historical average temperature on date t:

$$\bar{\Delta}^T = \mathbb{E}(\Delta_t^T) = \rho_1^T\Delta_{t-1}^T+\rho_2^T\Delta_{t-1}^T.$$

The spot price depends on the amount of interruption, and, given the interruption of load l^* for each electricity retailer other than retailer i, and of load $l^{(i)}$ for retailer i, and is given by:

$$\begin{aligned}p_{\text{spot}}(n,l^*,l^{(i)}) &= \beta_{S,l}\left(\beta_L\left(\bar{T}+\bar{\Delta}^T\right)+\alpha_L-(n-1)l^*-l^{(i)}+\beta_L\sigma_T\varepsilon_T+\sigma_L\varepsilon_L+\sigma_S\varepsilon_S\right)+\alpha_{S,l}\\
&\quad+\Theta\left(\beta_L\left(\bar{T}+\bar{\Delta}^T\right)+\alpha_L-(n-1)l^*-l^{(i)}+\beta_L\sigma_T\varepsilon_T+\sigma_L\varepsilon_L+\sigma_S\varepsilon_S-S_b\right)\\
&\qquad\times\left[\left(\beta_{S,h}-\beta_{S,l}\right)\left(\beta_L\left(\bar{T}+\bar{\Delta}^T\right)+\alpha_L-(n-1)l^*-l^{(i)}+\beta_L\sigma_T\varepsilon_T+\sigma_L\varepsilon_L+\sigma_S\varepsilon_S\right)+\alpha_{S,h}-\alpha_{S,l}\right],\end{aligned}$$

where Θ is the step function, with $\Theta(x)=0$, if $x\leq 0$, and $\Theta(x)=1$, if $x>0$.

Setting $y=\beta_L\left(\bar{T}+\bar{\Delta}^T\right)+\alpha_L$; i.e., equal to the expected load without any interruption, the benefit from interruption is equal to:

$$\begin{aligned}\text{Benefit}^{(i)}(n,l^*,l^{(i)}) =& \frac{\beta_{S,l}}{n}\left(\beta_L^2\sigma_T^2+\sigma_L^2\right)+\left(\beta_{S,l}\left(y-(n-1)l^*-l^{(i)}\right)+\alpha_{S,l}\right)\left(\frac{y}{n}-L^{(i)}_{\text{generation}}-l^{(i)}\right)\\
&+\mathbb{E}\Big[\Theta\left(y-(n-1)l^*-l^{(i)}+\beta_L\sigma_T\varepsilon_T+\sigma_L\varepsilon_L+\sigma_S\varepsilon_S-S_b\right)\\
&\quad\times\left(\left(\beta_{S,h}-\beta_{S,l}\right)\left(y-(n-1)l^*-l^{(i)}+\beta_L\sigma_T\varepsilon_T+\sigma_L\varepsilon_L+\sigma_S\varepsilon_S\right)+\alpha_{S,h}-\alpha_{S,l}\right)\\
&\quad\times\left(\frac{y}{n}+\frac{1}{n}\left(\beta_L\sigma_T\varepsilon_T+\sigma_L\varepsilon_L\right)-L^{(i)}_{\text{generation}}-l^{(i)}\right)\Big],\end{aligned} \tag{2}$$

where we used that:

$$\mathbb{E}\left(\varepsilon_T^2\right) = \mathbb{E}(\varepsilon_L^2) = 1,$$
$$\mathbb{E}(\varepsilon_T\varepsilon_L) = \mathbb{E}(\varepsilon_T\varepsilon_S) = \mathbb{E}(\varepsilon_L\varepsilon_S) = 0.$$

Calculating the expected value in Equation (2) above, can be done in a way similar to Appendix B. The total benefit, to all of the retailers, is given by:

$$\begin{aligned} n\text{Benefit}^{(i)}\left(n, l^*, l^{(i)}\right) =& \beta_{S,l}\left(\beta_L^2\sigma_T^2 + \sigma_L^2\right) + \left(\beta_{S,l}\left(y-(n-1)l^* - l^{(i)}\right) + \alpha_{S,l}\right)\left(y - nL^{(i)}_{\text{generation}} - nl^{(i)}\right) \\ &+ \mathbb{E}\Big[\Theta\left(y-(n-1)l^* - l^{(i)} + \beta_L\sigma_T\varepsilon_T + \sigma_L\varepsilon_L + \sigma_S\varepsilon_S - S_b\right) \\ &\times\left(\left(\beta_{S,h} - \beta_{S,l}\right)\left(y-(n-1)l^* - l^{(i)} + \beta_L\sigma_T\varepsilon_T + \sigma_L\varepsilon_L + \sigma_S\varepsilon_S\right) + \alpha_{S,h} - \alpha_{S,l}\right) \\ &\times\left(y + \beta_L\sigma_T\varepsilon_T + \sigma_L\varepsilon_L - nL^{(i)}_{\text{generation}} - nl^{(i)}\right)\Big] \end{aligned} \tag{3}$$

