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Stochastic taxation and asset pricing in dynamic general equilibrium

Clemens Sialm[a,b,*]

[a]Stephen M. Ross School of Business, University of Michigan, 701 Tappan Street, Ann Arbor, MI 48109-1234, USA
[b]NBER, USA

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Abstract

Tax rates have fluctuated considerably since federal income taxes were introduced in the U.S. in 1913. This paper analyzes the effects of stochastic taxation on asset prices in a dynamic general equilibrium model. Stochastic taxation affects the after-tax returns of both risky and safe assets. Whenever taxes change, bond and equity prices adjust to clear the asset markets. These price adjustments affect assets with long durations, such as equities and long-term bonds, more than short-term assets. Under plausible conditions, investors require higher term and equity premia as compensation for the risk introduced by tax changes.

JEL classification: G12; H20; E44

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1. Introduction

One of the few certain forecasts about the tax system is that it will change. Since federal income taxes were introduced in 1913, the tax system of the U.S. has been

*Corresponding author. Stephen M. Ross School of Business, University of Michigan, 701 Tappan Street, Ann Arbor, MI 48109, USA. Tel.: +1 734 764 3196; fax: +1 734 764 2557.
E-mail address: sialm@umich.edu.

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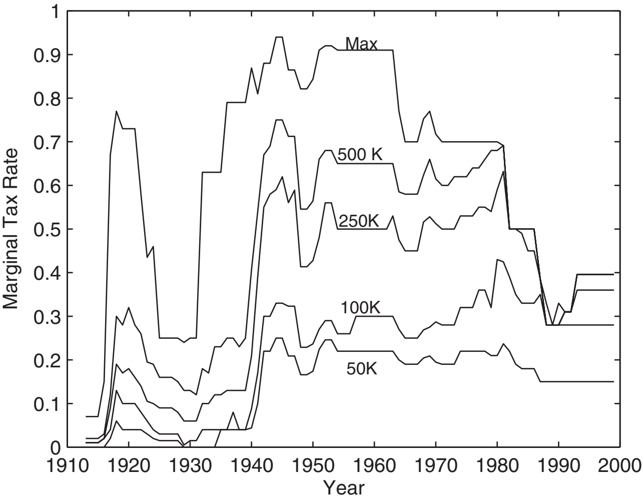

Fig. 1. Marginal income tax rates at different real income levels. The marginal income tax rates over the period from 1913 to 1999 are depicted for families with real income levels of 50, 100, 250, and 500 thousand U.S. dollars (with 1999 consumer prices), and the marginal tax rate for the highest tax bracket.

reformed several times. Tax rates have fluctuated considerably over this period, as depicted in Fig. 1, which shows the federal marginal income tax rates for individuals in five different tax brackets.[1] Besides marginal tax rate changes, other provisions of the tax code also changed, adding to the overall uncertainty of tax law. This paper investigates how stochastic tax policies affect asset prices and whether they introduce an additional risk factor in the economy, which changes equity and term premia.

This paper analyzes the effects of stochastic taxation on asset prices in a dynamic general equilibrium model. The theoretical model generalizes the Lucas (1978) asset pricing model by introducing a flat consumption tax that follows a two-state Markov chain. Whenever taxes change unexpectedly, stock and bond prices adjust instantaneously to clear the asset markets. The price adjustments are larger for assets with long durations, such as equities and long-term bonds, than for assets with shorter durations. This paper demonstrates that individuals require higher expected returns for holding the assets with larger price changes. Hence, long-term bonds and equities tend to pay on average higher returns than short-term bonds.

Stochastic taxes affect asset prices in three ways. First, they change the level of disposable income over time (*income effect*). Frequent tax changes increase the variability of consumption for a given production process. A higher variability of consumption significantly affects asset prices and leads to a higher equity premium, as previously shown in the asset pricing literature. Second, time-varying tax rates distort the relative price of consumption over time and affect the incentives to save

[1] A detailed description of the data is given in Appendix A.1.

and invest (*substitution effect*). Even if all the tax revenues were to be rebated to taxpayers and the consumption process remained completely unaffected by tax changes, stochastic taxes would still affect asset prices and equity and term premia. Third, taxes can influence the rate of growth of the economy and thereby affect asset prices (*growth effect*). Some tax regimes might be more conducive to economic growth than others.

This paper is related to the literature in finance that addresses the high equity premium and to the literature in public economics that analyzes the effects of taxes on savings decisions and portfolio choice. The papers in the finance literature show that conventional asset pricing models cannot generate equity premia, as observed in the U.S. during the last century. The literature has focused on three related puzzles. Mehra and Prescott (1985) show that extremely high levels of risk aversion are necessary to explain the large equity premium (*equity premium puzzle*). Weil (1989) demonstrates that the risk-free rate increases dramatically at higher levels of risk aversion (*risk-free rate puzzle*). And Shiller (1981) argues that stock prices tend to be more volatile than the underlying uncertainty in the economy (*excess volatility puzzle*). Many alternative explanations have been identified as potential explanations for these puzzles.[2] These generalizations of conventional asset pricing models contribute to a resolution of the equity premium puzzle but the puzzle can still not be resolved completely. This paper sheds light on the effects of stochastic taxes on asset prices and equity and term premia.

Many papers in public economics analyze the effects of taxes on saving decisions and portfolio choice.[3] However, the literature in public economics has only peripherally dealt with random taxes. Stiglitz (1982) discusses the welfare effects of random taxation. Bizer and Judd (1989) present a dynamic general equilibrium model where taxpayers understand the uncertainty in tax policy when making their portfolio decisions. On the other hand, Auerbach and Hines (1988) and Hassett and Metcalf (1999) analyze the pattern of U.S. corporate investment incentives and study the impact of tax policy uncertainty on firm-level and aggregate investment. This paper does not discuss the efficiency and the welfare implications of random taxation. Instead, this paper studies the effects of uncertain taxes on the distribution of asset prices and on term and equity premia, which have not been previously studied.

[2]Some proposed resolutions include more general preferences and expectations of individuals (e.g., Epstein and Zin, 1989; Abel, 1990; Constantinides, 1990; Campbell and Cochrane, 1999; Hansen et al., 1999); incomplete markets (e.g., Mankiw, 1986; Constantinides and Duffie, 1996; Heaton and Lucas, 1996; Storesletten et al., 2001; Constantinides et al., 2002); trading and transaction costs and other frictions (Mankiw and Zeldes, 1991; Jermann, 1998; Alvarez and Jermann, 2000, 2001; Vissing-Jørgensen, 2002); and rare events and survivorship bias (Rietz, 1988; Jorion and Goetzmann, 1999). Kocherlakota (1996), Campbell (1999), and Mehra and Prescott (2003) are surveys of this literature.

[3]The effect of taxation on portfolio allocation in a partial equilibrium model was first discussed by Domar and Musgrave (1944) and Stiglitz (1969). More recently, Eaton (1981), Gordon (1985), Judd (1985), Hamilton (1987), and Kaplow (1994) analyze the effects of different tax systems on risk taking in more general models.

An important insight into the role of the tax system on the equity premium puzzle has recently been provided by McGrattan and Prescott (2003, 2004). They find that changes in the tax and legal environment in the U.S., especially the introduction of tax-qualified retirement saving vehicles and the decrease in the top marginal income tax rate, account for the high return on corporate equity between 1960 and 2000 relative to long-term debt.[4] They compare the return on equities relative to the return on long-term debt, because short-term debt provides important liquidity services and has therefore relatively low rates of return. My paper shows that uncertain taxes have the largest impact for long-duration assets, such as equity and long-term bonds. Thus, uncertain taxes increase the equity and the term premium even if there is no differential taxation between different asset classes.

The remainder of the paper is divided into six sections. Section 2 describes the representative agent model. Section 3 derives closed-form solutions for equity prices. The pricing of zero-coupon bonds with different maturities is explored in Section 4. Section 5 summarizes the necessary conditions for an increasing term structure of interest rates and Section 6 proves that, if certain conditions are satisfied, the equity premium increases in an environment with tax reforms. Sections 7 and 8 check the sensitivity of the results using a numerical example.

2. The model

This paper generalizes the Lucas (1978) representative agent asset pricing model by introducing a flat consumption tax that follows a two-state Markov chain. This subsection describes the technology, the tax system, the preferences of the representative agent, and the equilibrium conditions.

2.1. Technology

The output in the exchange economy is exogenous and perishable. Aggregate output $y > 0$ follows a geometric random walk with drift, where z_t denotes its stochastic growth rate. Output growth z_{t+1} has a mean of μ_t and a standard deviation of σ_t. The moments of the growth rate can be time-dependent. The distribution of the growth rate is known one period in advance.

$$y_{t+1} = y_t \exp(z_{t+1}), \quad z_{t+1} \sim \mathrm{N}(\mu_t, \sigma_t^2). \tag{1}$$

Two asset classes are traded in this economy: risk-free zero-coupon bonds with different maturities (B) and one risky equity security (S). A zero-coupon bond with remaining maturity $m \in \{0, 1, 2, 3, \ldots\}$ pays a dividend of $d_t^{B,m} = 1$ if $m = 0$ and $d_t^{B,m} = 0$ otherwise. Each individual can issue or buy these bonds. There is no net aggregate supply of any bonds.

[4] Sialm (2004) studies empirically the relationship between stock valuations and effective marginal tax rates and confirms a negative relationship between marginal tax rates and stock valuations.

The equity security corresponds to the market portfolio and pays a dividend of $d_t^S = y_t$ at the beginning of each period t.[5] The prices of the two asset classes p_t^S and $p_t^{B,m}$ are defined 'ex-dividend.' The dividend and the price vectors at time t are abbreviated with $d_t \in \Re_+^n$ and $p_t \in \Re_+^n$, where n denotes the total number of assets traded in the economy ($n-1$ bonds and 1 equity security). Assets can be traded without incurring any transaction costs and investors face no borrowing or short-selling constraints.

2.2. Tax system

The government imposes a flat consumption tax and all assets face the same effective tax rates.[6] Future tax rates τ are stochastic and follow a two-state Markov-Chain with $0 \leqslant \tau_L \leqslant \tau_H < 1$. The transition probabilities ϕ_{ij} between the two states are defined as

$$\phi_{ij} = \text{Prob}(\tau_{t+1} = \tau_j | \tau_t = \tau_i). \tag{2}$$

Time-varying tax rates may reflect unpredictable changes in the balance of power among different groups of taxpayers. The tax revenues are exactly identical to the outlays of the government. The government uses a fixed proportion $\omega \in [0, 1]$ of the aggregate tax revenues T_t to finance a public good $g_t = \omega T_t$ and rebates the remaining resources to the individuals as a lump-sum payment, which can be used to purchase the aggregate consumption good without incurring any additional taxes.

The growth rate of the output and the dividends $z_{t+1} = \ln d_{t+1}^S - \ln d_t^S$ can depend on the current tax rate τ_t to allow for the possibility of distortionary taxes. The growth rate of dividends z_{t+1} is independent of the following period's tax rate τ_{t+1}. This is a natural assumption because the future tax rate τ_{t+1} (which is announced at the beginning of period $t+1$) cannot affect the dividend d_{t+1}^S (which accumulated during period t and is paid to shareholders at the beginning of period $t+1$).

2.3. Utility

The representative consumer purchases the n available assets in quantities $x_t \in \Re^n$ to maximize expected life-time utility. Utility is time-separable and the period utility is separable in the private consumption good c and the public good g. The coefficient ω is a measure of the separability between the private consumption good and the outlays of the government. If $\omega = 0$ (no separability), then all the tax revenues are rebated to the taxpayers and the outlays of the government and the private consumption good are perfect substitutes. If $\omega = 1$ (full separability), then all the tax

[5]This paper follows the equity premium literature by assuming that equity is a claim on the aggregate resources of the economy. In reality, stocks are a levered claim on the resources and exhibit considerably more risk. The return premium of an asset increases with its leverage. Thus, this assumption understates the actual premium of levered assets.

