REAL OPTIONS IN LEASING: THE EFFECT OF IDLE TIME

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We study options on short-term leases for capital-intensive equipment performing specific functions and services, such as leases for semi-submersible drilling rigs, marine seismic services, corporate real estate leasing, retail space leasing, and apartment leasing. We quantify the effect of an important factor in pricing options on these services: *idle time* between consecutive lease contracts. We show that while the expected, discounted value for a contract with options is unique, option prices and option exercise prices must be given with respect to a payment structure for the whole contract. We prove that there exist payment schemes in which prices do not depend on exercise probabilities. We use a simple analytic model to derive closed-form solutions for option prices and illustrate our methodology by pricing options for leasing oil-drilling services in the North Sea.

Repeated short-term leases are a feature of many businesses, such as apartment leasing or services that involve capital-intensive equipment performing specific functions. Examples include marine drilling and three-dimensional marine seismic exploration. In lease contracts, operators (lessees) require services that are short term relative to the life of the equipment and may be repeated, possibly at different locations, many times. Leases often include options affecting lease length, particularly extension, termination, and assignment or sublet options. Options on leasing contracts have previously been studied by Grenadier (1995) and Trigeorgis (1996).

In this paper we identify a new, critical factor in pricing leasing options: *idle time* between consecutive lease contracts, relative to the market state; i.e., the level of equipment or service utilization, the current demand, and the expected future demand. While in previous work valuation was based on exercise decisions that were made solely on whether the option was in or out of the money, certain options in leasing are specifically designed to make the exercise decision independent of the leasing rate. For example, an extension option on a lease may be specified as floating or at market rate, meaning that if the extension option is exercised, the renewal leasing rate will be the prevailing leasing rate at the time of exercise. However, the owner of the asset may still be affected by the existence of the option because of the uncertainty regarding the time when the asset will be available. We contrast the cash flows generated by a leased asset with those generated by a—fictitious—market resource, representing the average state of the market. The compensation to the owner of the asset for the options in the contract reflects differences in expected cash flows between the leased asset and the market resource—caused by the option presence and exercise—between the start of a lease and the start of the subsequent lease.

While the expected discounted value of a lease contract is unique, there may be many payment schemes with the same expected value. Instead of providing separate option prices, we present payment schemes that allow for different allocation of risk. Among these schemes, we discuss contingent payment schemes, which are independent of option exercise probabilities, and prove that they exist for all contracts.

The inclusion of idle time can result in qualitatively different effects on asset utilization rates. The presence of extension options, for example, can lead to *both* a decrease and an increase in the utilization rate of an asset. Consider the case of a tight market, with a large percentage of assets occupied, and a leasing contract with an extension option with a short notification time that is unlikely to be exercised. This contract leaves the lessor with an under-utilized asset, for which the lessor will demand compensation. On the other hand, in down markets, with only a few assets occupied, a lease with an extension option with a high probability of exercise would reduce under-utilization. Current business practice indicates that in such situations a discount may be offered as, for example, in the case of the extension of an existing apartment lease when the occupancy rate is low.

We place this work in the area of real options. Dixit and Pindyck (1994, p. 7) define a real option as

> " . . . the opportunities to acquire real assets are sometimes called 'real options.' "

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We extend this definition to encompass situations other than acquisitions as:

A real option provides the flexibility to affect future cash flows from real assets or investments,

where we borrow from Grinblatt anf Titman (1997, p. 319) the definition of a real investment:

"Real investments are expenditures that generate cash in the future and, as opposed to financial investments, like stocks and bonds, are not financial instruments that trade in the financial markets."

We stress that the flexibility mentioned in the definition of a real option may be with respect to the acquisition of an asset or with respect to the use of an asset. This flexibility may be re-packaged and re-sold, giving the holder the right, but not the obligation, to exercise the decision possibilities embodying the flexibility. For leasing purposes, the asset generates cash when leased or used to provide services; the options in the lease contract affect this cash flow. Other real options considered in the literature involve cash flow generated by oil fields (Smith and McCardle 1998, Paddock et al. 1988); mines (Trigeorgis 1990, Brennan and Schwartz 1985, Moel and Tufano 1998); real estate (Titman 1985, Grenadier 1996); other capital investments (Trigeorgis 1995, Dixit and Pindyck 1994); flexible manufacturing facilities (Chen et al. 1998); and partially finished goods in manufacturing (Cortazar and Schwartz 1993). Ingersoll and Ross (1992) have emphasized that under stochastic interest rate changes, even investments with a known fixed payoff have characteristics of options. A review of the real options literature can be found in Trigeorgis (1993) and introductions in Dixit and Pindyck (1994) and Trigeorgis (1996a).

While our work treats real options in leasing, it differs from previous work by Grenadier (1995) and Trigeorgis (1996), as well as from the combination of option pricing and decision analysis methodologies studied by Smith and Nau (1995) and Smith and McCardle (1998). The approach taken in those papers was to develop an equilibrium market model, with option valuation based on replication arguments. Because lease contracts are typically held with a view to consumption, prices based on replication arguments may give only one-sided bounds for option values (Hull 1997). We also point out that in the framework of our model, described in §2, the prices of the types of options we study would be zero when priced using the previous work, which ignores the effect of idle time.

The paper is organized as follows. Section 1 presents the framework of incorporating expected idle time in leasing contracts, first for the case of fixed-length contracts and subsequently for contracts with options. Section 2 presents a simple model that describes the market situation. The model is tractable and allows for closed-form solutions for option prices in a static market. Section 3 presents alternative contract designs with associated options and discusses the prices and payment schedules associated with each design. For certain types of options we show that particular contract designs can eliminate the need for the option exercise probability to price the options. Section 4 estimates the model parameters for the case of the market for semi-submersible rigs in the North Sea and presents the effect of options on the pricing of typical leasing contracts. Section 5 summarizes and concludes the paper.

1. GENERAL FRAMEWORK

We approach valuation of lease contracts with options, including the effect of idle time, in three steps. We first consider valuing contracts of different lengths without options. We construct a market rate for a continuously leased resource, which we term a market resource, starting from the value of a fixed-term contract, and modify this rate according to the market utilization. From the leasing rate of the market resource we can construct contract values, or equivalently the lease rates, for other fixed-term contracts with different lengths. The pricing is based on the expected value of the cash flows, generated by the market resource, from the start of the contract until the start of the next contract.

The second step is to generalize from contracts of different fixed lengths to contracts with arbitrary sets of options affecting asset utilization. The options we are considering are those that affect the length of the contract and the idle time and asset utilization rate until the next contract. This extension is achieved by taking the expectation for the value of the contract over the joint distribution of all option exercise decisions. Comparison of the same base contract with and without options gives a value for the bundle of options under consideration. Marginal values for additional options can also be generated from similar comparisons.

The third step to price lease options is to specify a payment scheme. Because many different payment schemes are possible with the same expected value, option prices and option exercise prices are well defined only with respect to a specific payment scheme. Payment schemes for contracts and specific option prices and exercise prices are given in §3.

1.1. Market Rates for Fixed-Length Contracts

Consider a contract of length l, with lessees willing to set up new contracts that start, at the earliest, $a \geqslant 0$ ahead and at the latest $b \geqslant a$ ahead. For reference we will use a—fictitious—market resource. The assumption on the market resource is that it is continuously leased at a rate modified by the average market utilization rate, i.e., the ratio of assets that are leased over the total number of assets in the market. Our pricing framework is based on the premise that for any contract specification, the expected value of the contract should be the same as that for the market resource over the same period; i.e., from the beginning of a contract to the beginning of the subsequent contract.

Based on the above, the leasing rate for a fixed-length contract includes compensation for the expected time that the resource will be idle at the end of the contract while waiting for the next contract. The leasing rate, over the time from the contract start to the next contract start, will be equal to the rate for the market resource multiplied by the average market utilization rate.