We define the expected load after interruption by all the retailers,

$$x = y - (n-1)l^* - l^{(i)}$$

Then, Equation (2) above, becomes identical to Equation (1) from Appendix B, under the transformation

$$\text{Benefit}(x) = n\text{Benefit}(n, l^*, l^{(i)})$$
$$L_{\text{generation}} = nL^{(i)}_{\text{generation}} + (n-1)\left(l^{(i)} - l^*\right)$$

where the Benefit function and the variable $L_{\text{generation}}$ on the left hand side correspond to the definitions in Appendix B, while the Benefit function and the variables $n, l^*, L^{(i)}_{\text{generation}}, l^{(i)}$ on the right hand side correspond to the definitions in this Appendix.

The calculation of the expected value, as well as its derivatives proceeds similar to the calculation in Appendix B.

D. Description of the Numerical Algorithm

The value function for either pay-in-advance or pay-as-you-go contracts solves the maximization problem defined in Equation (5) from the paper, which we simplify here to:

$$\pi_t(\Delta_t^T, \Delta_{t-1}^T, L_{\text{remaining}})$$
$$= \beta \max_{0 \le l \le \min\left(L_{\text{daily}}, L_{\text{remaining}}\right)} \left\{ \mathbb{E}\left[\Delta\pi\left(L_{t+1}, p_{\text{spot},t+1}, l\right) \middle| \mathcal{F}_t \right] + \mathbb{E}\left[\pi_{t+1}\left(\Delta_{t+1}^T, \Delta_t^T, L_{\text{remaining}} - l\right) \middle| \mathcal{F}_t \right] \right\}$$

In this equation, $L_{\text{remaining}}$ is either $L_{\text{advance, remaining}}$ or $L_{\text{pago, remaining}}$, depending on whether we are considering a pay-in-advance or a pay-as-you-go contract. For a pay-in-advance contract, $L_{\text{pago, remaining}} = 0$, while for a pay-as-you-go contract, $L_{\text{advance, remaining}} = 0$. The first term in the maximization on the right hand side, in the case of pay-in-advance contracts, can be rewritten as

$$\frac{\mathbb{E}\left[\Delta\pi\left(L_{t+1}, p_{\text{spot},t+1}, l\right) \middle| \mathcal{F}_t \right]}{16} = L_{\text{under\_contract}} p_{\text{reduced}} - L_{\text{generation}} p_{\text{generation}} - l\ p_{\text{reduced}} - \text{Benefit}\left(y_{t+1}\right)$$

and, in the case of pay-as-you-go contracts, can be written as:

$$\begin{aligned}
&\frac{\mathbb{E}\left[\Delta\pi\left(L_{t+1}, p_{\text{spot},t+1}, l\right) \middle| \mathcal{F}_t \right]}{16} \\
&= p_{\text{retail}} \mathbb{E}\left[L_{t+1} \middle| \mathcal{F}_t \right] - L_{\text{under_contract}} p_{\text{retail}} - L_{\text{generation}} p_{\text{generation}} \\
&\qquad - l\left(p_{\text{fine}} + p_{\text{retail}}\right) - \text{Benefit}\left(y_{t+1}\right) \\
&= \left(\beta_L \left(\bar{T}_{t+1} + \bar{\Delta}_{t+1}^T \right) + \alpha_L - L_{\text{under_contract}} \right) p_{\text{retail}} - L_{\text{generation}} p_{\text{generation}} \\
&\qquad - l\left(p_{\text{fine}} + p_{\text{retail}}\right) - \text{Benefit}\left(y_{t+1}\right),
\end{aligned}$$

where the Benefit function is defined in Appendix B and y_{t+1} is the expected load on date $t+1$, after interruption l, $y_{t+1} = \beta_L \left(\bar{T}_{t+1} + \bar{\Delta}_{t+1}^T \right) + \alpha_L - l$. Dropping constant terms (independent of l) we can see that the optimization problem for the pay-in-advance contract is similar to the problem for the pay-as-you-go contract.