[6]The tax is levied on the tax-inclusive asset prices, whereas sales and value-added taxes are traditionally levied on the tax-exclusive price. It is straightforward to redefine the taxes as levied on the tax-exclusive price.

revenues are used to finance the public good. The asset pricing results with full separability are identical to the case where the government throws away the tax revenues. The discount factor is denoted by $\beta \in (0, 1)$. The period utilities of the two goods are denoted by $u(c)$ and $v(g)$, where $u'(c) > 0$ and $u''(c) \leqslant 0$.

The consumer's problem is to maximize

$$\mathrm{E}_t \sum_{i=0}^{\infty} \beta^i (u(c_{t+i}) + v(g_{t+i})), \tag{3}$$

where

$$c_t = (1 - \tau_t)[(p_t + d_t)' x_{t-1} - p_t' x_t] + (1 - \omega) T_t, \tag{4}$$

$$g_t = \omega T_t. \tag{5}$$

Consumers are assumed to have a power-utility function with a coefficient of relative risk aversion $\alpha \in [0, \infty)$.

$$u(c_t) = \frac{c_t^{1-\alpha} - 1}{1 - \alpha}. \tag{6}$$

Individuals with $\alpha = 0$ are risk-neutral and individuals with $\alpha = 1$ have logarithmic-preferences $u(c_t) = \ln(c_t)$. The risk aversion coefficient equals the reciprocal of the elasticity of intertemporal substitution.

The first-order conditions for the two asset classes are

$$p_t^{B,m} u'(c_t)(1 - \tau_t) = \beta^m \mathrm{E}_t(u'(c_{t+m})(1 - \tau_{t+m})), \tag{7}$$

$$p_t^S u'(c_t)(1 - \tau_t) = \beta \mathrm{E}_t(u'(c_{t+1})(1 - \tau_{t+1})(p_{t+1}^S + d_{t+1}^S)). \tag{8}$$

An optimal solution to the agent's maximization problem must also satisfy the following transversality condition:

$$\lim_{i \to \infty} \beta^i \mathrm{E}_t \left(u'(c_{t+i})(1 - \tau_{t+i}) p_{t+i}^S x_{t+i}^S \right) = 0. \tag{9}$$

2.4. Market equilibrium

For market equilibrium, the quantities of each asset demanded must equal the exogenous supply. The zero-coupon bonds have zero aggregate supply and the aggregate supply of equity is normalized to one. In equilibrium, the tax revenues, the consumption of the representative agent, and the provision of the publicly provided good amount to

$$T_t = \tau_t[(p_t + d_t)' x_{t-1} - p_t' x_t] = \tau_t d_t^S, \tag{10}$$

$$c_t = (1 - \tau_t) d_t^S + (1 - \omega) \tau_t d_t^S = (1 - \omega \tau_t) d_t^S, \tag{11}$$

$$g_t = \omega T_t = \omega \tau_t d_t^S. \tag{12}$$

The (ex-post) marginal rate of intertemporal substitution of the private good is in equilibrium:

$$\frac{u'(c_{t+i})}{u'(c_t)} = \left(\frac{c_{t+i}}{c_t}\right)^{-\alpha} = \left(\frac{1-\omega\tau_{t+i}}{1-\omega\tau_t}\frac{d^S_{t+i}}{d^S_t}\right)^{-\alpha}. \tag{13}$$

The first-order conditions (7) and (8) give the relationship determining the prices of bonds and equity securities. A zero-coupon bond with maturity $m \in \{0, 1, 2, 3, \ldots\}$ and unit face value $d^{B,0}_t = 1$ should trade in equilibrium at the following price:

$$p^{B,m}_t = \beta^m \mathrm{E}_t\left(\frac{u'(c_{t+m})}{u'(c_t)}\frac{1-\tau_{t+m}}{1-\tau_t}\right). \tag{14}$$

The price of the risky asset can be expressed as follows if the transversality condition holds:

$$\begin{aligned} p^S_t &= \sum_{i=1}^{\infty} \mathrm{E}_t\left(\beta^i \frac{u'(c_{t+i})}{u'(c_t)}\frac{1-\tau_{t+i}}{1-\tau_t} d^S_{t+i}\right), \\ &= d^S_t \sum_{i=1}^{\infty} \mathrm{E}_t\left(\beta^i \frac{1-\tau_{t+i}}{1-\tau_t}\left(\frac{1-\omega\tau_{t+i}}{1-\omega\tau_t}\right)^{-\alpha}\left(\frac{d^S_{t+i}}{d^S_t}\right)^{1-\alpha}\right) = d^S_t \delta_t. \end{aligned} \tag{15}$$

The current tax regime determines the probability distribution of future tax rates and of future growth rates $z_{t+i} = \ln d^S_{t+i} - \ln d^S_{t+i-1}$. The price-dividend ratio of equity δ_t and the price of the bond $p^{B,m}_t$, therefore, do not depend on the level of the equity dividends d^S_t. They depend only on the current tax regime and the maturity of the bonds.

In summary, taxes affect asset prices in three ways. First, the level of taxation influences consumption if some portion of the tax revenues are used to fund the public good (i.e., $\omega > 0$). This income effect changes the marginal rate of intertemporal substitution in Eq. (13). Tax changes result in a higher variability of consumption. Second, taxes distort the price of the private consumption good in different periods. Thus, Eqs. (14) and (15) include the ratio of the tax rates. This substitution effect is important even if all the tax revenues are rebated to the representative individual. Third, taxes influence the growth rate of the economy. This growth effect determines the mean and the variance of the growth rate of dividends in Eq. (1).

A special case of this model occurs if the tax rate does not vary over time (i.e., $\tau_L = \tau_H$). In this case, a flat consumption tax has no effect on asset returns. The tax terms in the marginal rate of intertemporal substitution in Eq. (13) cancel if $\tau_t = \tau_{t+i}$ for $i \in \{0, 1, 2, \ldots\}$. The tax factors in the pricing equations (14) and (15)) also cancel and taxes do not influence the distribution of the growth rate of the economy. A constant tax decreases consumption in all time periods by the same proportion and does not affect the marginal rate of intertemporal substitution. However, constant taxes have an impact on the welfare of the representative agent. Section 8.5 shows that the main conclusions of this paper are not affected if the Constant Relative Risk Aversion (CRRA) utility function is replaced with a Constant Absolute Risk

Aversion (CARA) utility function. The asset valuations and the expected returns using a CARA utility function depend on the tax level even if taxes are constant over time.

3. Equity valuation

This section derives closed-form solutions of the equity prices in the two tax regimes.

The price-dividend ratio of equity is denoted by $\delta_t = p_t^S / d_t^S$. The first-order condition (8) can be expressed as

$$\begin{aligned}\delta_t = \frac{p_t^S}{d_t^S} &= \beta \mathrm{E}_t \left[\frac{u'(c_{t+1})}{u'(c_t)} \frac{1-\tau_{t+1}}{1-\tau_t} \frac{d_{t+1}^S}{d_t^S} (1+\delta_{t+1}) \right], \\ &= \beta \mathrm{E}_t \left[\frac{d_{t+1}^S}{d_t^S} \right]^{1-\alpha} \mathrm{E}_t \left[\frac{1-\tau_{t+1}}{1-\tau_t} \left(\frac{1-\omega\tau_{t+1}}{1-\omega\tau_t} \right)^{-\alpha} (1+\delta_{t+1}) \right]. \end{aligned} \tag{16}$$

The first factor is determined by the dividend process and is denoted by $\gamma_t = \beta \mathrm{E}_t[(d_{t+1}^S / d_t^S)^{1-\alpha}] = \beta \exp((1-\alpha)\mu_t + 0.5(1-\alpha)^2 \sigma_t^2)$. This factor depends on the current tax rate ($\gamma_t = \gamma_L$ if $\tau_t = \tau_L$ and $\gamma_t = \gamma_H$ otherwise). The second factor is determined by the tax process and depends on the future price-dividend ratio and current and future tax rates. I use the following abbreviation:

$$\rho_{t,t+1} = \frac{1-\tau_{t+1}}{1-\tau_t} \left(\frac{1-\omega\tau_{t+1}}{1-\omega\tau_t} \right)^{-\alpha} > 0. \tag{17}$$

The tax-pricing factor $\rho_{t,t+1}$ equals 1 if the tax rates do not change ($\rho_{HH} = \rho_{LL} = 1$). It is defined as ρ_{LH} if the tax rates increase and as ρ_{HL} if they decrease. Note that ρ_{LH} is the reciprocal of ρ_{HL}. The expected value of $\rho_{t,t+1}$ equals $\rho_H = \phi_{HH} + \phi_{HL}\rho_{HL}$ in the high- and $\rho_L = \phi_{LL} + \phi_{LH}\rho_{LH}$ in the low-tax regime. The price-dividend ratio depends only on the current tax regime and is denoted by $\delta_t = \delta_H$ if $\tau_t = \tau_H$ and $\delta_t = \delta_L$ otherwise.

$$\delta_H = \gamma_H[\phi_{HH}(1+\delta_H) + \phi_{HL}\rho_{HL}(1+\delta_L)], \tag{18}$$

$$\delta_L = \gamma_L[\phi_{LL}(1+\delta_L) + \phi_{LH}\rho_{LH}(1+\delta_H)]. \tag{19}$$

Solving the system of linear equations for the two price-dividend ratios yields

$$\delta_H = \frac{\gamma_H[\rho_H + \gamma_L(1-\phi_{HH}-\phi_{LL})]}{1-[\phi_{HH}\gamma_H + \phi_{LL}\gamma_L + \gamma_H\gamma_L(1-\phi_{HH}-\phi_{LL})]}, \tag{20}$$

$$\delta_L = \frac{\gamma_L[\rho_L + \gamma_H(1-\phi_{HH}-\phi_{LL})]}{1-[\phi_{HH}\gamma_H + \phi_{LL}\gamma_L + \gamma_H\gamma_L(1-\phi_{HH}-\phi_{LL})]}. \tag{21}$$

To ensure that the transversality condition (9) holds, it is required that $0 < \gamma_i < 1$ for $i \in \{L, H\}$. In this case, the price-dividend ratios in Eqs. (20) and (21) are positive as shown in Appendix B.1.

The following proposition shows the relationship between the tax regime and the price-dividend ratio of equity securities in the special case where the dividend growth rate is independent of the current tax rate (i) and in the general case (ii).

Proposition 1. (i) *If the tax and the dividend processes are independent* (*i.e.*, $\mu_L = \mu_H$ *and* $\sigma_L = \sigma_H$), *then the price-dividend ratio is higher in the high-tax regime if* $\alpha < \tilde{\alpha}$ *and lower in the high-tax regime if* $\alpha > \tilde{\alpha}$:

$$\delta_H \gtreqless \delta_L \quad \text{if} \quad \alpha \lesseqgtr \tilde{\alpha},$$

where the critical risk aversion $\tilde{\alpha}$ *is given by*

$$\tilde{\alpha} = \frac{\ln(1-\tau_L) - \ln(1-\tau_H)}{\ln(1-\omega\tau_L) - \ln(1-\omega\tau_H)}. \tag{22}$$

(ii) *The price-dividend ratios in the two tax regimes have the following relationship with dependent dividend and tax processes*:

$$\delta_H \gtreqless \delta_L \quad \text{if} \quad \gamma_H \rho_H \gtreqless \gamma_L \rho_L.$$

Proof. All proofs can be found in Appendix B. □

If tax rates are not expected to change over time (i.e., $\tau_H = \tau_L$), then the price-dividend ratios are equal in the two states (i.e., $\delta_H = \delta_L = \gamma_H/(1-\gamma_H) = \gamma_L/(1-\gamma_L)$).