Even though the leasing rate for the market resource is not observable, it can be calculated from any given contract rate for a fixed length contract. Assuming that interest rates are constant and zero (this assumption is not critical and can be relaxed at the expense of losing some of the analytic tractability of the model), the spot leasing rate $s(l)$ for a fixed length contract of length l is given by

$$s(l) \times l = (c_\infty \times u) \times (l + E[w(l)]), \tag{1}$$

where $w(l)$ is the time a contract of length l is idle (waiting) between the end of the current contract and the beginning of the next contract, $E[\ ]$ is the usual expectation operator, c_∞ is the (unobserved) rate for the market resource that is continuously leased for a 100% market utilization rate, and u is the market utilization, i.e., the percentage of assets in the market that are being utilized.

The left-hand side of Equation (1) corresponds to the income received for the lease contract from the lessor: lease rate times length of contract. The right-hand side of Equation (1) describes the equivalent cash flow over the period from the beginning of the contract until the beginning of the next contract, where $c_\infty \times u$ is the leasing rate for the market resource and $l + E[w(l)]$ the expected length of time between contract starts. In general, c_∞ may be a function of the market utilization and the demand level, which can themselves be random variables. As contract lengths increase, the leased asset approaches the market resource, and its leasing rate approaches the leasing rate of the market resource

$$\lim_{l\to\infty} s(l) = c_\infty \times u.$$

The distribution and first moment of the waiting, or idle, time $w(l)$, can be determined given a model for the market dynamics, i.e., the stochastic process governing new contract arrivals, distributions of contract types, and assignment of contracts to resources. We present such a model in §2.

Given the expected idle time, the utilization rate, and the spot rate $s(l)$ for a fixed-length contract of length l we can calculate c_∞, as well as $s(l')$ for any other fixed-length contract of length l'. For a stationary (nontime-dependent) market situation our construction predicts a declining leasing rate for longer contracts because the cost of the same expected idle time is spread over a longer contract period.

Allowing nonzero interest rates and time-dependent market utilization rate u, the value V of a fixed term contract of length l, $V(l)$ is given by

$$\mathrm{V}(l) = \int_{t=0}^{t=l} s(l) d(0,t)\, dt,$$

$$\begin{aligned}\int_{t=0}^{t=l} s(l) d(0,t)\, dt = E_u\Bigg[&\int_{t=0}^{t=l} c_\infty(u(t),t)\; u(t) d(0,t)\, dt \\ &+ \int_{x=0}^{x=\infty} P\{w(l)=x\} \\ &\times \int_{y=0}^{y=x} c_\infty(u(l+y), l+y) \\ &\times u(l+y)\, d(0,l+y) dy\, dx\Bigg],\end{aligned} \tag{2}$$

where $d(0,t)$ is the discount factor from time zero to time t, $P\{w(l)=x\}$ is the probability density that the idle time between contracts is x when the first contract has length l, and E_u is the expectation with respect to u. Equation (2) links a given or desired spot rate $s(l)$ with the market rate for a continuously leased resource c_∞. We have assumed that the lease rate $s(l)$ is constant, but it would be easy to extend to the case where the leasing rate follows a predetermined schedule.

The basis for Equation (2) is that resource utilization is computed from the start of the contract under consideration until the start of the next contract. The time of this next start is a stochastic variable. This specification has advantages over alternative specifications[1] because it does not depend on the details of subsequent contracts, other than their arrival time.

1.2. Contracts with Options

The value V of a lease contract with options, starting at time t in the future is

$$\begin{aligned}&V(t, \Gamma, \Phi, \mathcal{P}_\Phi, \Theta, \mathcal{P}_\Theta, \mathcal{I}, \mathcal{P}_\mathcal{I}) \\ &\quad = E_{t_{\text{next}}, \mathcal{P}_\Phi, \mathcal{P}_\Theta, \mathcal{P}_\mathcal{I} \mid \Gamma}\left[\int_{x=t}^{x=t_{\text{next}}} d(0,x)\; c_\infty u\; dx\right],\end{aligned}$$

where the expectation $E[\ ]$ is taken with respect to the random variables conditioned by the current market state Γ and where the remaining symbols are:

Φ	set of parameters of the demand process
$\mathcal{P}_\Phi$	joint probability density function of the parameters of the demand process
Θ	set of option exercise decisions
$\mathcal{P}_\Theta$	joint probability density function of the exercise decisions
$\mathcal{I}$	set of parameters for the interest rate model
$\mathcal{P}_\mathcal{I}$	joint probability density function of the parameters of the interest rate model
$d(x_1, x_2)$	discount factor from time x_1 to time x_2
c_∞	market rate for a continuously leased resource for 100% market utilization rates
u	market utilization rate
t_{next}	random variable representing the start time of the next contract

Note that $d(x_1, x_2)$, t_{next}, c_∞, and u are functions of the random variables in Φ, Θ, and $\mathcal{J}$. This formulation is sufficiently general to include contracts with stochastic lengths as well as fixed-length services. That is, a contract may be for a specific service with options on other services where the service length is itself stochastic. For example, in drilling-rig services, if a rig is leased to perform a particular drilling task then it may not be known exactly how long this service will take. Bad weather could shut down operations for a time, or the subsurface rock profile could be unexpectedly hard or soft. Examples of fixed-length services include cases when a drilling rig is leased for a set period or when apartments are let by the month or year.

2. A SIMPLE MARKET MODEL

To illustrate the basic ideas in our option pricing formalism we develop a simple analytical model for lease contracts. Because we are interested in highlighting the effect of idle time on contract pricing, the model assumes a constant availability level and constant demand, thus constant prices. This quasi-static analysis may be applicable over short periods even when market parameters change over longer times. The model is chosen for its simplicity and ability to capture the features relevant to real lease contracts. We make the following assumptions.

ASSUMPTION 1. *Contracts are on one asset against the backdrop of the rest of the market.*

ASSUMPTION 2. *The market utilization rate u is constant and is not significantly affected by changes in the working status of a single asset.*

ASSUMPTION 3. *Market lease rates are constant, i.e., constant rate c_∞.*

ASSUMPTION 4. *The demand is described by a Poisson arrival process for contracts for the single asset with constant average arrival rate λ.*

ASSUMPTION 5. *No queuing is allowed, i.e., contract requests to the asset that are not fulfilled are lost.*

ASSUMPTION 6. *There is limited flexibility regarding contract start times, i.e., a contract request arriving at time t may start at any time in the interval $[t+a, t+b]$, $a \geqslant 0$, $b \geqslant a$.*

ASSUMPTION 7. *Contract start times once fixed are not changed, i.e., if you arrange to start a contract at a certain time and then find that you could start it earlier you do not change the start time but have an idle period, which might be filled with a separate contract.*

Assumptions 1 through 7 define a simple example in which we can analytically include for the impact of idle time on valuing contracts. While the qualitative results remain unchanged for less restrictive assumptions, allowing for example random variation in utilization and market rates as well as for economic depreciation of the leased asset, we would need to use numerical methods to value the contracts.

For any contract, we denote by l the length of the original lease. For an extension option, we denote by n_e the notification date for option exercise and by o the length of the extension. For a termination option, n_t is the notification time required prior to termination.

We use $w(l, n)$ to denote the idle (waiting) time from the end of the contract to the beginning of the next contract, for a contract with length l and notification time n. The notification time is the length of the time interval between the time when the expiration date of the contract is known with certainty and the expiration date itself. For a contract with no options the notification time is equal to the length of the contract, i.e., $n = l$.

Let

$$a' = \min(l, n, a), \tag{3}$$

$$b' = \min(l, n, b). \tag{4}$$

To determine w, let T be the end date of the contract, and define the following events:

A: at least one contract request arrived in the period between a' before the end of the contract and the end of the contract, i.e., between $[T - a', T]$.

B: at least one contract request arrived in the period between b' and a' before the end of the contract, i.e., between $[T - b', T - a']$.
We also define $\overline{A}$ and $\overline{B}$ as the complementary events to A, B, respectively.