The algorithm for calculating the value function is the following:

- We set the value function on the terminal date equal to zero

$$\pi_{t_{\text{final}}} = 0$$

- On the date immediately prior to the terminal date, we discretize the state space ($\Delta^T_{t_{\text{final}}-1}$, $\Delta^T_{t_{\text{final}}-2}, L_{\text{remaining}}$), to an $N_T \times N_T \times N_L$ grid, where both $\Delta^T_{t_{\text{final}}-1}$ and $\Delta^T_{t_{\text{final}}-2}$ varies between $-D^T$ and D^T and $L_{\text{remaining}}$ varies between 0 and L_{yearly}. For each point on the grid, we solve the constrained optimization problem:

$$\pi_{t_{\text{final}}-1} = \beta \max_{0 \leq l \leq \min\left(L_{\text{daily}}, L_{\text{remaining}}\right)} \left\{ \mathbb{E}\left[\Delta\pi\left(L_{t+1}, p_{\text{spot},t+1}, l\right) \middle| \mathcal{F}_t \right] \right\}$$

  The number of one-dimensional constrained optimization problems that are solved is equal to the number of points on the grid, $N_T \times N_T \times N_L$.

- On every date $t < t_{\text{final}-1}$, we compute $\pi_t\left(\Delta^T_t, \Delta^T_{t-1}, L_{\text{remaining}}\right)$ on the same $N_T \times N_T \times N_L$ grid as in the previous step. For each grid point in the state space ($\Delta^T_t, \Delta^T_{t-1}, L_{\text{remaining}}$), we need to solve a constrained optimization problem. We express:

$$\mathbb{E}\left[\pi_{t+1}\left(\Delta^T_{t+1}, \Delta^T_t, L_{\text{remaining}} - l\right) \middle| \mathcal{F}_t\right] = \mathbb{E}\left[\pi_{t+1}\left(\rho^T_1 \Delta^T_t + \rho^T_2 \Delta^T_{t-1} + \sigma^T_{t+1}\varepsilon^T, \Delta^T_t, L_{\text{remaining}} - l\right)\right]$$

  Since we only know π_{t+1} on the grid points, we need to perform several interpolations.

  - We interpolate π_{t+1} along the Δ^T_{t+1} direction using cubic splines with natural boundary conditions,

$$\pi_{t+1}\left(\cdot, \Delta^T_t, L_{\text{remaining}}\right) \to f(x)$$

    for each point in the ($\Delta^T_t, L_{\text{remaining}}$) directions. This step results in the calculation of $N_T \times N_L$ cubic splines over N_T points.

- For each grid point at time t, $(\Delta_t^T, \Delta_{t-1}^T, L_{\text{remaining}}$, we calculate the value of $\mathbb{E}\left[\pi_{t+1}\left(\rho_1^T \Delta_t^T + \rho_2^T \Delta_{t-1}^T + \sigma_{t+1}^T \varepsilon^T, \Delta_t^T, L_{\text{remaining}}\right)\right]$ by performing the one-dimensional integration

$$\mathbb{E}\left[\pi_{t+1}\left(\rho_1^T \Delta_t^T + \rho_2^T \Delta_{t-1}^T + \sigma_{t+1}^T \varepsilon^T, \Delta_t^T, L_{\text{remaining}}\right)\right] = \int_{-\infty}^{\infty} \frac{d\varepsilon\, e^{-\frac{\varepsilon^2}{2}}}{\sqrt{2\pi}} f\left(\rho_1^T \Delta_t^T + \rho_2^T \Delta_{t-1}^T + \sigma_{t+1}^T \varepsilon\right)$$

This step results in $N_T \times N_T \times N_L$ one dimensional integrations.

- We define the value of a function g at time t as

$$g\left(\Delta_t^T, \Delta_{t-1}^T, L_{\text{remaining}}\right) = \mathbb{E}\left[\pi_{t+1}\left(\rho_1^T \Delta_t^T + \rho_2^T \Delta_{t-1}^T + \sigma_{t+1}^T \varepsilon^T, \Delta_t^T, L_{\text{remaining}}\right)\right]$$

From the previous steps, we have already calculated the value of the function g on all the grid points at time t. For the case of the pay-in-advance contract, the value function at time t is given by

$$\begin{aligned}
&\pi_t\left(\Delta_t^T, \Delta_{t-1}^T, L_{\text{remaining}}\right) \\
&\quad = 16\beta\left(L_{\text{under_contract}} p_{\text{reduced}} - L_{\text{generation}} p_{\text{generation}}\right) \\
&\qquad + \beta \max_{0 \le l \le \min\left(L_{\text{daily}}, L_{\text{remaining}}\right)} \left(g\left(\Delta_t^T, \Delta_{t-1}^T, L_{\text{remaining}} - l\right) - 16l\ p_{\text{reduced}} - 16\text{Benefit}\left(y_{t+1}\right)\right)
\end{aligned}$$

The case of the pay-as-you-go contract is similar.