The coefficient $\tilde{\alpha}$ is larger than 1 and decreasing in ω. $\tilde{\alpha}$ is defined for all $\omega \in (0, 1]$. If all the tax revenues are rebated to the tax payers as a lump-sum distribution ($\omega = 0$), then $\tilde{\alpha} = \infty$. If the tax revenues are used to finance a separable public good ($\omega = 1$), then $\tilde{\alpha} = 1$.

The special case where the dividend process does not depend on the current tax regime (i) is discussed first. The price of equity can be higher in the high-tax regime. To better understand this result, it helps to analyze the different effects that determine asset prices. First, the government taxes a higher proportion of the aggregate dividends in high-tax regimes (income effect). Individuals would like to compensate for this tax by saving less and by decreasing their demand of risky assets. In equilibrium, the supply of assets cannot adjust and the price of equity has to *decrease* due to the income effect. Second, consumption is relatively more expensive in periods with high taxes since future taxes are expected to be equal to or lower than current taxes (substitution effect). Individuals want to consume less and invest more during these periods. In equilibrium, the price of the risky asset has to *increase* as a consequence of this substitution effect. The elasticity of intertemporal substitution determines which of the two effects is more important. The second effect is stronger for individuals who are more willing to substitute consumption intertemporally (i.e., $\alpha < \tilde{\alpha}$), whereas the first effect is stronger for individuals with a low elasticity (i.e., $\alpha > \tilde{\alpha}$). The price-dividend ratios are identical in the two states if $\alpha = \tilde{\alpha}$. In this case, the two effects exactly offset each other because the expenditure elasticity equals zero.

If all the tax revenues are used to finance the separable public good (i.e., $\omega = 1$), then $\tilde{\alpha} = 1$ and equity valuations are higher in the high-tax regime only if individuals are less risk-averse than a log-utility individual. If all the tax revenues are rebated to the taxpayers (i.e., $\omega = 0$), then $\tilde{\alpha} = \infty$ and equities are always valued higher in the high-tax regime. In this case, the aggregate consumption level does not depend on the tax rate and is equal to the before-tax dividend. Thus, the first effect of taxes on equity valuation is completely eliminated with full redistribution. However, the second effect is still important, because individuals have an incentive to consume less in periods where the tax on consumption is higher. Valuations in the high-tax regime tend to be higher than those in the low-tax regime at low levels of risk aversion and at low levels of separability.

A dependence between the two processes adds a third effect of taxes on equity valuation. The current tax regime affects the distribution of future output levels (growth effect).

The expected gross return of equity at time t can be separated into the following two components:

$$\mathrm{E}_t(r_{t+1}) = \mathrm{E}_t\left(\frac{p^S_{t+1} + d^S_{t+1}}{p^S_t}\right) = \xi_t \mathrm{E}_t\left(\frac{1+\delta_{t+1}}{\delta_t}\right), \tag{23}$$

where $\xi_t = \mathrm{E}_t(d^S_{t+1}/d^S_t) = \exp(\mu_t + 0.5\sigma_t^2)$.

4. Bond valuation

The prices of zero-coupon bonds can be derived from Eq. (7)

$$p_t^{B,m} = \beta^m \mathrm{E}_t\left[\frac{u'(c_{t+m})}{u'(c_t)}\frac{1-\tau_{t+m}}{1-\tau_t}\right] = \lambda_t \mathrm{E}_t[\rho_{t,t+1}(p_{t+1}^{B,m-1} + d_{t+1}^{B,m-1})]. \tag{24}$$

The second equality uses the definition $\lambda_t = \beta \mathrm{E}_t[(d^S_{t+1}/d^S_t)^{-\alpha}]$. The separation into two components is possible because the price of the bond $p_t^{B,m}$ does not depend on the level of the dividend d_t^S.

The equilibrium prices of a zero-coupon bond with maturity m can be expressed recursively using the initial conditions $p_H^{B,0} = p_L^{B,0} = 0$ and $d_H^{B,0} = d_L^{B,0} = 1$. Note that $d_t^{B,m} = 0$ if $m \in \{1, 2, 3, \ldots\}$.

$$p_H^{B,m+1} = \lambda_H[\phi_{HH}(p_H^{B,m} + d^{B,m}) + \phi_{HL}\rho_{HL}(p_L^{B,m} + d^{B,m})], \tag{25}$$

$$p_L^{B,m+1} = \lambda_L[\phi_{LL}(p_L^{B,m} + d^{B,m}) + \phi_{LH}\rho_{LH}(p_H^{B,m} + d^{B,m})]. \tag{26}$$

The next proposition proves in which tax regime the valuations of zero-coupon bonds with a maturity of m are higher.

Proposition 2. (i) *Suppose that the dividend process is independent of the tax process. The price of a zero-coupon bond with a maturity of m years is higher in the high-tax regime if $\alpha < \tilde{\alpha}$ and lower in the high-tax regime if $\alpha > \tilde{\alpha}$:*

$$p_H^{B,m} \gtreqless p_L^{B,m} \quad \text{if} \quad \alpha \lesseqgtr \tilde{\alpha}.$$

(ii) *Bond prices with a maturity of m years have the following relationship with dependent dividend and tax processes*:

$$p_H^{B,m} \gtreqless p_L^{B,m} \quad \text{if} \quad \lambda_H \rho_H \gtreqless \lambda_L \rho_L.$$

If the dividend growth rate does not depend on the current tax rate (i), then the condition for bond prices is exactly identical to the condition for equity securities from Proposition 1. In this case, valuations of both assets are higher in the high-tax regime if $\alpha < \tilde{\alpha}$. The intuition of the income and the substitution effects is similar for bonds as for equity securities.

The growth effect, which is relevant if the dividend and the tax processes are dependent, differs for equity and one-period zero-bonds. The growth rate affects the discount factor of bonds λ but not the future payoffs of the bonds. For equity, both the discount factor γ and the future dividends are affected. This difference in the growth effect on bonds and equity explains why condition (ii) for bonds differs from the one for equity.

The properties of bond prices are discussed in more detail in Appendix B.3. If tax regimes are persistent ($\phi_{HH} + \phi_{LL} > 1$), then the ratio of the bond prices in the two tax regimes converges monotonically towards a steady state value as the maturity of the bonds increases. If tax regimes are transitory ($\phi_{HH} + \phi_{LL} < 1$), then the ratio of the two bond prices oscillates around the steady-state level and converges if either $\phi_{HH} > 0$ or $\phi_{LL} > 0$. If the tax rate switches deterministically between the two regimes ($\phi_{HH} = \phi_{LL} = 0$), then the ratio of the bond prices fluctuates between two different values and does not converge.

5. Term structure of interest rates

In an environment without tax changes, the expected gross return of a bond with maturity m is given by

$$\mathrm{E}(r^{B,m}) = \frac{p^{B,m-1}}{p^{B,m}} = \frac{p^{B,m-1}}{\lambda p^{B,m-1}} = \frac{1}{\lambda}. \tag{27}$$

Corollary 1. *The expected return of zero-coupon bonds does not depend on the maturity m if tax rates do not vary over time.*

Next, I discuss the term structure in an environment with tax rate changes. The expected gross returns of a bond with maturity m in the two tax regimes is given by

$$\mathrm{E}(r_H^{B,m}) = \frac{\phi_{HH} p_H^{B,m-1} + \phi_{HL} p_L^{B,m-1}}{p_H^{B,m}}, \tag{28}$$

$$\mathrm{E}(r_L^{B,m}) = \frac{\phi_{LL} p_L^{B,m-1} + \phi_{LH} p_H^{B,m-1}}{p_L^{B,m}}. \tag{29}$$

The next proposition summarizes the necessary conditions for an increasing or decreasing term structure of interest rates. A necessary condition for a monotonically increasing term structure is that tax regimes are persistent (i.e., $\phi_{HH} + \phi_{LL} > 1$). This condition is empirically relevant because tax rates are more likely to remain unchanged than to change in each period.

Proposition 3. *Suppose that tax regimes are persistent ($\phi_{HH} + \phi_{LL} > 1$).*

(i) *Longer-term bonds have a higher expected return than shorter-term bonds if the dividend process is independent of the tax process.*
(ii) *The slope of the term structure of interest rates is determined by the following condition with dependent dividend and tax processes:*

$$\mathrm{E}(r_i^{B,m+1}) \gtreqless \mathrm{E}(r_i^{B,m}) \text{ for } i \in \{L, H\} \text{ if } (\lambda_L \rho_L - \lambda_H \rho_H)(1 - \rho_H) \gtreqless 0.$$

(i) states that the term premium increases with the maturity of the bonds if the distribution of dividends does not depend on the current tax rate ($\lambda_H = \lambda_L$). Stochastic taxes add an additional source of uncertainty to the economy and require higher returns to long-duration bonds that are exposed more to tax uncertainty. This result does not depend on the uses of the tax revenues.

(ii) allows the growth rate of the economy to be correlated with the current tax regime. A decreasing term structure is possible if the distribution of the dividend growth rate is sufficiently different between the two tax regimes. The term structure is decreasing if the tax changes are such that they reduce the aggregate uncertainty investors are exposed to.

The expected returns oscillate around the long-term values if tax regimes are transitory ($\phi_{HH} + \phi_{LL} < 1$). In this case, the expected returns are not monotonic in the maturity. If tax regimes switch deterministically ($\phi_{HH} = \phi_{LL} = 0$), then expected returns of bonds with different maturities fluctuate between two values and do not converge. The properties of bond returns is described in more detail in Appendix B.4.

6. Equity premium

The equity premium compares the expected return of equity to the return of risk-free one-period zero-coupon bonds. The following proposition states the conditions under which the equity premium increases in an environment with tax changes.

Proposition 4. *The equity premium π_i for $i \in \{L, H\}$ equals the sum of a premium due to dividend uncertainty π_i^D and a premium due to tax changes π_i^T:*

$$\pi_i = \mathrm{E}_t(r_{t+1}^S) - r_t^{B,1} = \pi_i^D + \pi_i^T. \tag{30}$$

(i) *If the dividend process is independent of the tax process, then both premia are positive.*
(ii) *The sign of the tax premium is determined by the following condition with dependent tax and dividend processes:*

$$\pi_i^T \gtreqless 0 \text{ for } i \in \{L, H\} \text{ if } (\gamma_L \rho_L - \gamma_H \rho_H)(1 - \rho_H) \gtreqless 0.$$

The excess return of stocks over short-term bonds is due to two premia. The first premium π_i^D equals the equity premium in an environment without tax changes and is always positive. The second premium π_i^T is due to tax changes and is positive if the condition in (ii) is satisfied. It is possible for the equity premium to become negative if the distribution of the dividend growth rate is sufficiently different between the two tax regimes. The equity premium decreases if the tax changes are such that they reduce the aggregate uncertainty investors are exposed to. (i) states that the tax premium is always positive if the distribution of dividends does not depend on the current tax rate. The sign of the tax premium does not depend on whether tax regimes are transitory or persistent.

The conditions in (ii) of Propositions 3 and 4 look very similar, but they depend on λ and γ, respectively. These two factors differ because the growth rate of dividends affects only the payoffs of equity and not the payoffs of bonds. It is possible, therefore, to observe an increasing term structure of interest rates and a negative tax premium. If the growth rate is independent of the tax rate, then the equity and the term premia are higher with stochastic taxes.