The expected idle time from the end of the contract until the next contract starts is given by

$$E[w(l, n)] = P\{B\}\ E[w|B] + P\{\overline{B}\}\ E[w|\overline{B}], \tag{5}$$

and the expected utilization rate $u^r(l, n)$ by

$$E[u^r(l, n)] = P\{B\}E[u^r|B] + P\{\overline{B}\}\ E[u^r|\overline{B}]. \tag{6}$$

Note that $E[w|B] = 0$ and $E[u^r|B] = 1$.

In Appendix A we show that, under the assumptions made, the idle time $w(l, n)$ and the utilization rate for an asset $u^r(l, n)$ are given by

$$E[w(l, n)] = e^{-\lambda(b'-a')}\left(e^{-\lambda a'}\left(a + \frac{1}{\lambda}\right) + \left(1 - e^{-\lambda a'}\right)\left(a - \frac{a'}{2}\right)\right),$$

$$E[u^r(l, n)] = P\{B\} + P\{\overline{B}\} \times \left(P\{A\}\int_{a-a'}^{a} \frac{1}{a'}\frac{l}{l+x}\,dx + P\{\overline{A}\}\int_0^\infty \lambda e^{-\lambda x}\frac{l}{l+a+x}\,dx\right).$$

3. OPTION PRICING AND CONTRACT DESIGN

The price of a particular option affecting the length of a lease contract depends on the payment scheme for the contract as a whole. As we mentioned in §1, many different payment schemes exist with the same expected value for the contract. Because we are interested in the effect of idle time, we will make the assumption that option exercise probabilities are independent of lease rates. This assumption is realistic for cases in which the leasing costs are a small percentage of a larger, inflexible, capital expenditure or when transaction costs are high.[2]

Instead of focusing on option prices, we focus on the expected cash flows from the market resource, over the time period between two successive contracts. As discussed in §1.2, if options are present, the expectation of the period of time between subsequent contracts is modified to cover all possible outcomes from option exercise decisions.

It is important to notice how the payment scheme can affect the option price and option exercise price. The option price is the price paid to have the option available, and the option exercise price is the price to exercise the option. Either or both of these may be positive or negative, i.e., there could be a bonus (incentive) to exercise an option or a penalty (disincentive) on exercise. Consider, for example, a three-month contract with a three-month extension option, with exercise notification one month prior to the expiration of the initial three-month period. We will make the additional simplifying assumptions that the expected idle time $\mathcal{I}$ is independent of notification time, and that the option exercise probability is 1/2. The expected value of the cash flows from the contract, for zero interest rates, is

$$0.5(3+\mathcal{I})+0.5(6+\mathcal{I})(c_\infty u)=(4.5+\mathcal{I})(c_\infty u).$$

If the initial leasing rate is set to the three-month leasing rate, no adjustment is necessary in the event that the option is not exercised. However, if the option is exercised, then over the second three months the rate has to be adjusted to reflect the increase in the utilization rate of the asset. Indeed, if the renewal were made at the three-month leasing rate, the asset would generate cash flows that would dominate those of the market resource, because compensation for the idle time $\mathcal{I}$ would be received twice. Similarly, if the initial leasing rate was set at the six-month leasing rate, then a penalty would need to be paid in the event that the extension option is not exercised. Clearly, the option prices and exercise prices cannot be separated from the contract design.

There exist contract designs that can eliminate the need to know the exercise probability of the option to determine a payment scheme for a contract. We will call a pricing scheme where pricing is independent of the option exercise probabilities a "contingent pricing scheme."

THEOREM 1 (Existence of Contingent Pricing Schemes). *If the fair value of a contract is the value that matches the expected cash flows of the market resource between subsequent contracts, then there exists a contingent pricing scheme for a contract with an arbitrary set of options that affect contract length and the expected start of the next contract.*

PROOF. By construction. The pricing scheme is the following: There is no payment until the time the last option decision is made (this decision could correspond to the last option or to the time that a decision to terminate at a known future time is made). At that time, calculate the expected idle time for the asset up to the start of the next contract. Add this expected idle time to the time between the start and the then-known end of the contract, and charge the market resource leasing rate over that period. This scheme does not depend on the option exercise probabilities, so it is a contingent pricing scheme. □

While Theorem 1 demonstrates the existence of contingent pricing schemes, the scheme proposed is far from the ones used in practice. Many lease contracts have cash flows made at fixed intervals, with a possible rate revision, bonus, or penalty on the event of an option exercise. Two possible pricing schemes are the following:

Upfront payment. A fixed schedule for lease rates is determined at the initiation of the contract, and an additional payment is made up front to reflect the deviations between the expected cash flows of the asset and the market resource. For example, this pricing schedule could specify that in the extension option example given above, the leasing rate for the initial three months could be fixed at the three-month leasing rate with the renewal rate, in the event of option exercise, fixed at the three-month rate. A payment, in this case from the lessor to the lessee, would be made to compensate for the discrepancy between the asset expected cash flows and the market resource expected cash flows[3].

Contingent payment. A schedule for lease payments is made for the fixed length of the contract to match the expected cash flows from the market resource in the event that no option is exercised. In the event of option exercise, adjustment to the lease rate, or penalties, are specified to bring the expected value of the contract in line with that for the market resource.

3.1. Extension Option

An extension option is an option to extend the length of a contract. The option exercise takes place sometime prior to the original contract termination, at the notification date. We will assume that the probability of the option exercise is independent of the leasing rate of the market resource as well as of the pricing scheme used to value the contract.

In the context of the model presented in §2, with constant market conditions, for a single extension option for a length of time o, with notification time n_e prior to the expiration of the original contract, and probability of extension exercise p, the expected idle time, until the start of the following contract, is given by

$$E[w_e]=(1-p)E[w(l,n_e)]+pE[w(l+o,n_e+o)], \qquad (7)$$

where we have used the notation of §2. The expected utilization rate for the asset, u_e^r is similarly given by

$$E[u_e^r]=(1-p)E[u^r(l,n_e)]+pE[u^r(l+o,n_e+o)]. \qquad (8)$$

Notice that for low probabilities of exercise and a short notification time, the utilization rate for a contract with an extension option would be lower than the utilization rate for a contract without the option.[4] On the other hand, for high probabilities of exercise in a market situation with relatively long expected idle times, utilization rates are higher for a contract with an extension option than a contract without the option.[5]

The expected value of the contract is given by

$$E[v_e] = (1-p)s(l, n_e)l + ps(l+o, n_e+o)(l+o). \tag{9}$$

Given the expected value, a possible upfront pricing scheme is to offer the leasing rate for the longer period $s(l+o, n_e+o)$ as the leasing rate before and after the extension exercise, and charge an upfront fee

$$\text{Fee} = (1-p)l(s(l, n_e) - s(l+o, n_e+o)).$$

A possible contingent pricing scheme is to charge the shorter term leasing rate $s(l, n_e)$ for the original period, and in the event of exercise of the extension option, charge an adjusted leasing rate for the extension period. The adjusted leasing rate for the extension period would be

$$s_{\text{extension}} = \frac{1}{o}(s(l+o, n_e+o)(l+o) - s(l, n_e)l).$$

This scheme has the advantage that it is independent of the probability of option exercise p.

3.2. Termination Option

A termination option is the option to terminate a contract earlier that originally agreed upon. Early termination is usually allowed at any point during a contract with prior notification required. There are three main differences between termination and extension options: (1) A contract with a termination option is expected to have a high probability that the option will not be exercised; (2) the lessor may, knowing the final date at which the asset will be available, proceed to arrange a new leasing contract starting as close after that date as possible; (3) the notification can be given at any time during the contract life. Similar to the case of contracts with extension options, we will assume that the probability density of early termination is independent of the market conditions and of the pricing scheme used.