- For each grid point at time t, in the $(\Delta_t^T, \Delta_{t-1}^T)$ directions, we find an interpolating cubic spline with natural boundary conditions for

$$g\left(\Delta_t^T, \Delta_{t-1}^T, \cdot\right) \rightarrow h(x)$$

This step results in the calculation of an additional $N_T \times N_T$ cubic splines over N_L points.

– For each grid point at time t, $(\Delta_t^T, \Delta_{t-1}^T, L_{\text{remaining}})$, we calculate the value function π_t by solving the constrained maximization problem

$$\max_{0 \leq l \leq \min\left(L_{\text{daily}}, L_{\text{remaining}}\right)} \left(h\left(L_{\text{remaining}} - l\right) - 16l\, p_{\text{reduced}} - 16\text{Benefit}\left(y_{t+1}\right)\right)$$

This step results in an additional $N_T \times N_T \times N_L$ one dimensional constrained optimization problems.

- We repeat the previous step until $t = 0$.

Overall, to find the optimal policy at time t, we calculate $N_T \times N_L$ cubic splines over N_T points and $N_T \times N_T$ cubic splines over N_L points, as well as $N_T \times N_T \times N_L$ one dimensional integrals, and solve $N_T \times N_T \times N_L$ constrained maximizations.

To estimate the accuracy of the approximations, once we have the optimal interruption policy from the dynamic programming algorithm, we perform Monte-Carlo simulation following the prescribed interruption policy. We can get a measure of the accuracy of the interpolations by comparing the estimate of the value function calculated from the Monte-Carlo simulation and the value function calculated from dynamic programming. In all the results we report, the value of the value function calculated from dynamic programming was within two standard errors of the mean of the average value of the value function calculated by Monte-Carlo simulation.

As noted in the text, in our numerical experiments we took $N_T = 21$, $N_L = 20$, $D^T = 10$. The algorithm was programmed in C using the GNU Scientific Library for interpolations, integrations and maximizations. Running on an 1.7 GHz Pentium 4 processor, the program computes the value of a 90 day contract in 90 seconds and performs 10,000 Monte Carlo simulations in 3 seconds.

Table 1
Approximation of the functions f_0, f_1, f_2

i	j	a^i_j	b^i_j
0	0	$3.1694680E-08$	$1.0000000E+00$
0	1	$6.5146886E-01$	$-3.8168394E-01$
0	2	$-2.4864378E-01$	$2.1239734E-01$
0	3	$6.5930468E-02$	$-4.9218606E-02$
0	4	$-4.3010262E-03$	$1.3075845E-02$
0	5	$1.8398374E-04$	$-1.3997635E-03$
0	6	$6.8124638E-05$	$1.4880846E-04$
0	7	$1.4816344E-05$	$1.3044717E-05$
1	0	$6.5146999E-01$	$1.0000000E+00$
1	1	$-3.8826942E-01$	$-5.9599249E-01$
1	2	$3.0697238E-02$	$3.8049119E-01$
1	3	$3.4874636E-02$	$-1.4533331E-01$
1	4	$-1.3650616E-02$	$5.0873548E-02$
1	5	$2.3832165E-03$	$-1.2563172E-02$
1	6	$-2.2830602E-04$	$2.5248551E-03$
1	7	$1.1699271E-05$	$-3.1379021E-04$
1	8	$-2.5225764E+00$	$2.4213828E-05$
2	0	$-2.9502545E-09$	$1.0000000E+00$
2	1	$-4.3431318E-01$	$-5.2710566E-01$
2	2	$2.2892650E-01$	$3.4825716E-01$
2	3	$-6.4638843E-03$	$-1.2582758E-01$
2	4	$-2.1729456E-02$	$4.5417004E-02$
2	5	$6.7240331E-03$	$-1.1095136E-02$
2	6	$-9.2026844E-04$	$2.3693399E-03$
2	7	$6.2239652E-05$	$-3.0583503E-04$
2	8	$-1.6976963E-06$	$2.7110737E-05$