The three effects that drive the asset valuation give an intuition of the effect of tax changes on the equity premium. The first effect (income effect) increases the equity premium because tax changes increase the variability of consumption over time. It is well-known that an increase in consumption volatility increases the required risk-premia. The first effect disappears if all the tax revenues are rebated to the representative agent. In this case, the consumption process is not affected by tax changes. The second effect (substitution effect) remains important in the case with a full rebate. Varying tax rates affect the relative price of consumption over time. The third effect (growth effect) can increase or decrease the equity premium, depending on the correlation between taxes and productivity growth. Individuals require higher expected returns for holding long-duration assets, such as stocks and long-term bonds, compared to short-term bonds. The increase of the equity premium occurs because equities have a relatively long duration.[7]

7. Numerical example

This section illustrates the impact of stochastic taxes on asset valuations using a numerical example.

7.1. Numerical assumptions

The model is solved for plausible underlying parameter values to determine whether stochastic taxation is economically significant. In this example, the two tax rates are assumed to be $\tau_L = 0.3$ and $\tau_H = 0.4$ and the transition probabilities are $\phi_{LL} = \phi_{HH} = 0.8$. This implies an average duration of a tax regime of five years. The average tax rate equals 35 percent and has a standard deviation of 5 percent. These

[7] Abel (1999) divides the equity premium into a term premium and a risk premium.

Table 1
Numerical assumptions for base case

Coefficient	α	β	ω	τ_L	τ_H	ϕ_{LL}	ϕ_{HH}	μ	σ
Value	2.50	0.98	1	0.30	0.40	0.80	0.80	0.01	0.05

α denotes the coefficient of relative risk aversion, β the discount factor, ω the ratio of the tax revenues, which are used to fund a separable public good, τ_L and τ_H the tax rates in the low- and high-tax regimes, ϕ_{LL} and ϕ_{HH} the transition probabilities between the two regimes, and μ and σ the mean and the standard deviation of the logarithm of the growth rate of dividends.

values correspond roughly to past tax changes of an investor in the $250,000 tax bracket. For example, between 1940 and 1999, tax rates increased four times and decreased six times by more than five percent. The average increase equaled 11.06 percent and the average decrease equaled 8.22 percent. The distribution of average tax changes is similar for the longer period between 1914–1999.[8]

The dividend growth rate has a mean of 1 percent and a standard deviation of 5 percent. The distribution of the growth rate is assumed to be independent of the tax regime, because there is no significant correlation between tax rates and the growth rate of the economy over the period between 1914–1999, using the tax series from Fig. 1. Sensitivity analyses summarized in Section 8 show that changes in these assumptions do not affect the results much. The base case assumes a discount factor of $\beta = 0.98$. The analyses in this section will concentrate on risk-aversion coefficients in the range between 0 and 5. The tax revenues are used to finance separable public goods ($\omega = 1$). Table 1 summarizes all the base-case assumptions.

7.2. Asset prices

Fig. 2 shows how the price-dividend ratio depends on the coefficient of relative risk aversion. The dashed curve shows the price-dividend ratio in an environment without tax changes. In this case, the price-dividend ratio does not depend on the tax rate. The price-dividend ratio without tax rate changes equals 30.38 at a risk aversion of $\alpha = 2.5$. The ratio increases considerably as individuals become less risk-averse. The solid curve depicts the price-dividend ratios in the low-tax and the high-tax regime in an environment with tax rate changes. The valuations are higher in the high-tax regime at low-levels of risk aversion (i.e., $\alpha < \tilde{\alpha} = 1$) and higher in the low-tax regime at high-levels of risk aversion. The valuations are identical if individuals have logarithmic utility. In this case, tax regime changes have no effect on asset valuations, and the price-dividend ratio is exactly what it would be in an environment without tax changes. At a risk-aversion level of $\alpha = 2.5$, the price-dividend ratio equals 27.38 in the high-tax state and 34.15 in the low-tax state. Stock

[8] I concentrate on the tax rates of relatively wealthy individuals because those individuals hold a significant portion of financial assets. Poterba (2000) shows that the top one percent of equity holders own 52.2 percent of household holdings of corporate stock according to the 1998 Survey of Consumer Finances.

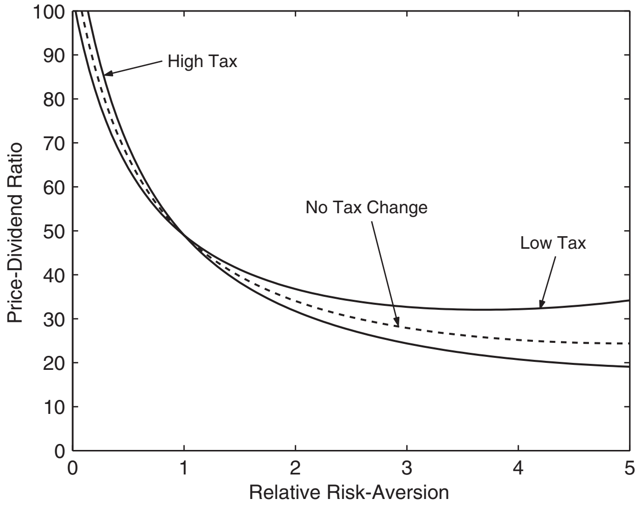

Fig. 2. Price-dividend ratios of equity. The dashed curve shows the price-dividend ratios in an environment without tax rate changes, and the solid curve shows the price-dividend ratios in the low-tax and the high-tax regimes in an environment with tax rate changes.

prices fall in this case by 19.82 percent whenever taxes increase, and increase by 24.73 percent whenever taxes decrease. The model assumes that the timing of tax regime changes is not anticipated by the investors. In reality, market participants learn gradually about possible future tax reforms and the price changes occur over longer time horizons as investors adjust their expectations about future tax changes.

Fig. 3 depicts the expected returns of short-term bonds and equity in an environment with and without tax rate changes. The curves correspond to the average returns over both tax regimes. If tax rates do not change over time, then the return of the short-term bond equals 3.81 percent at a risk aversion of $\alpha = 2.5$ and increases at higher levels of risk aversion. If tax rates are stochastic, the mean return of the risk-free one-period bond equals 3.48 percent if $\alpha = 2.5$. Panel B depicts the expected returns of equity securities in environments with and without tax changes. The mean return of equity increases from 4.46 percent to 4.95 percent if $\alpha = 2.5$.

Next, I compute the term premium and the equity premium given the parametric assumptions from Table 1. Panel A of Fig. 4 depicts the expected returns of zero-coupon bonds with different maturities. In an environment without tax changes, the term structure of interest rates is flat with a yield of 3.81 percent. With tax regime shifts, the expected returns of zero-coupon bonds increase with their maturity. All the bond returns with a maturity of more than 1 year are higher in a model with stochastic taxes than the bond returns without tax changes. The expected returns of the bonds converge to 4.37 percent as $m \to \infty$. The average term premium, which is defined as the difference between the yield of bonds where the maturity converges to infinity and the yield of a 1-year bond, amounts to 0.89 percent.

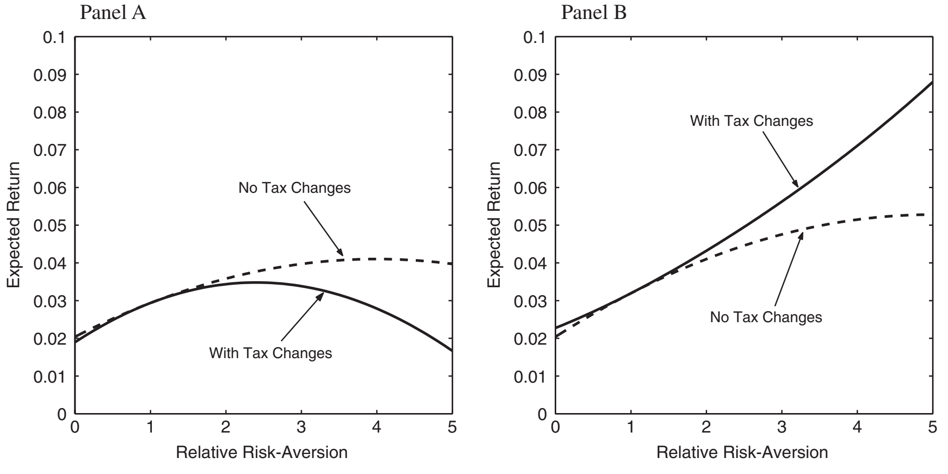

Fig. 3. Expected returns of (Panel A) short-term bonds and (Panel B) equity: the dashed curves show the expected returns in an environment without tax rate changes, and the solid curves show the expected returns with tax rate changes.

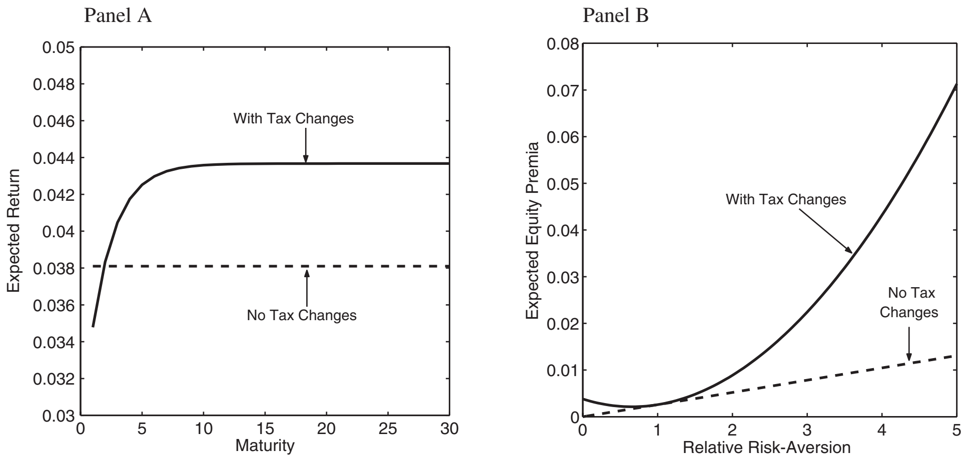

Fig. 4. Panel A: Term and Panel B: Equity premium. The solid curves correspond to an environment with stochastic taxes and the dashed curves correspond to an environment with constant taxes.

Panel B depicts the equity premium at different levels of risk aversion. If tax rates do not change, then the equity premium equals only 0.65 percent at a coefficient of risk aversion of $\alpha = 2.5$. This is considerably lower than the equity premium of between 2.55 and 4.32 percent estimated by Fama and French (2002). The equity premium is highly sensitive to changes in the coefficient of relative risk aversion. The equity premium is lowest at a risk aversion of $\alpha = 0.65$. An equity premium of 7.13 percent results at a coefficient of relative risk aversion of 5. The equity premium at this level of risk aversion without tax rate changes would have been only 1.31 percent.

The higher term premium accounts for a large portion of the equity premium. The effect of tax rate changes on assets depends primarily on the duration of the assets. Both equity and bonds have long durations and are highly sensitive to changes in tax rates. For tax changes to have a substantial effect on the equity premium, risk aversion has to be sufficiently large.

The qualitative implications of tax changes do not change if the tax revenues are rebated to the tax payers. However, the quantitative effects decrease. At a risk-aversion level of $\alpha = 2.5$, the term premium decreases from 0.89 to 0.38 percent, while the equity premium decreases from 1.48 to 1.02 percent if all the revenues are rebated.

8. Robustness tests

This section tests whether the previous results are robust to alternative specifications and different calibrations.