We break down the probability density function of termination as the probability p that the operator at some point during the contract has no further use for the rig (similar to an option exercise probability) and the conditional probability density function f of the termination time, given that the termination option was exercised. The argument of f, x denotes the time at which the termination decision was taken. So for a given x, the contract actually terminates at $x+n_t$. Let l be the original length of the contract and n_t be the termination notice. The idle time satisfies

$$E[w_t] = (1-p)E[w(l, l)] + p\int_0^{l-n_t} f(x)\alpha(x)\,dx, \tag{10}$$

where $\alpha(x)$ is the expected idle time incurred if the termination decision is taken at time x from the start of the contract. A description of α is given in Appendix B.

The expected resource utilization is given by

$$E[u_t^r] = (1-p)E[u^r(l, l)] + p\int_0^{l-n_t} f(x)\beta(x)\,dx, \tag{11}$$

where $\beta(x)$ is the expected utilization rate given that the decision to terminate the contract was made at x. A description of β is given in Appendix B.

The expected value of the contract is found by the expected value of the cash flows of the market resource between two subsequent contracts

$$E[v_t] = (1-p)ls(l, l) + p\int_0^{l-n_t} f(x)\gamma(x)\,dx, \tag{12}$$

where $\gamma(x)$ is the expected value of the cash flows for the market resource, given that the contract termination decison was made at x after the start of the original contract. A description of γ is given in Appendix B.

Given the expected value of the contract, a possible upfront pricing scheme is to offer a leasing rate equal to the leasing rate of the fixed-length contract $s(l, l)$ and charge an upfront fee

$$\text{Fee} = E[v_t] - \Big((1-p)ls(l, l) + p\int_0^{l-n_t} f(x)s(l, l)(x+n_t)\,dx\Big).$$

A contingent pricing scheme would be to charge the leasing rate $s(l, l)$ and a penalty in the event of early exercise. The penalty would compensate for the lower expected utilization rate.

3.3. Other Options

Other options that may have a direct effect on the effective length of contracts include combinations of multiple extension and termination options as well as assignment or sublet options. Combinations of extension and termination options can be dealt with by the methods presented above.

In the assignment option, the lessee has the right to sell his remaining rights to the asset to another party. Because cash flows to the lessor are not different than promised and because market conditions, including market leasing rates, are constant, the assignment option seems to have no effect on the expected cash flows. There is, however, a possible effect of granting an assignment option in that a contract that could start after the end of the current contract is removed from the market. This will increase the expected idle time and decrease the expected utilization rate, but the effect will in general be very small. If, on the other hand, market lease rates could change significantly, the assignment option might be quite valuable.

4. EMPIRICAL RESULTS: OPTION VALUATION IN THE NORTH SEA MARKET FOR SEMI-SUBMERSIBLE RIGS 1989–1998

We demonstrate our contract and option valuation framework in the specific case of the North Sea market for semi-submersible rigs. Of the world's oil production, 35% comes from offshore; see Brandt et al. (1998). In this market options are not priced separately, and there has been no systematic or analytic effort by market participants to value the options present although they are ubiquitous. Explicit option pricing has recently been suggested in trade journals, but no details have been proposed (Gooch 1997, Moomjian 1998). One feature of note is that options are suppressed by the sellers when the market is tight, indicating an expectation of a relatively high value under these circumstances, together with an unwillingness by market participants to price the options. The North Sea semi-submersible rig market also displays nonstationary market utilization and prices. For example, prices increased almost linearly in 1997–1998 with no change in utilization, which was effectively at 100%. We will use our option pricing method, which assumes stationarity, as a quasi-static approximation for relatively short base contracts (three months) and look at single extension and termination options in detail.

Contract data was provided by Stevens (1998). We intentionally neglect certain details of the actual North Sea market, such as the fact that there are two main sectors (NOR and UK) and that some rigs are certified to work in only one. We also assume that all rigs are equally suitable for all jobs although there are significant differences in specification (e.g., blow out preventer rating of 10k versus 15k) and generation (generations 1 through 5 exist with generations 2.5 through 4 currently operating in the North Sea).

4.1. Market History 1989–1998

The rig market in the North Sea is relatively small, with 47 working rigs (October 1998). The recent history shows four phases in terms of rig counts (Figure 1) and rig day rates (Figure 2) (day rate is the daily rate paid for using a rig):

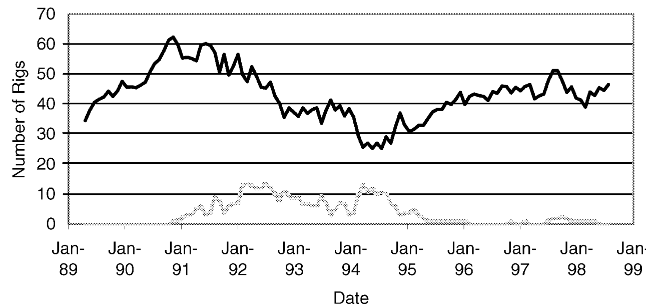
Build up, 89 & 90. An initial increase from 35 rigs in mid-1989 to a peak of 62 in late 1990 with few idle rigs. The day rate went from around 25KUSD (thousand US dollars) to around 40KUSD.

Decline, 91 to 94. A steady decline until late in 1994 with a nadir of 28 working rigs accompanied by a level of idle rigs of around 5 to 10. In this period there was an initial decrease in the day rate followed by a slower decline to around 28KUSD.

Second build up, 95. Starting in late 1994 the number of idle rigs dropped to nearly zero, and the number of working rigs gradually increased to just over 40. Simultaneously, rig day rates nearly doubled to over 60KUSD.

Undersupply, 96 to 98. The rigs supply increased slowly to 47 rigs while the day rate continued its increase from around 65KUSD at the end of the previous period, reaching around 130KUSD in late 1998 with the high end at over 200KUSD.

Figure 1. Working rigs (dark line) and idle rigs (light line) in the North Sea 1989–1998 for semi-submersible rigs. Working includes mobilization time, i.e., the time to transport the rig from one location to another as well as to make any necessary alterations.



The fourth phase of the market is perhaps most easily understood by looking at the total work starting in each year in terms of rig-years, in Figure 3. The undersupply started with a surge in demand for rig time in 1996, but demand was unexceptional thereafter. Thus the undersupply was caused by one exceptional year and not by a change in the base level of demand. This explains why contractors may have been reluctant to add new capacity to the supply side. We note that new semi-submersible rigs cost from 200MUSD (million US dollars) to 400MUSD and take around two years to build.

4.2. Model Parameter Estimation

To calculate contract and option values over time we need to obtain model parameters consistent with the market. We need values for:

a: minimum time before a contract can be started.

Figure 2. Starting day rates for semi-submersible rig contracts in the North Sea 1989–1998. Missing data for existing contracts are plotted as zeros. Note that rates after the start depend on the contract structure and may be fixed, ramp, or float.

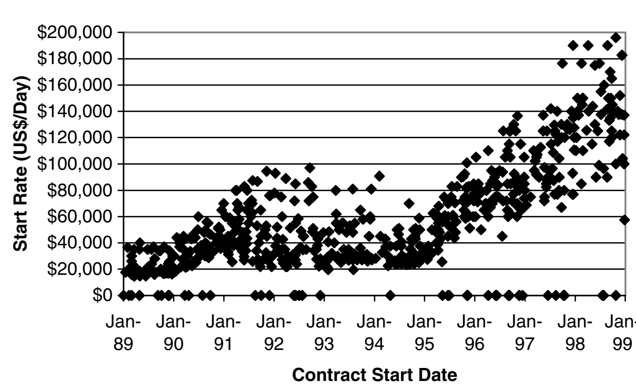
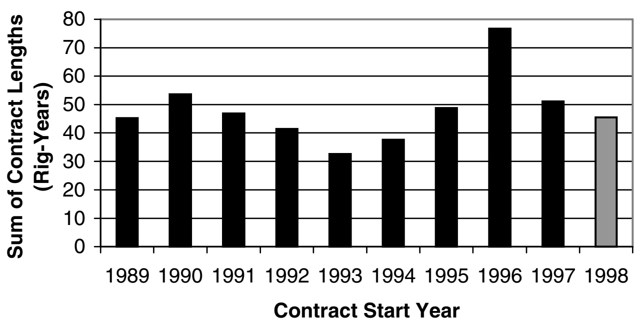


Figure 3. Total work starting in each year in terms of rig-years. The column for 1998 is lighter colored, indicating that the data are potentially incomplete because the dataset was collected August 1998. However, most contracts for the rest of the year are expected to have been fixed by that point.



b: maximum time that operators will wait before starting contract.