8.1. Persistence and tax differences

To check the robustness of the numerical example in the previous section, the numerical assumptions are changed. Panel A of Fig. 5 depicts the expected returns of short- (1-year bonds) and long-term (30-year bonds) bonds and equity at different levels of persistence of the tax regimes with a symmetric transition matrix (i.e., $\phi_{HH} = \phi_{LL}$) and at a risk aversion of $\alpha = 2.5$. If tax rates are permanent

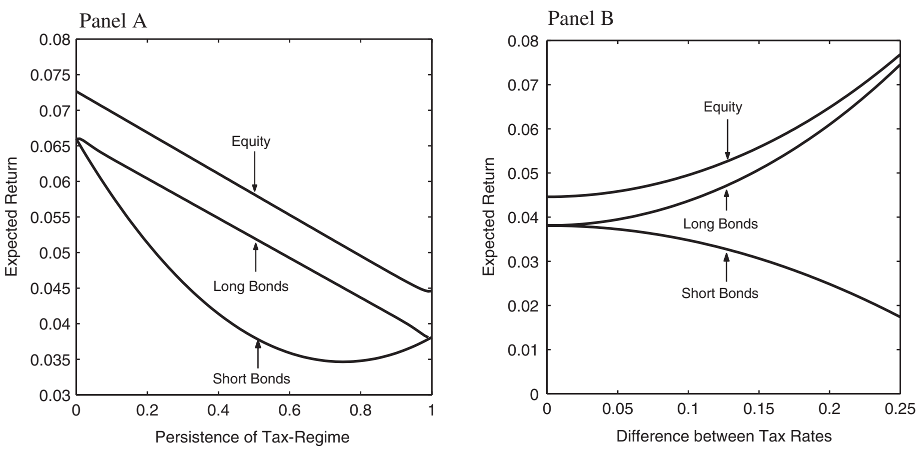

Fig. 5. Asset returns with changing persistence levels and tax rates. The expected returns of 1 and 30-year zero-coupon bonds and equity securities are depicted at different persistence levels (Panel A) and at different tax-rate differentials (Panel B).

($\phi_{HH} = \phi_{LL} = 1$), then the term premium due to tax changes equals zero. The equity premium equals the premium in an environment without tax changes. The price changes are large and infrequent at high persistence levels and small and frequent at low persistence levels. The term premium is largest for intermediate persistence levels, when tax changes are common and price changes are relatively large.

Panel B of Fig. 5 shows the dependence of the returns of the two assets on the difference between the tax rates in the two states. The average tax rate is kept constant at its average level of 35 percent. As the difference between the tax rates in the two tax regimes increases, the mean return of equity securities and long-term bonds increases and the mean return of short-term bonds decreases. Most of the equity premium is due to the term premium if the tax difference is relatively large.

8.2. Dependence between growth rate and tax regime

The previous numerical exercises assume that the distribution of the growth rate is identical in the two tax regimes. It is plausible that a higher tax burden will stifle the growth rate of the economy.[9] The results in Section 6 demonstrate that the premium due to tax changes can be either positive or negative if the growth rate of the economy depends on the tax rate.

Fig. 6 depicts the equity premium if the distribution of the dividend growth rate differs between the two tax regimes. In the base case, the mean and the standard deviation of the growth rate are 1 and 5 percent, respectively. I change the mean and the standard deviation of the growth rate by increasing the moments in one regime and decreasing the moments by the same amounts in the other regime. Thus, if the difference in the mean growth rates is $\mu_H - \mu_L = -0.04$, then the growth rates are as follows: $\mu_H = -0.01$ and $\mu_L = 0.03$. The equity premium decreases with the difference in the means of the dividend growth rates. Thus, the equity premium tends to be larger in the more plausible case where the economy grows at a slower pace during time periods of relatively high taxes.

The differences between the moments of the growth rates in the two regimes have to be very large to generate a negative tax premium. If $\sigma_H = \sigma_L$, then the growth rate in the low-tax regime would need to exceed the growth rate in the high-tax regime by more than 6.2 percent to generate a negative tax premium.

Fig. 6 also shows that there is a 'U'-shaped relationship between the equity premium and the difference in the standard deviations of the growth rates in the two regimes. The equity premium tends to increase as one regime becomes relatively more risky. However, the impact of changes in the difference between the standard deviations on the equity premium is relatively small.

8.3. Additional states

The previous results assume that tax rates follow a two-state Markov chain. The results do not change much if additional states are introduced. Alternative

[9] See for example Prescott (2004) for a study of the effects of taxes on the labor supply.

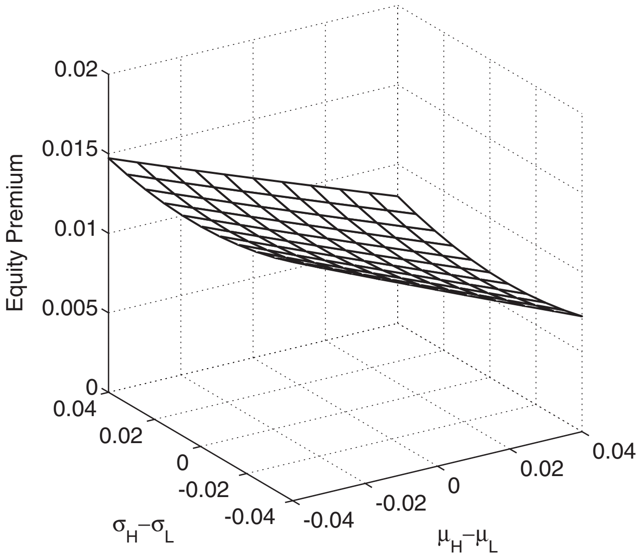

Fig. 6. Equity premium with dependence between growth rate and tax regime. The equity premium is depicted for cases where the mean μ and the standard deviation σ of the dividend growth rate differ between the two tax regimes.

specifications of the Markov chain can either increase or decrease the equity premium relative to the base case with two states. For example, a three-state Markov chain with $\tau \in \{0.275, 0.350, 0.425\}$ and with a probability of remaining in the current state of 0.8222 and equal probabilities to switch to any of the two other states has the same unconditional mean and variance of the tax rates and the same expected tax change in each period as the two-state Markov chain used previously. In this example, the equity premium increases from 1.48 to 1.51 percent. The equity premium increases slightly because larger tax changes become possible. The equity premia are considerably larger in the highest and the lowest tax state compared to the middle tax state.

8.4. Heterogeneity

To check the robustness of the results with a representative individual, I also compute numerically term and equity premia in a model with heterogeneous agents, where agents differ in their wealth levels. The government decreases income inequality by imposing net taxes on the wealthy and paying net transfers to the poor. This redistribution policy affects the portfolio choices of the individuals and changes asset returns and the equity premium in equilibrium because asset markets are incomplete. Tax changes will generally benefit some individuals and harm others. People with diverging interests can use the available assets to insure each other against tax rate changes.[10] The introduction of heterogeneous agents does not affect asset prices significantly. The equity premium decreases only slightly as the wealth distribution becomes more unequal.

8.5. Alternative utility specifications

The finance and the macroeconomic literature often uses a power-utility specification which implies constant relative risk aversion (CRRA) because the asset prices do not depend on the wealth level of the investor and because closed form solutions are usually obtainable. One potential problem of a power utility function is that asset prices and returns do not depend on consumption tax rates if tax rates do not change over time, as discussed previously in Section 3. In this subsection, I compute asset prices and equity premia using a Constant Absolute Risk Aversion Utility (CARA). The asset prices and the asset returns depend in this case on the output and the tax levels.

Consumers are assumed to have a utility with a constant coefficient of absolute risk-aversion $\alpha \in (0, \infty)$.

$$u(c_t) = -\frac{1}{\alpha}\exp(-\alpha c_t). \tag{31}$$

Since an unbounded dividend process creates undesired results in a model with CARA utility, I change the dividend process by bounding the dividend levels between $[0, 2]$. Moreover, I assume that the dividend changes are normally distributed with a mean of 1 percent and a standard deviation of 5 percent per year. The boundaries of the dividend ranges are absorbing states, such that dividends remain permanently at $d = 0$ or $d = 2$ once they reach these boundaries. Households are assumed to have an absolute risk-aversion coefficient of 2.5 percent. The other assumptions remain identical to the ones from the CRRA specification.

The prices and the returns using this alternative specification cannot be obtained analytically. Instead, I compute the price functions numerically. The dividend range is discretized using a grid distance of 0.005. To find the stock price for each dividend level, the pricing (15) is iterated until convergence is attained. The prices of the bonds are computed following Eq. (24).

Fig. 7 plots the stock price and the equity premia at different dividend levels. The solid curves correspond to environments where tax rates are stochastic as described in Section 2.2 and the dashed curves correspond to environments where tax rates do not change over time and equal 35 percent. The top figure indicates that the value of the stocks increases monotonically with the dividend level. The value is zero if dividends reach the absorbing state of $d = 0$. Similarly to the specification with CRRA utility, the asset valuations are not always higher in the low-tax state. For example, if the initial dividend is below 0.62, then the stock valuations are lower in the low-tax regime.

[10] Dumas (1989) formulates a two-person dynamic model with capital markets and shows that the two investors interact to share their risks.

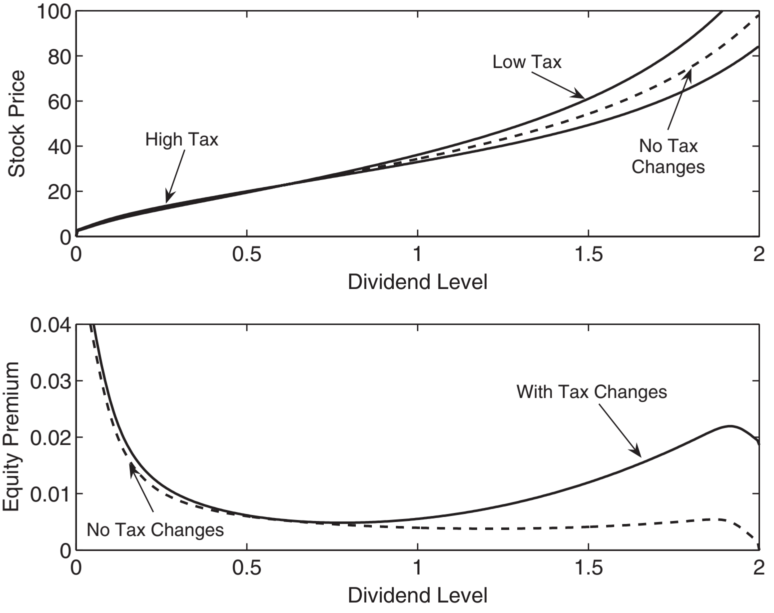

Fig. 7. Stock prices and equity premia with CARA utility. The stock price and the equity premium is depicted for different dividend levels using a CARA utility specification. The solid curves correspond to an environment with stochastic taxes and the dashed curves correspond to an environment with constant taxes.

The bottom figure depicts the equity premium in stochastic (solid) and constant (dashed) tax environments, respectively. In an environment without tax changes, the equity premium is zero if $d = 2$, because dividends are assumed to remain permanently at this level. On the other hand, the equity premium increases significantly as the dividend level approaches $d = 0$, because holding equity becomes relatively more risky. If the dividend reaches $d = 0$, then the value of equity will immediately drop to zero. Households are only willing to hold equity securities if equity securities provide a large premium compared to default-free fixed-income securities as the dividend level approaches zero.

The equity premium in a stochastic tax environment is always at least as large as the equity premium in a constant tax environment. If the stock valuations are identical in the two tax regimes ($d = 0.62$), then the equity premium in an environment with stochastic taxes is identical to the equity premium in an environment with constant taxes.

9. Conclusions

This paper generalizes the Lucas (1978) asset pricing model by introducing a flat consumption tax, which follows a two-state Markov chain. This tax does not merely affect equity securities: it affects all assets symmetrically. Whenever taxes change, asset prices need to adjust instantaneously to clear asset markets. These price changes increase the variability of expected and actual asset returns. The price adjustments are more severe for assets with long durations, such as equity and long-term bonds, than for assets with shorter durations. Individuals require higher expected returns for holding the assets with more severe price changes under plausible conditions.