λ: effective contract arrival rate that we will distinguish from the total arrival rate λ_{tot} below.

H: cumulative distribution of contract lengths (or the density h).

$c_\infty u$: market day rate of a continuously leased rig.

We also validated that the contract interarrival distribution was consistent with a Poisson process.

We found a and b by taking the 20th and 80th percentiles of the cumulative distribution of contract start date minus bid-in date intervals (Figure 4 shows the cumulative distribution). The values found were $a = 45$ days and $b = 170$ days. We considered contracts starting from 1989 through 1994 because after this time most contracts were arranged privately—most tenders were canceled—so very little data were available. The contracts that went through a public tender process after 1994 were mostly for unrepresentative situations. We assumed that the parameters were unchanged throughout the study period on the basis that this appeared to be true through the first build-up and decline phases of the market.

We estimated the effective arrival rate for a single rig by taking the market utilization as the average utilization of the rig and then calculated what effective arrival rate was required to observe that average utilization. To find the effective arrival rate, we used the equation for the expected utilization for a rig together with the observed distribution of contract lengths. At each point in solving for the effective arrival rate we calculate an expected utilization of the rig given the observed distribution of contract lengths. We adjusted the test arrival rate until the expected utilization matched the observed utilization. One point that required attention was that if the observed utilization is 100%, then the arrival rate is not unique. In that case we use the lowest arrival rate that is consistent with 100% utilization. In fact, given that there is a minimum time before a new contract can be started, for observed distributions of contract lengths we found that the utilization rate reached only up to 95%. To adjust for this effect, we scaled observed utilizations by 0.95. The effective arrival rate of contract requests for a single rig is shown in Figure 5. The dips in arrival rates in 1992 and 1994 correspond to low market utilizations for these years of 80% and 78%, respectively. Figure 6 shows a comparison between the actual contract interarrival distribution, for all years in the study period, and a fitted exponential distribution (mean 5.2 days). While we do not claim that the arrival (demand) process is Poisson, we do observe that the data are consistent with such an assumption.

Figure 6 also shows the distribution of all contract lengths in the study period and a fitted lognormal distribution (mean 9.84 months, standard deviation 17.9 months). Thus we used a lognormal distribution for the distribution of contract lengths. The mean contract length each year is plotted in Figure 7.

The market day rate of a continuously leased rig as modified by utilization ($c_\infty u$) was calculated from the median contract starting rates for contracts between 60 and 120

Figure 4. Cumulative distribution of contract start date minus bid-in date intervals. Arrows indicate 20th and 80th percentiles. These were used for estimating contract start date flexibility and minimum time before contract start parameters.

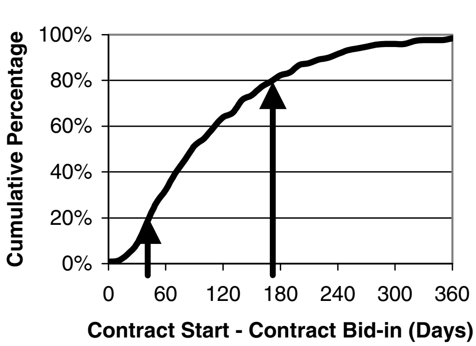


Figure 5. Effective arrival rate of contract requests for a single rig. This is the arrival rate required to produce the observed average utilization.

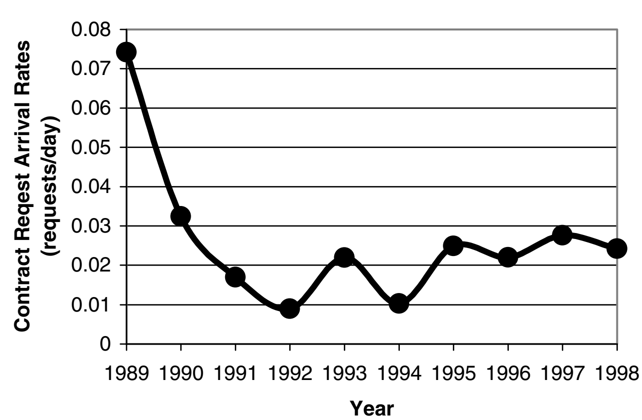
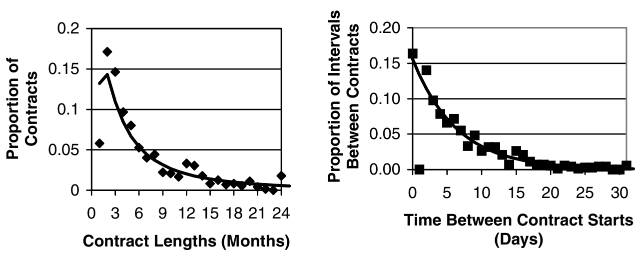


Figure 6. Contract length and interarrival request distributions for 1989–1998. Left panel shows (discretized) lognormal distribution fit for all contract lengths from start to finish, including any exercised options. Lognormal distribution had a mean of 9.84 months and a standard deviation of 17.9 months. Right panel shows exponential distribution fit for contract interarrival times (mean 5.2 days). Parameter estimates were maximum likelihood estimates.



days. These rates are shown in Figure 8. In our data, the market rate, $c_\infty u$, turned out to be always less than the spot rate. Their proportion indicates the expected idle time until the next contract for contracts of the lengths considered. We choose this range of contract lengths because in our examples we use a base contract of 90 days. As an illustration of the discount offered for longer contracts, Figure 9 shows a curve of day rates versus contract length for 1998 calculated based on a spot rate of 140KUSD for a 90-day contract. Figure 10 illustrates the effect of changing the amount of time available to look for a new contract for the same 90-day contract. With no options, the notification period is effectively the length of the contract. However, options change this period and the time between the notice and extension or termination is typically 30 days.

4.3. Extension Option Pricing

We consider a 90-day base contract with a single 90-day extension option. The decision to exercise the option is declared 30 days before the end of the contract (i.e., this

Figure 7. Mean contract length each year. These were found from fitted lognormal distributions for the contracts starting each year. Note the peak in 1996.

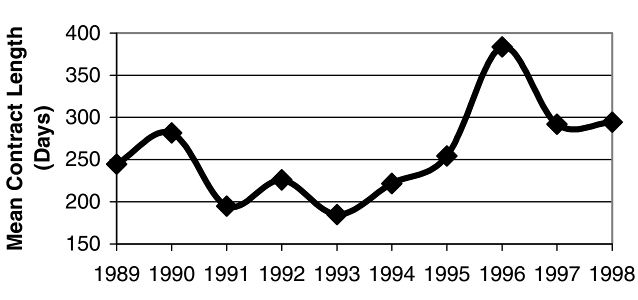
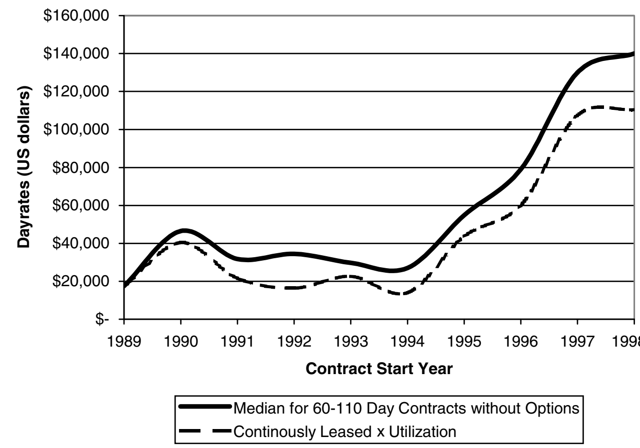


Figure 8. Median day rates for short (60–120-day) contracts and the implied lease rate for a continuously leased rig $c_\infty u$.



is a European style option). Figure 11 shows the historical presence and exercise probability of extension options. In 1989, roughly 60% of the contracts had at least one extension option. This percentage decreased gradually; in 1996 the percentage was less than 40%; in 1997 and 1998 10% or less of contracts had extension options. Up to 1996 there was roughly a 40% probability of at least one extension option exercise. In 1997 and 1998 no options were exercised. This last statistic was probably for two reasons: very few extension options, and little time for them to be exercised up to the end of the dataset in August 1998.

We consider two payment schemes: single payment at start; and a mixed contingent amortized payment scheme. In the first scheme the expected value of the contract is paid at the start of the contract, and there are no further

Figure 9. Calculated dependence of day rate on contract length in 1998 based on a spot rate of 140KUSD for a 90-day contract. Discounts are offered for longer contracts, given the greater expected utilization. Vertical lines indicate model parameters: minimum time to start a new contract (45 days); longest time between contract bin-in and start (170 days).

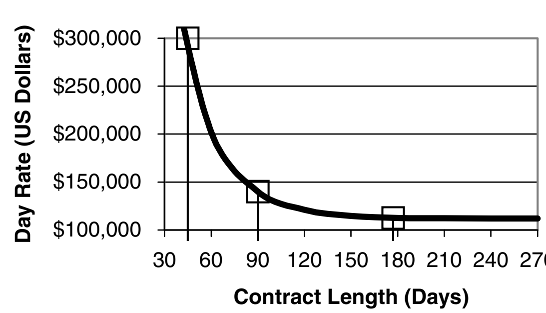
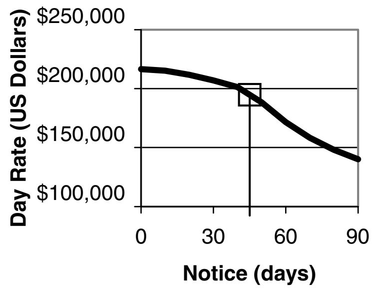


Figure 10. Effect of changing the amount of time available to look for a new contract (notice) on day rate in 1998 based on a spot rate of 140KUSD for a 90-day contract. Vertical line indicates minimum time to start a new contract.



payments. In the second the operator makes the following payments:

- the difference between the value of the 90-day contract with no options and the 90-day contract with no options but with a notice period of 30 days,
- the day rate for a 90-day contract during the 90 days of the base contract,
- and a day rate for the extension, if exercised, so that the total payments come to that for a 180-day contract with a 120-day notice.

Figure 12 shows the expected value of a 90-day contract without options and a contract without options but with 30-day notice (equivalent to an option exercise probability of zero, i.e., $p = 0$). It also shows the expected value of a 180-day contract with 120-day notice ($p = 1$) and a 90-day contract with a 40% chance of option exercise. Note that up to the end of 1996, the contract value with options was almost independent of the exercise probability. The discount for the longer contract when exercised was balancing the increase in day rate for the shorter notice when not

Figure 11. Extension option presence and exercise. Darkest gray is for contracts with no extension options; next darkest is for one extension option, then two; and finally white for contracts with more than two extension options. Continuous line gives percentage of extension options exercised (on same scale).

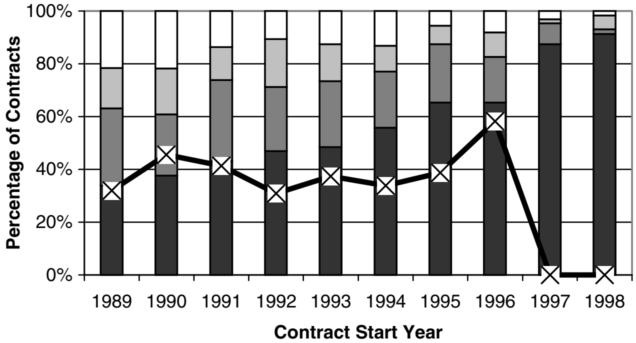
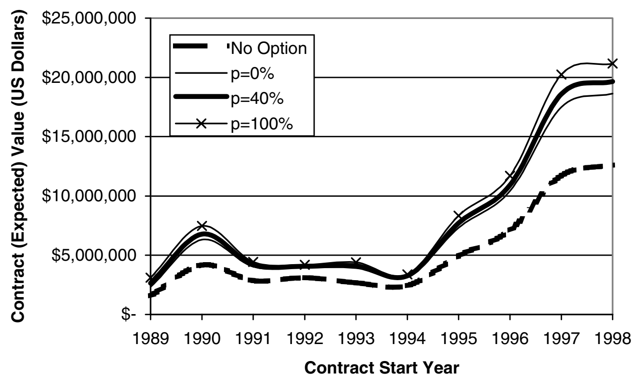


Figure 12. Expected value of a 90-day contract without options (dashed line) and without options and with 30-day notice (equivalent to an option exercise probability of zero, i.e., $p = 0\%$). It also shows the expected value of a 180-day contract with 120-day notice ($p = 100\%$) and a 90-day contract with a 40% chance of option exercise.



exercised. However, in 1997 and 1998 a significant dependence developed, with the gap between $p = 0$ and $p = 1$ reaching around 2.5MUSD.

Figure 13 shows the first extension option payment in the contingent payment scheme. Note that the expected option price is the value of this payment times one minus the extension option exercise probability. Effectively, there is a payment for the option only when it is not exercised.

Figure 13. Extension option price (solid line) and expected cost (dashed line) in the contingent/amortized payment scheme. See text for scheme details. Price is the amount paid for the option. However, if the option is exercised the price paid is factored in to the extension day rate and is effectively zero. Hence if the extension options are exercised 40% of the time on average, their expected long-run cost is 60% of their price.

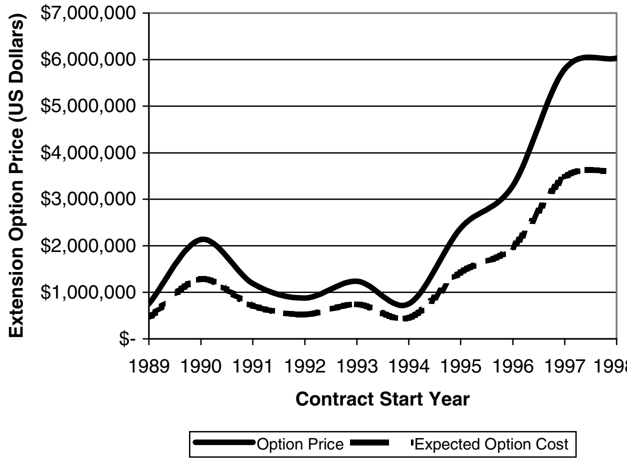
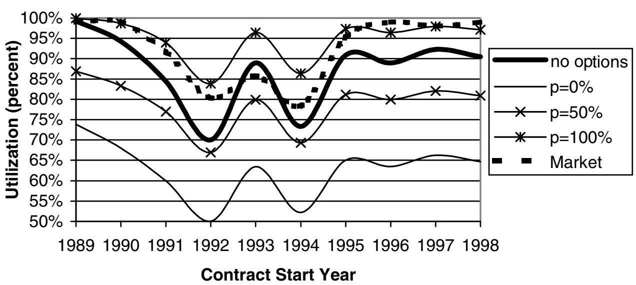


Figure 14. Expected utilization of the market (dashed line) and expected utilization for a rig, given a contract with no options and a contract with an extension option and different option exercise probabilities. Rig utilization scales linearly with option exercise probability between 0% and 100%.



Thus the figure also shows the expected option price with a 40% exercise probability. We see that the expected option cost increases continuously from 1994 to 1997 and then reaches a plateau at 3.6MUSD. The highest previous value was little more than 1MUSD.