Tax rate changes affect asset prices even if all the tax revenues are rebated to the representative individual and the consumption process remains completely unaffected by tax changes. Stochastic taxes remain important in this case because tax changes distort the price of consumption over time and affect investment incentives.

This paper makes several simplifying assumptions which could be relaxed in future work. First, the model uses a simple exchange economy without real investment opportunities to illustrate the effects of tax changes. Endogenizing real investment choices will result in a more realistic model of the economy. Second, the current tax system in the U.S. is not a flat consumption tax system. It is a progressive income tax system where some income sources are exempt from taxes (e.g., tax-deferred accounts, municipal bonds). In particular, stocks and bonds face different effective tax rates. The effects of tax reforms will differ if the effective tax on stock returns is smaller than the tax on bond returns and if the variability of the tax rates of the two assets differs. The analysis under a more realistic tax system would be interesting.

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Appendix A. Data

A.1. Marginal tax rates

Taxable income was derived for five real income levels after deducting exemptions for a married couple filing jointly with two dependent children from the income levels. The proportion of total deductions relative to the adjusted gross income was assumed to equal the proportion of total deductions in the whole population for each year as reported by the Internal Revenue Service. The marginal income tax brackets and exemptions were determined using the Statistics of Income of the Internal Revenue Service (1954) for the years 1913–1943, Pechman (1987) for the years 1944–1987, and different issues of the Instructions to Form 1040 from the IRS for the remaining years between 1988 and 1999. The values of the Consumer Price Index from 1913 to 1957 were taken from Mitchell (1983) and for the other years from the U.S. Government Printing Office (2000). Total deductions as a proportion of adjusted gross income (AGI) were derived from different issues of the Statistics of Income of the IRS.

Appendix B. Proofs

B.1. Conditions for positive price-dividend ratios

This proof shows that the price-dividend ratios in Eqs. (20) and (21) are positive if $0<\gamma_i<1$ for $i \in \{L, H\}$. The numerator of Eq. (20) N_H is positive if $0<\gamma_i<1$ for $i \in \{L, H\}$:

$$\begin{aligned} N_H &= \gamma_H[\rho_H + \gamma_L(1-\phi_{HH}-\phi_{LL})] \\ &= \gamma_H[\phi_{HH} + \phi_{HL}\rho_{HL} + \gamma_L(1-\phi_{HH}-\phi_{LL})] \\ &= \gamma_H[\phi_{HH}(1-\gamma_L) + \phi_{HL}\rho_{HL} + \gamma_L(1-\phi_{LL})]>0. \end{aligned}$$

The denominator D_H of Eq. (20) is positive if $0<\gamma_i<1$, because

$$\begin{aligned} 1-D_H &= \phi_{HH}\gamma_H + \phi_{LL}\gamma_L + \gamma_H\gamma_L(1-\phi_{HH}-\phi_{LL}) \\ &= \gamma_H[\phi_{HH} + \phi_{HL}\gamma_L] + \gamma_L[\phi_{LL} + \phi_{LH}\gamma_H] - \gamma_H\gamma_L \\ &\leqslant \gamma_H + \gamma_L - \gamma_H\gamma_L = \gamma_H + (1-\gamma_H)\gamma_L < 1. \end{aligned}$$

Similar operations show that (21) is positive. □

B.2. Proof of Proposition 1

(i) If $\mu_L = \mu_H$ and $\sigma_L = \sigma_H$, then $\gamma_L = \gamma_H$. In this case only the first product in the numerator differs between Eqs. (20) and (21). $\rho_{HL} = [(1-\tau_L)/(1-\tau_H)][(1-\omega\tau_L)/(1-\omega\tau_H)]^{-\alpha}$. $\rho_{HL} = 1$ if $\alpha = \tilde{\alpha}$ and $\partial\rho_{HL}/\partial\alpha \leqslant 0$. Thus, $\rho_{HL} \gtreqless 1$ if $\alpha \lesseqgtr \tilde{\alpha}$. Because $\rho_{LH} = 1/\rho_{HL}$, $\rho_{LH} \lesseqgtr 1$ if $\alpha \lesseqgtr \tilde{\alpha}$. $\rho_H = \phi_{HH} + \phi_{HL}\rho_{HL}$ and $\rho_L = \phi_{LL} + \phi_{LH}\rho_{LH}$ are simply weighted averages of ρ_{HL} and ρ_{LH} with 1, respectively. Thus, $\rho_H \gtreqless 1 \gtreqless \rho_L$ if $\alpha \lesseqgtr \tilde{\alpha}$. $\delta_H \gtreqless \delta_L$ if $\rho_H \gtreqless \rho_L$, because the denominators in Eqs. (20) and (21) are positive as proved in Appendix B.1 and because by assumption $0<\gamma_L = \gamma_H$. Thus, $\delta_H \gtreqless \delta_L$ if $\alpha \lesseqgtr \tilde{\alpha}$.

(ii) Eqs. (20) and (21) differ only in the first product of their numerators. The denominators in Eqs. (20) and (21) are positive as proved in Appendix B.1. Therefore, $\delta_H \gtreqless \delta_L$ if $\gamma_H\rho_H \gtreqless \gamma_L\rho_L$. □

B.3. Proof of Proposition 2

This proof shows first the necessary conditions for bond prices to be higher in the high-tax regime for the general case (ii). It is then straightforward to demonstrate the conditions for the special case (i), where tax rates and growth rates are independent.

The bond price ratio ψ^m with maturity $m > 1$ is

$$\psi^m = \frac{p_H^{B,m}}{p_L^{B,m}} = f(\psi^{m-1}) = \frac{\lambda_H[\phi_{HH}\psi^{m-1} + \phi_{HL}\rho_{HL}]}{\lambda_L[\phi_{LL} + \phi_{LH}\rho_{LH}\psi^{m-1}]}. \tag{32}$$

The bond price ratio is a function of its lagged value $\psi^m = f(\psi^{m-1})$. Its first derivative is

$$\frac{d\psi^m}{d\psi^{m-1}} = \frac{\lambda_H\lambda_L[\phi_{HH} + \phi_{LL} - 1]}{[\lambda_L(\phi_{LL} + \phi_{LH}\rho_{LH}\psi^{m-1})]^2}. \tag{33}$$

Thus, $d\psi^m/d\psi^{m-1} \gtreqless 0$ and $d^2\psi^m/d\psi^{m-1^2} \lesseqgtr 0$ if $\phi_{HH} + \phi_{LL} \gtreqless 1$. f is increasing and concave if the tax regimes are persistent and decreasing and convex if the tax regimes are transitory. Moreover, ψ^m is positive for all maturities m.

Fig. 8 depicts four different cases, depending on whether tax rates are persistent and on the size of the discount factors. In Case 1.1, tax regimes are persistent ($\phi_{HH} + \phi_{LL} > 1$) and the discount factor is higher in the high-tax regime ($\lambda_H\rho_H > \lambda_L\rho_L$). In the first period, $\psi^1 = f(1) = \lambda_H\rho_H/\lambda_L\rho_L > 1$. Because $f(\psi)$ is increasing and concave, $\psi^2 = f(\psi^1) > \psi^1$. The bond price ratio increases monotonically and converges to its steady state $\psi^* > 1$. Thus, $\psi^m > 1$ for all maturities. In Case 1.2, tax regimes are persistent and the discount factor is lower in the high-tax regime. In the first period, $\psi^1 = f(1) = \lambda_H\rho_H/\lambda_L\rho_L < 1$. Because $f(\psi)$ is increasing and concave, $\psi^2 = f(\psi^1) < \psi^1$. The bond price ratio decreases monotonically and converges to its steady state $\psi^* < 1$. Thus, $\psi^m < 1$ for all maturities.

These explanations prove that Proposition 2 holds if tax rates are persistent. The following discussion shows that the proposition also holds in Cases 2.1 and 2.2. In Case 2.1, tax regimes are transitory and the discount factor is higher in the high-tax regime. In the first period, $\psi^1 = f(1) = \lambda_H\rho_H/\lambda_L\rho_L > 1$. Because $f(\psi)$ is decreasing and convex, $\psi^2 = f(\psi^1) < \psi^*$. The bond price ratio oscillates around its steady state $\psi^* > 1$. In Case 2.2, tax regimes are transitory and the discount factor is lower in the high-tax regime. In the first period, $\psi^1 = f(1) = \lambda_H\rho_H/\lambda_L\rho_L < 1$. Because $f(\psi)$ is decreasing and convex, $\psi^2 = f(\psi^1) > \psi^*$. The bond price ratio oscillates around its steady state $\psi^* < 1$.

Next, I analyze whether the price ratios converge to the steady-state level ψ^*. Suppose, first, that $\phi_{HH} = \phi_{LL} = 0$. In this case, the function f is identical to its inverse f^{-1}:

$$f(\psi) = f^{-1}(\psi) = \frac{\lambda_H\rho_{HL}}{\lambda_L\rho_{LH}\psi}. \tag{34}$$

Thus, the bond ratio ψ follows in this case a cycle of $\psi^m = \psi^1 = \lambda_H\rho_H/\lambda_L\rho_L$ if m is odd and $\psi^m = \psi^0 = 1$ if m is even. With $\phi_{HH} = \phi_{LL} = 0$, the price ratio ψ does not

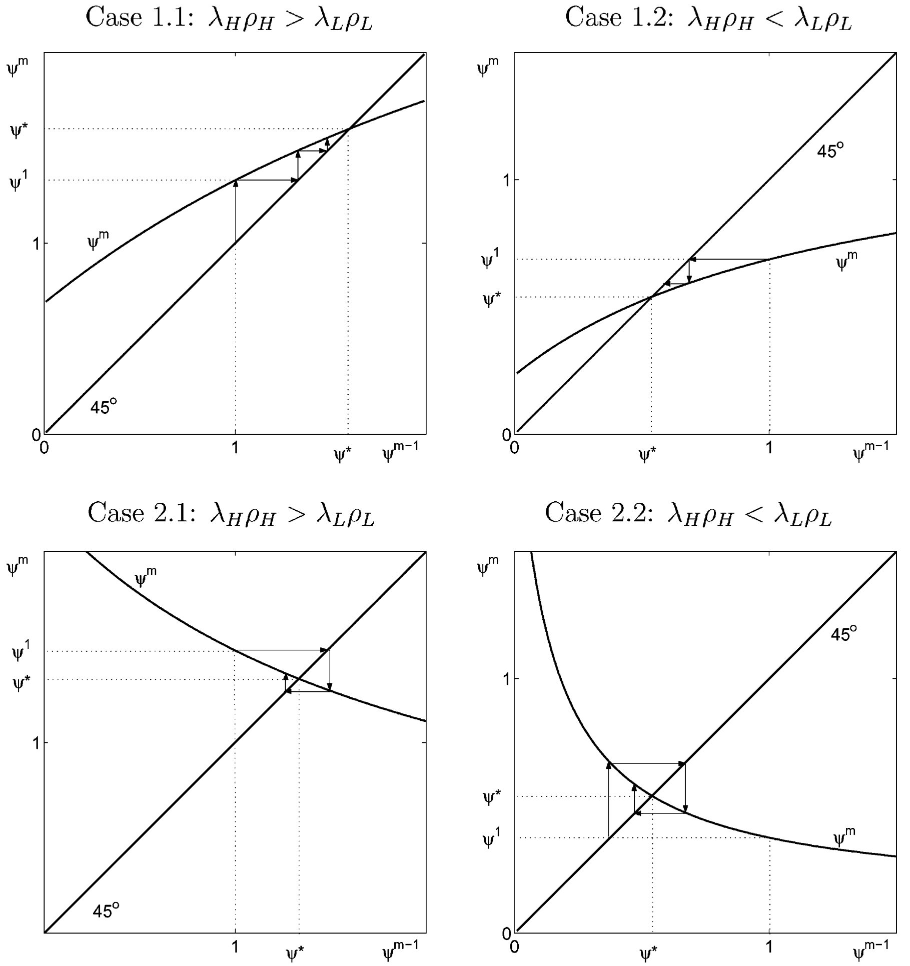

Fig. 8. Dynamics of price ratio ψ. Case 1: Persistent tax regimes ($\phi_{HH} + \phi_{LL} > 1$) and Case 2: Transitory tax regimes ($\phi_{HH} + \phi_{LL} < 1$). These figures depict the bond price ratios ψ in four different cases according to Eq. (32). If tax regimes are persistent, then the price ratio ψ converges monotonically to its steady state (Cases 1.1 and 1.2). If tax regimes are transitory, then the price ratio oscillates around its steady state (Cases 2.1 and 2.2). The bond-price ratio ψ is larger than 1 if $\lambda_H\rho_H > \lambda_L\rho_L$ (Cases 1.1 and 2.1) and smaller than 1 if $\lambda_H\rho_H < \lambda_L\rho_L$ (Cases 1.2. and 2.2).

converge to ψ^*. Proposition 2 still holds in this case because $\psi^m \geqslant 1$ for all m if $\lambda_H\rho_H > \lambda_L\rho_L$ and $\psi^m \leqslant 1$ for all m if $\lambda_H\rho_H < \lambda_L\rho_L$.