Figure 14 shows the expected rig utilization resulting from the range of contracts and the market utilization. Only in 1993 does a 90-day contract result in greater expected utilization than the market utilization. The contract with no options has a utilization rate between the option contract with 50% and 100% exercise probabilities.

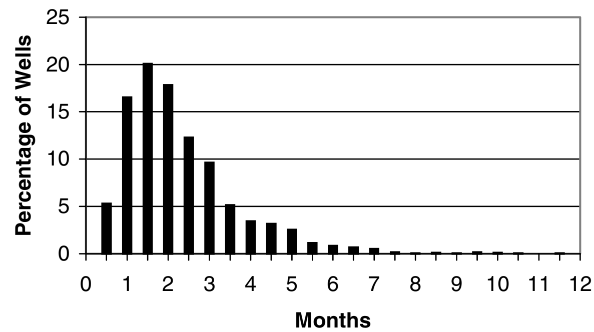
4.4. Termination Option Pricing

Termination options are exercised when the operator has no further use for the rig. This is specified in two steps. The first is an overall probability of deciding at some point during the contract that the operator will have no further use for the rig. The second step is the specification of when the rig is no longer needed. We will model the time when the rig is no longer needed as being after the first well is drilled and specify a 30-day termination notice. Figure 15 shows the density function for all the wells in the time period; the average is 61 days and the standard deviation 56 days. We note in passing that there was no difference between the exploration or production well length distributions. The very shortest wells (0.5–1 month) are mostly workovers or other services rather than new wells.

We will consider a 180-day contract for the calculation of termination option values. If the decision to terminate is based on the results of a first well, then for contracts with lengths less than the length of one well plus the termination notice the value of a termination option is very low. A 180-day contract represents a two-to three-well contract, so is of reasonable length for the termination option to have practical value.

Figure 16 shows the value of a contract with a termination option and its dependence on option exercise,

Figure 15. Probability density function of well (not contract) length 1989–1998. Both exploration and development wells are included because no difference was found between them. Wells shorter than 1 month are mostly interventions and services rather than new wells. Average is 2 months.



i.e., early termination probability. Note that if the termination probability is zero then the contract reduces to a standard contract because we assume that the contractor will start looking for new work for the rig as soon as possible. If a new contract has been fixed and the original contract is then terminated, there will be idle time because of the termination. This is the setup for which the equations in §3 were developed. If the termination option is certain to be exercised, then on average the expected value of the contract is 90% of the value without the option. Thus the value of a contract is little affected by the presence of a termination option. However, the price to exercise the termination option may be substantial.

To price the termination option we use the following contingent payment structure: The day rate is standard, i.e., as though the option were not present. On termination the operator pays a penalty dependent on whether or not a new contract has already been fixed.

• If a new contract has been fixed and cannot be started earlier, the operator pays for the idle time incurred as a

Figure 16. Expected value of a 180-day contract without options (dashed line) and with a termination option (30-day notice). Termination is modeled to occur after the first well. Well duration is a random variable.

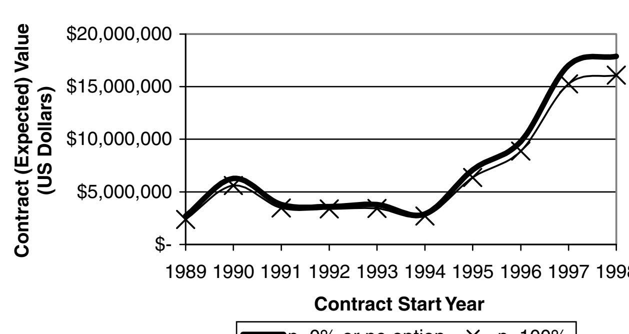
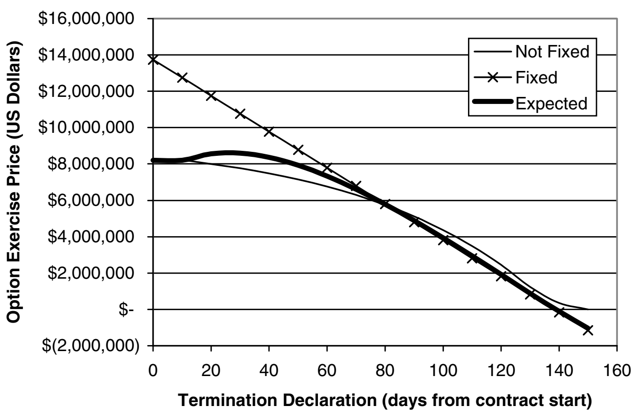


Figure 17. Termination option exercise price for 1998 for a 180-day contract with day rate 100KUSD (corresponds to a 140KUSD day rate for a 90-day contract). The setup is as follows. The operator starts looking for a new contract for the rig as soon as possible and may have fixed a new contract before the termination option is exercised. The figure shows both cases: exercise price with new contract already fixed and exercise price when no contract has yet been fixed. An expected value is shown based on the probability of a contract having been fixed by the termination declaration. Note that if a new contract has not been fixed by the termination declaration the amount of time the contractor had to set one up is still factored in to the termination price.

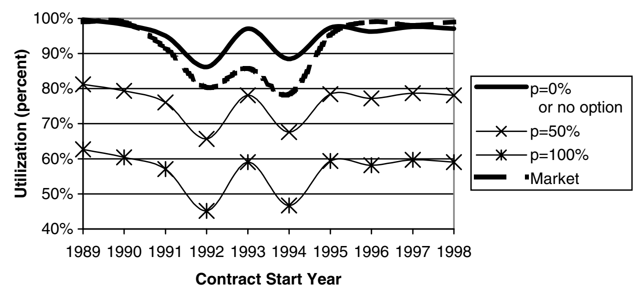


result of the early termination $l \times c_\infty \times u - s(l, l)(x + n_t)$, where x is the time interval between the original end of the contract and the new end of the contract and n_t is the notification time for termination.

- If no contract has been fixed, the exercise price is the same as before, with the additional expected idle time, $(x + n_t + E[w(l, l)]) \times c_\infty \times u - (x + n_t) \times (l, l)$.

Figure 17 shows option exercise prices together with the expected exercise price for a contract in 1998 with respect to the time the termination was declared. Note that if only 10% of contracts are terminated, then the expected cost of a termination option in the long run is 1/10th of the values shown. At 60 days the expected termination price is about 7.7MUSD; recall that the day rate for this period, for a 180-day contract, is around 100KUSD for a contract value of 18MUSD. Figure 18 shows the expected utilization rates for different termination option exercise probabilities. These rates are calculated based on termination after the first well. Because termination options in this scheme are paid for upon termination, it is the 100% termination probability that is the most relevant. This probability results in a large drop in expected utilization of around 20%, which is roughly constant over the period 1989–1998,

Figure 18. Expected utilization of the market (dashed line) and expected utilization for a rig given a contract with no options and a contract with a termination option and different option exercise probabilities. Rig utilization scales linearly with option exercise probability between 0% and 100%.



which indicates why the option exercise prices are approximately one-third of the contract value above.

5. SUMMARY AND FUTURE RESEARCH

The value of flexibility is a rapidly expanding subject area that has moved from pure financial options to options on real investments; see Trigeorgis (1996a) and Dixit and Pindyck (1994). It has also been seen in contract design for supply chain coordination (Tsay et al. 1999). Our work continues this trend by considering options on cash flows from real assets where asset utilization and idle time are key factors. We have shown that the flexibility embodied in the contract options may be priced by specific payment schemes, and that there exists a payment scheme that is independent of the option exercise probabilities. We applied our pricing and payment models to the case of leases and options on semi-submersible rigs in the North Sea market of 1989–1998.

Our pricing methods require two inputs: the dependence of contract price on contract length, and a stochastic model to provide the expected idle time before the next contract. Although we used expected values for pricing, our framework can be extended to include risk-premia. For example, the risk-premia could be based on downside risk, mean-variance, value-at-risk, or whatever combination was of interest to the parties arranging the contract.

In our application to semi-submersible rig leases, for calibrated parameters and specific contracts we found that an extension option added roughly 52% to the expected value of the contract, whereas a termination option reduced the expected value by roughly 10%. Thus option presence and exercise have potentially major effects in this market.

Our analysis applies to lease contracts that are short relative to the useful life of the leased asset. In this framework we have been able to realistically describe the behavior of lease rates without accounting for the economic depreciation of the assets. Allowing for economic depreciation and the flexibility associated with maintenance decisions would increase the complexity of the problem, leading to the need for numerical solutions. However, our approach is appropriate for longer term contracts in which the maintenance schedule has already been agreed upon, thus ensuring the future quality of the leased asset. This is a possible extension of our current framework.

Other future research possibilities include explicit modeling of nonstationary markets. This could be done with time-dependent prices and utilization rates and using queuing theory to allow resource reservation and demand backlogs. It is doubtful whether much could be done analytically in this framework for contract and option pricing because transient behavior rather than asymptotic limits is important. Such situations are more suitable for simulation techniques where much more complexity can be handled; see for example Law and Kelton (1991). Another possible extension is to allow for competition between asset owners for a particular contract. This is an area where game and auction theory could be of use for bidding on contracts.

APPENDIX A—CALCULATION OF EXPECTED IDLE TIME AND UTILIZATION RATE

The expected waiting time if a contract arrives at least a' before the end of the contract and not more than b', i.e., $E[w|B]$ is zero because the next contract can start immediately after the current one.

From the properties of Poisson processes (Ross 1997), we have

$$P\{A\} = 1 - e^{-\lambda a'}, \tag{A1}$$

$$P\{B\} = 1 - e^{-\lambda(b'-a')}. \tag{A2}$$

The expected waiting time, conditional on event $\overline{B}$, is given by

$$E[w(l,n)|\overline{B}] = P\{A\}E[w|\overline{B}\cup A] + P\{\overline{A}\}E[w|\overline{B}\cup\overline{A}], \tag{A3}$$

where we have additionally conditioned on the event A. The expected idle times $E[w|\overline{B}\cup A]$, $E[w|\overline{B}\cup A]$ are given by

$$E[w|\overline{B}\cup\overline{A}] = a + 1/\lambda, \tag{A4}$$

$$E[w|\overline{B}\cup A] = \frac{1}{a'}\int_{x=0}^{x=a'} (a-x)\,dx, \tag{A5}$$

where $1/a'$ is the probability density that, given that a contract will arrive between a' prior to the end of the contract and the end of the contract, it will arrive in any subinterval dx. The integrand $a-x$ corresponds to the earliest time that the contract would start, given that it arrives at time x. From the above we have

$$E[w(l,n)] = e^{-\lambda(b'-a')}\left(e^{-\lambda a'}\left(a+\frac{1}{\lambda}\right) + (1-e^{-\lambda a'})\left(a-\frac{a'}{2}\right)\right). \tag{A6}$$

We can similarly compute the expected utilization rate $u^r(l,n)$ for an asset with a contract of length l and notification time n

$$\begin{aligned} E[u^r(l,n)] &= P\{B\}\times 100\% + P\{\overline{B}\}\times E[u^r|\overline{B}] \\ &= P\{B\} + P\{\overline{B}\} \\ &\quad \times \left(P\{A\}E[u^r|\overline{B}\cup A] + P\{\overline{A}\}E[u^r|\overline{B}\cup\overline{A}]\right) \\ &= P\{B\} + P\{\overline{B}\} \\ &\quad \times \left(P\{A\}\int_{a-a'}^{a}\frac{1}{a'}\frac{l}{l+x}\,dx \right. \\ &\quad \left. + P\{\overline{A}\}\int_0^\infty \lambda e^{-\lambda x}\frac{l}{l+a+x}\,dx\right), \end{aligned} \tag{A7}$$

where the integrals can be expressed in terms of exponential integral function $Ei(x) = \int e^{-x}/x\,dx$ and its associated series expansion (Tuma 1987).

APPENDIX B—CALCULATIONS FOR THE CASE OF A TERMINATION OPTION

We present descriptions for the functions α, β, γ, used in the calculations of the expected idle time, expected utilization rate, and expected value of a contract with a termination option.

Given that the termination option is exercised, we define the event A as having already fixed a new contract to begin as soon as possible after the original end of contract date T. The new contract was fixed prior to notification of the termination and we will assume that its start date cannot be changed.

Note that we use x for the time of the termination decision after the start of the original contract. Hence the contract terminates at $x+n_t$ after it starts given that the termination decision is at x. Recall that the length of the base contract is l.

For the calculation of α, if $(l-x) > b$, then $\alpha(x) = w(n_t, n_t)$. For $(l-x) \leqslant b$ we have

$$\alpha(x) = \begin{cases} \text{if } \overline{A}, & w(n_t,n_t) \\ \text{if } A, & \begin{cases} \text{if } a < l-x < b, & l-(x+n_t), \\ \text{if } a > l-x, & a - 0.5(a-(l-x)), \end{cases} \end{cases} \tag{B1}$$

where the probability of the event A, i.e., the probability of having found a contract $l-x$ prior to the end of the current contract, is given by

$$P\{A\} = 1 - e^{\lambda(b'-(l-x))},$$

where $b' = \max(b, l-x)$.

The function β for the expected utilization rate is similarly given by $u^r(n_t, n_t)$ if $l-x > b$ and, for $l-x \leqslant b$, by

$$\beta(x) = \begin{cases} \text{if } \overline{A}, & u^r(n_t,n_t), \\ \text{if } A, & \begin{cases} \text{if } a < l-x < b, & \dfrac{\min(x+n_t, l)}{l}, \\ \text{if } a > l-x, & \dfrac{\min(x+n_t, l)}{l+a-0.5(a-(l-x))}. \end{cases} \end{cases} \tag{B2}$$

Finally, the function γ, for the expected cash flows for the market resource, is given by the market resource rate uc_∞ times the expected idle time, given exercise of the termination option at time x prior to the original contract end date, i.e.,

$$\gamma(x) = uc_\infty(x + n_t + \alpha(x)). \tag{B3}$$

ENDNOTES

[1]Alternative specifications could be to calculate resource utilization from the start of the contract under consideration to some other event, e.g., the start of the tenth contract afterward, or simply for a fixed time interval, e.g., for a multiple of the current contract length or the average contract length, etc. However, both these alternatives have disadvantages. The main disadvantage is that they would make the price of a contract depend on the details of subsequent contracts. For the fixed time interval approach, final time effects become important. A given time interval may end in the middle of a contract, which may bias the results unless the time interval is made sufficiently large. Unfortunately, for sufficiently large time intervals the effect of the option itself may be swamped. Moreover, if pricing depends on details of subsequent contracts, then taking an expectation will require integration over all possible contract specifications, which would greatly increase the dimensionality of the problem.

[2]For example, the cost of leasing a semi-submersible rig is typically 5%–10% of the total cost of exploration. Because interrupting exploration and rescheduling work crews and other equipment can be very expensive, the decision on exercising an extension option embedded in a rig leasing contract is based on the outcome of the exploration.

[3]This scheme is very similar to schemes used in residential apartment leasing contracts. While renewal is typically made at then current lease rates, a discount might be given at the beginning, possibly in the form of a perk, such as half a month of rent off, at the initiation of the contract.

[4]For $p \to 0$, the utilization rate for the contract with the extension option tends to $E[u^r(l, n_e)]$, which is less or equal to $E[u^r(l, l)]$, which is the utilization rate for a contract without the option.

[5]For $p \to 1$ the utilization rate for a contract with an extension option tends to $E[u^r(l+o, n_e+o)]$ which, for markets with long expected idle time, is greater than $E[u^r(l, l)]$, the utilization rate for a contract without options.

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