The next sections show that the bond price ratio ψ^m converges to ψ^* as m goes to ∞ as long as either $\phi_{HH} > 0$ or $\phi_{LL} > 0$. First, I look at the case where $0 < \psi^m < \psi^*$. The price ratio converges to ψ^* if $\psi^{m+1} < \psi^{m-1}$ for all possible m. Note that both ψ^{m+1} and ψ^{m-1} are larger than ψ^* if $\psi^m < \psi^*$ whenever the price ratios are oscillating around the steady-state value.

The function $f(\psi)$ and its inverse $f^{-1}(\psi)$ are defined as follows:

$$f(\psi) = \frac{\lambda_H(\phi_{HH}\psi + \phi_{HL}\rho_{HL})}{\lambda_L(\phi_{LL} + \phi_{LH}\rho_{LH}\psi)}, \tag{35}$$

$$f^{-1}(\psi) = \frac{\lambda_L\phi_{LL}\psi - \lambda_H\phi_{HL}\rho_{HL}}{\lambda_H\phi_{HH} - \lambda_L\phi_{LH}\rho_{LH}\psi}. \tag{36}$$

Note that $\psi^{m+1} = f(\psi^m)$ and $\psi^{m-1} = f^{-1}(\psi^m)$ by the definition of the functions f and f^{-1}. The price ratio ψ converges to its steady-state level ψ^* if $f(\psi) < f^{-1}(\psi)$ for all possible $\psi < \psi^*$. Thus, the price ratio converges if

$$\frac{\lambda_H(\phi_{HH}\psi + \phi_{HL}\rho_{HL})}{\lambda_L(\phi_{LL} + \phi_{LH}\rho_{LH}\psi)} < \frac{\lambda_L\phi_{LL}\psi - \lambda_H\phi_{HL}\rho_{HL}}{\lambda_H\phi_{HH} - \lambda_L\phi_{LH}\rho_{LH}\psi}. \tag{37}$$

The function f is decreasing in Cases 2.1 and 2.2 and takes its lowest level if $\psi \to \infty$

$$\lim_{\psi\to\infty} f(\psi) = \frac{\lambda_H\phi_{HH}}{\lambda_L\phi_{LH}\rho_{HL}}. \tag{38}$$

The denominator of the right-hand side of Eq. (37) is negative, because $\lambda_H\phi_{HH} - \lambda_L\phi_{LH}\rho_{LH}\psi \leqslant \lambda_H\phi_{HH} - \lambda_L\phi_{LH}\rho_{LH}(\lambda_H\phi_{HH})/(\lambda_L\phi_{LH}\rho_{HL}) = 0$.
Simplifying Eq. (37) gives

$$[\lambda_H\phi_{HH} + \lambda_L\phi_{LL}]g(\psi) < 0, \tag{39}$$

where

$$g(\psi) = \psi^2(\lambda_L\phi_{LH}\rho_{LH}) + \psi(\lambda_L\phi_{LL} - \lambda_H\phi_{HH}) - (\lambda_H\phi_{HL}\rho_{HL}). \tag{40}$$

The first factor of Eq. (39) is strictly positive as long as either ϕ_{HH} or ϕ_{LL} is strictly positive. The quadratic (40) is exactly identical to the equation that solves for the steady-state price ratio ψ^*. To derive the equation for the steady-state price, simply set $\psi^* = f(\psi^*)$ in Eq. (35). The solutions to the quadratic equation are

$$\psi^*_{1,2} = \frac{\lambda_H\phi_{HH} - \lambda_L\phi_{LL} \pm \sqrt{(\lambda_H\phi_{HH} - \lambda_L\phi_{LL})^2 + 4\lambda_L\lambda_H\phi_{HL}\phi_{LH}}}{2\lambda_L\phi_{LH}\rho_{LH}}. \tag{41}$$

One solution to Eq. (41) is positive and the other is negative. Since bond prices are always positive we can ignore the negative solution. The quadratic (40) is $g(\psi) < 0$ if $0 \leqslant \psi < \psi^*$ and $g(\psi) > 0$ if $\psi^* < \psi < \infty$. Thus, the second factor $g(\psi)$ of Eq. (39) is negative if $0 \leqslant \psi < \psi^*$. Thus, inequality (37) is satisfied if $0 < \psi < \psi^*$

Next, I look at the case where $\psi^* < \psi^m < \infty$. The price ratio ψ converges to its steady-state level ψ^* if $f(\psi) > f^{-1}(\psi)$ for all possible ψ. Thus, the price ratio converges if:

$$\frac{\lambda_H(\phi_{HH}\psi + \phi_{HL}\rho_{HL})}{\lambda_L(\phi_{LL} + \phi_{LH}\rho_{LH}\psi)} > \frac{\lambda_L\phi_{LL}\psi - \lambda_H\phi_{HL}\rho_{HL}}{\lambda_H\phi_{HH} - \lambda_L\phi_{LH}\rho_{LH}\psi}. \tag{42}$$

Following similar steps as for the case where $0 \leqslant \psi < \psi^*$, it can be demonstrated that inequality (42) is satisfied whenever $g(\psi) > 0$, which holds if $\psi^* < \psi < \infty$.

These arguments show that the bond price ratios ψ converge to the steady-state value $\psi*$ if either $\phi_{HH}>0$ or $\phi_{LL}>0$. In Case 2.1, where $\lambda_H\rho_H>\lambda_L\rho_L$, $\psi^0=1<\psi^m<\psi^{m+2}<\psi^*$, where m is even and $\psi^m>\psi^{m+2}>\psi^*>1$, where m is odd. Thus, $\psi^m>1$ for all $m>0$. In Case 2.2, where $\lambda_H\rho_H<\lambda_L\rho_L$, $\psi^0=1>\psi^m>\psi^{m+2}>\psi^*$, where m is even and $\psi^m<\psi^{m+2}<\psi^*<1$, where m is odd. Thus, $\psi^m<1$ for all $m>0$. This proves case (ii) of the proposition.

Note that the condition $\lambda_H\rho_H \gtreqless \lambda_L\rho_L$ in (ii) simplifies to $\rho_H \gtreqless \rho_L$ if $\mu_L=\mu_H$ and $\sigma_L=\sigma_H$. As shown in the proof to Proposition 1, $\rho_H \gtreqless \rho_L$ if $\alpha \lesseqgtr \tilde{\alpha}$. □

B.4. Proof of Proposition 3

This proposition assumes that tax regimes are persistent ($\phi_{HH}+\phi_{LL}>1$). I will briefly characterize the term structure if tax regimes are transitory at the end of this section. The expected bond return in the high-tax regime is

$$\mathrm{E}(r_H^{B,m})=\frac{\phi_{HH}\psi^{m-1}+\phi_{HL}}{\lambda_H(\phi_{HH}\psi^{m-1}+\phi_{HL}\rho_{HL})}. \tag{43}$$

The expected bond return is a function of the bond price ratio: $R_H(\psi^m)$, where $\psi^m=p_H^{B,m}/p_L^{B,m}$. The first derivative with respect to ψ is

$$\frac{\mathrm{d}R_H}{\mathrm{d}\psi}=\frac{\phi_{HH}\phi_{HL}\lambda_H(\rho_{HL}-1)}{[\lambda_H(\phi_{HH}\psi+\phi_{HL}\rho_{HL})]^2}. \tag{44}$$

$\mathrm{d}R_H/\mathrm{d}\psi \gtreqless 0$ if $\rho_{HL}\gtreqless 1$. I demonstrated in Section B.3 that $\psi^m \gtreqless \psi^{m-1} \gtreqless 1$ if $\lambda_H\rho_H-\lambda_L\rho_L \gtreqless 0$ as long as tax regimes are persistent.

Next I discuss the term structure in four cases: First, if $\lambda_H\rho_H>\lambda_L\rho_L$ and $\rho_{HL}>1$, then ψ^m increases with m and the expected bond return $R_H(\psi^m)$ increases monotonically with the maturity m. Second, if $\lambda_H\rho_H>\lambda_L\rho_L$ and $\rho_{HL}<1$, then ψ^m increases with m and the expected bond return $R_H(\psi^m)$ decreases monotonically with the maturity m. Third, if $\lambda_H\rho_H<\lambda_L\rho_L$ and $\rho_{HL}>1$, then ψ^m decreases with m and the expected bond return $R_H(\psi^m)$ decreases monotonically with the maturity m. Fourth, if $\lambda_H\rho_H<\lambda_L\rho_L$ and $\rho_{HL}<1$, then ψ^m decreases with m and the expected bond return $R_H(\psi^m)$ increases monotonically with the maturity m. Thus, the bonds with longer maturity have a higher expected return in the high-tax state if $(\lambda_H\rho_H-\lambda_L\rho_L)(\rho_{HL}-1)>0$. It can also be shown that bonds with longer maturity have a higher expected return in the low-tax state if $(\lambda_L\rho_L-\lambda_H\rho_H)(\rho_{LH}-1)$. Note that this condition is identical to $(\lambda_H\rho_H-\lambda_L\rho_L)(\rho_H-1)>0$. This proves (ii) of the Proposition.

In (i), it is assumed that $\lambda_H=\lambda_L$. The condition from (ii) is equivalent to $(\rho_H-\rho_L)(\rho_{HL}-1)>0$. Note that $\rho_H=\phi_{HH}+(1-\phi_{HH})\rho_{HL}$. Thus, ρ_H is between 1 and ρ_{HL} and ρ_L is between 1 and ρ_{LH}. The sign of $(\rho_H-\rho_L)$ is therefore identical to the sign of $(\rho_{HL}-1)$. This implies that $(\lambda_H\rho_H-\lambda_L\rho_L)(\rho_{HL}-1)$ is always positive. This concludes the proof. □

Next, I characterize the expected returns of bonds with maturity m if tax regimes are transitory. $\mathrm{d}R_H/\mathrm{d}\psi \gtreqless 0$ holds again if $\rho_{HL}\gtreqless 1$. With transitory regimes $\psi^m \gtreqless \psi^{m-2} \gtreqless 0$

and $\psi^m \gtreqless \psi^* \gtreqless \psi^{m-1} \gtreqless 0$ if $\lambda_H \rho_H - \lambda_L \rho_L \gtreqless 0$. Thus, the bond price ratio fluctuates around its steady-state value.

The term structure has the following shape in the different cases: First, if $\lambda_H \rho_H > \lambda_L \rho_L$, then ψ^m increases with m if m is odd and decreases if even. Second, if $\lambda_H \rho_H < \lambda_L \rho_L$, then ψ^m decreases with m if m is odd and increases if even. The expected bond return $R_H(\psi^m)$ oscillates around its steady state and converges towards the steady state if either $\phi_{HH} > 0$ or $\phi_{LL} > 0$. If $\phi_{HH} = \phi_{LL} = 0$, then the bond price ratio ψ and the bond return follow a stable cycle.

B.5. Proof of Proposition 4

The following proof holds in the high-tax state. The proof for the low-tax state is similar. The return of equity in the high-tax state equals

$$\begin{aligned} \mathrm{E}(r_H^S) &= \xi_H \left(\phi_{HH} \frac{1+\delta_H}{\delta_H} + \phi_{HL} \frac{1+\delta_L}{\delta_H} \right), \\ &= \xi_H \frac{1 - \phi_{HH}\gamma_H - \phi_{LL}\gamma_L + \phi_{HH}\gamma_H\rho_H + \phi_{HL}\gamma_L\rho_L}{\gamma_H[\rho_H + \gamma_L(1 - \phi_{HH} - \phi_{LL})]}. \end{aligned} \tag{45}$$

Plugging the return of the risk-free asset with a maturity of 1 year during the high-tax-state into Eq. (45) and simplifying gives the following equation:

$$\mathrm{E}_t(r_H^S) = r_H^{B,1} \frac{\lambda_H \xi_H}{\gamma_H} (1 + \upsilon_H), \tag{46}$$

where:

$$\upsilon_H = \gamma_H \frac{\phi_{HH}(1-\rho_H)\left(\frac{\gamma_L}{\gamma_H}\rho_L - \rho_H\right)}{\phi_{HH}(1-\gamma_L) + \phi_{HL}\rho_{HL} + \gamma_L \phi_{LH}}.$$

The one-period interest rate $r_H^{B,1} = 1/(\lambda_H \rho_H) > 0$ is defined as the gross return and is therefore always strictly positive. The second factor ($\lambda_H \xi_H / \gamma_H = \exp(\alpha \sigma_H^2) \geqslant 1$, because $\alpha \geqslant 0$) results from the uncertainty of dividend payments. The third factor $1 + \upsilon_H$ results from tax rate changes. This factor equals 1 in an environment without tax rate changes (i.e., $\phi_{HH} = \phi_{LL} = 1$ or $\tau_H = \tau_L$), because $\rho_H = \rho_L = 1$.

The premium due to dividend uncertainty π_H^D is positive

$$\pi_H^D = r_H^{B,1} \frac{\lambda_H \xi_H}{\gamma_H} - r_H^{B,1} = \frac{\exp(\alpha\sigma_H^2) - 1}{\lambda_H \rho_H} \geqslant 0.$$

The premium due to tax uncertainty π_H^T is

$$\pi_H^T = r_H^{B,1} \frac{\lambda_H \xi_H}{\gamma_H} \upsilon_H = \frac{\exp(\alpha\sigma_H^2)\upsilon_H}{\lambda_H \rho_H}.$$

The sign of π_H^T is identical to the sign of υ_H. The factor υ_H is negative if $(\rho_L\gamma_L - \rho_H\gamma_H)(1-\rho_H) < 0$. The tax premium π_i^T is negative if $\rho_H\gamma_H < \rho_L\gamma_L$ whenever $\alpha \leqslant \tilde{\alpha}$ and if $\rho_H\gamma_H > \rho_L\gamma_L$ whenever $\alpha > \tilde{\alpha}$, because $\alpha \leqslant \tilde{\alpha}$ implies that $\rho_H \geqslant 1$ and $\alpha > \tilde{\alpha}$ implies that $\rho_H < 1$.

Note that the condition $(\rho_L\gamma_L - \rho_H\gamma_H)(1-\rho_H) < 0$ is equivalent to the condition $(\rho_H\gamma_H - \rho_L\gamma_L)(1-\rho_L) < 0$, because if $\rho_H \lesseqgtr 1$ then $\rho_L \gtreqless 1$.

If $\mu_L = \mu_H$ and $\sigma_L = \sigma_H$, then $\gamma_L = \gamma_H$ and υ_H simplifies to

$$\upsilon_H = \gamma_H \frac{\phi_{HH}(1-\rho_H)(\rho_L-\rho_H)}{\phi_{HH}(1-\gamma_L) + \phi_{HL}\rho_{HL} + \gamma_L\phi_{LH}}.$$

This term is always positive since the sign of $(1-\rho_H)$ is identical to the sign of $(\rho_L - \rho_H)$, because 1 lies between ρ_L and ρ_H. $\square$

References

Abel, A.B., 1990. Asset prices under habit formation and catching up with the Joneses. American Economic Review 80 (2), 38–42.

Abel, A.B., 1999. Risk premia and term premia in general equilibrium. Journal of Monetary Economics 43 (1), 3–33.

Alvarez, F., Jermann, U.J., 2000. Efficiency, equilibrium, and asset pricing with risk of default. Econometrica 68 (4), 775–797.

Alvarez, F., Jermann, U.J., 2001. Quantitative asset pricing implications of endogenous solvency constraints. Review of Financial Studies 14 (4), 117–1151.

Auerbach, A.J., Hines, J.R., 1988. Investment tax incentives and frequent tax reforms. American Economic Review 78 (2), 211–216.

Bizer, D.S., Judd, K.L., 1989. Taxation and uncertainty. American Economic Review 79 (2), 331–336.

Campbell, J.Y., 1999. Asset prices, consumption, and the business cycle. In: Taylor, J., Woodford, M. (Eds.), Handbook of Macroeconomics, vol. 1. North-Holland, Amsterdam, pp. 1231–1303 (Chapter 19).

Campbell, J.Y., Cochrane, J.H., 1999. By force of habit: a consumption-based explanation of aggregate stock market behavior. Journal of Political Economy 107 (2), 205–251.

Constantinides, G.M., 1990. Habit formation: a resolution of the equity premium puzzle. Journal of Political Economy 98 (3), 519–543.

Constantinides, G.M., Duffie, D., 1996. Asset pricing with heterogeneous consumers. Journal of Political Economy 104 (2), 219–240.

Constantinides, G.M., Donaldson, J.B., Mehra, R., 2002. Junior can't borrow: a new perspective on the equity premium puzzle. Quarterly Journal of Economics 117 (1), 269–296.

Domar, E.D., Musgrave, R.A., 1944. Proportional income taxation and risk-taking. Quarterly Journal of Economics 58 (3), 388–422.

Dumas, B., 1989. Two-person dynamic equilibrium in the capital market. Review of Financial Studies 2 (2), 157–188.

Eaton, J., 1981. Fiscal policy, inflation and the accumulation of risky capital. Review of Economic Studies 48 (3), 435–445.

Epstein, L.G., Zin, S.E., 1989. Substitution, risk aversion, and the temporal behavior of consumption growth and asset returns, I: a theoretical framework. Econometrica 57 (4), 937–969.

Fama, E.E., French, K.F., 2002. The equity premium. Journal of Finance 57 (2), 637–659.

Gordon, R.H., 1985. Taxation of corporate capital income: tax revenue versus tax distortions. Quarterly Journal of Economics 100 (1), 1–27.

Hamilton, J.H., 1987. Taxation, savings, and portfolio choice in a continuous time model. Public Finance/ Finances Publiques 42 (2), 264–281.

Hansen, L.P., Sargent, T.J., Tallarini, T.D., 1999. Robust permanent income and pricing. Review of Economic Studies 66 (4), 873–907.

Hassett, K.A., Metcalf, G.E., 1999. Investment with uncertain tax policy: does random tax policy discourage investment? Economic Journal 109 (457), 372–393.

Heaton, J., Lucas, D., 1996. Evaluating the effects of incomplete markets on risk sharing and asset pricing. Journal of Political Economy 104 (3), 668–712.

Internal Revenue Service (Ed.), 1954. Statistics of Income. U.S. Treasury Department. Washington DC.

Jermann, U.J., 1998. Asset pricing in production economies. Journal of Monetary Economics 41 (2), 257–275.

Jorion, P., Goetzmann, W.N., 1999. Global stock markets in the twentieth century. Journal of Finance 54 (3), 953–980.

Judd, K.L., 1985. Redistributive taxation in a simple perfect foresight model. Journal of Public Economics 28 (1), 59–83.

Kaplow, L., 1994. Taxation and risk taking: a general equilibrium perspective. National Tax Journal 47 (4), 789–798.

Kocherlakota, N.R., 1996. The equity premium: it's still a puzzle. Journal of Economic Literature 34 (1), 42–71.

Lucas, R.E., 1978. Asset prices in an exchange economy. Econometrica 46 (6), 1429–1445.

Mankiw, G.N., 1986. The equity premium and the concentration of aggregate shocks. Journal of Financial Economics 17 (1), 211–219.

Mankiw, G.N., Zeldes, S.P., 1991. The consumption of stockholders and non-stockholders. Journal of Financial Economics 29 (1), 97–112.

McGrattan, E.R., Prescott, E.C., 2003. Average debt and equity returns: puzzling? American Economic Review 93 (2), 392–397.

McGrattan, E.R., Prescott, E.C., 2004. Taxes, regulations, and the value of U.S. and U.K. corporations. Federal Reserve Bank of Minneapolis Research Department Staff Report 309.

Mehra, R., Prescott, E.C., 1985. The equity premium: a puzzle. Journal of Monetary Economics 15 (2), 145–161.

Mehra, R., Prescott, E.C., 2003. The equity premium in retrospect. In: Constantinides, G.M., Harris, M., Stulz, R.M. (Eds.), Handbook of The Economics of Finance, vol. 1B. North-Holland, Amsterdam, pp. 889–938.

Mitchell, B., 1983. International Historical Statistics: the Americas and Australasia. Gale Research, Michigan.

Pechman, J.A., 1987. Federal Tax Policy, fifth ed. Brookings, Washington DC.

Poterba, J.M., 2000. Stock market wealth and consumption. Journal of Economic Perspectives 14 (2), 99–118.

Prescott, E.C., 2004. Why do Americans work so much more than Europeans? Federal Reserve Bank of Minneapolis Quarterly Review 28 (1), 2–13.

Rietz, T.A., 1988. The equity risk premium: a solution. Journal of Monetary Economics 22 (1), 117–131.

Shiller, R.J., 1981. Do stock prices move too much to be justified by subsequent changes in dividends? American Economic Review 71 (3), 421–436.

Sialm, C., 2004. Tax changes and asset returns: an empirical investigation. University of Michigan, Mimeo.

Stiglitz, J.E., 1969. The effects of income, wealth, and capital gains taxation on risk-taking. Quarterly Journal of Economics 83 (2), 263–283.

Stiglitz, J.E., 1982. Utilitarianism and horizontal equity: the case for random taxation. Journal of Public Economics 18 (1), 1–33.

Storesletten, K., Telmer, C.I., Yaron, A., 2001. Asset pricing with idiosyncratic risk and overlapping generations. Carnegie Mellon, Mimeo.

U.S. Government Printing Office (Ed.), 2000. Economic Report of the President. Washington DC.

Vissing-Jørgensen, A., 2002. Limited stock market participation and the elasticity of intertemporal substitution. Journal of Political Economy 110 (4), 825–853.

Weil, P., 1989. The equity premium puzzle and the risk-free rate puzzle. Journal of Monetary Economics 24 (2), 401–421.